MARKUS KLUTE: Welcome back to special relativity, 8.20. After discussing energy and momentum and examples with collisions, we now want to talk about forces. And we get back to the example of Alice traveling to the center of the galaxy and asking what does it mean in terms of acceleration.

So we start from Newton's second law. We know that a force is a change in momentum. We can write this down as d dt m0 times u over the square root of 1 minus u squared over c squared, or just with a gamma factor.

And the thing to consider here is that now the gamma factor and the velocity are actually time dependent. So there's two components to this. We'll come back to this.

The kinetic energy is the work done by an external force. And you can get to the kinetic energy by just integrating, let's say, for a particle which is accelerated by an external force from the velocity 0 to some velocity v. That's the integral over the path of the particle, over the path of this object, times the force. If you just assume here uniform motion in x-direction, then this simplifies to just an f dx.

So, as the first activity, I want you to find the kinetic energy of an object with velocity v and the mass and mass m0. And, as the second part, I want you to test this result for velocities much, much smaller than the speed of light where you're used to doing this kind of calculation, and you're familiar with the outcome.

So we just have to integrate, just have to integrate. So this is a little bit involved here. So we have to integrate from 0 to v m0-- that's our constant; we can take that out-- d dt of this u times gamma dx. OK, so you find there's two components here. And then we do a trick where we introduce this du dx dx. And then the integral becomes an m times u du over 1 minus u squared over c squared to the third, to the third power half.

OK, and then you can just look up the integral or work it out. It's not that difficult actually, but you find that this is equal to m0 times c squared over square root 1 minus u squared over c squared, which you have to evaluate for velocities v and 0. And, when you do that, you find those two components here.

OK, the first one is m0 c squared times gamma. And the second one is m0 c squared. So that result is actually not too surprising, as we saw that we can write the energy equal to m0 c squared plus k.

And what we just calculated here from this example is k is equal to energy minus m0 c squared, OK? So that result already makes sense with respect to the discussion we had to this point. Or you can simplify this by saying, the kinetic energy is gamma minus 1 times m0 c squared, OK?

So, if we now evaluate this for small values of v, as we did before, we find that the kinetic energy is 1/2 m0 v squared. And I find it interesting, illustrative, to plot what this means now. So, if we plot the kinetic energy of a particle as a function of its velocity, we find that, for small values, those two curves basically overlap.

For small values, m0 c squared times gamma minus 1 is basically the same as 1/2 mv squared, which we just derived here from the Taylor expansion. But, for larger values, this diverges and especially when you get closer to the speed of light.

Just to get a quantitative example, I asked you to do another calculation here. I want you to reinvestigate Alice's journey to the center of the galaxy where she has a spacecraft, which moves with a gamma factor of 15,000, an acceleration of 10 meters per second squared, and the mass of the spacecraft, let's say, is 10,000 metric tons or 10,000-- sorry, 100 metric tons or 100,000 kilograms. So compare the kinetic energy using Newtonian mechanics or special relativity.

And you find that the difference is astonishingly large. So, if you just work this out, 1/2 mc squared-- we can just use c squared here because the velocity is basically c-- 5 times 10 to the 22 kilograms meter squared over seconds squared. And, in relativistic terms, the answer is 30,000 times larger, so 30,000 times larger than the classical case. So there's a huge difference between the classical evaluation and the evaluation with special relativity.

One more word on F equal ma, the question is, how does this transform under Lorentz transformation? It's something we already halfway figured out. So here you basically want to see how a transforms under Lorentz transformation.

We have started the discussion by saying, you know, in Galilean transformation, the acceleration is invariant, while, in Lorentz transformation, that's not the case. But, if you investigate again the second law of physics, the force as a change in momentum, you find that you get those two components here.

One is parallel to the acceleration, so m times gamma a. But the second one is not. The second one is m0 times u times the change in time of the gamma factor. And that's not parallel to F or to a.

And so you find that there's two-- the new vector or the new force of a particle is not parallel to the acceleration anymore. That's kind of counterintuitive.