MARKUS KLUTE: Welcome back to 8.20, Special Relativity. In this section, we're going to talk about the relativistic Doppler effect. And we make good use of our space-time diagrams, which we discussed earlier.

So the situation is as follows-- to simplify this, we have a source which is emitting pulses. So the waves are pulses. Every now and then there is a beep, and another beep, and another beep. And those pulses travel with their velocity-- with their wave velocity.

And they have a world line represented here in the space-time diagram. This is pulse number one, and this is pulse number two. The distance between those two pulses is our period, the period of our wave, which we call tau. The question now is, how is this being observed by an observer which is moving with a relative velocity v with respect to the source?

So let's analyze this. So if we want to characterize or find our position x1 and x2, we can do this by saying x1 is equal to ct1 or equal to x0, which is the distance of the observer to the source plus c times t1. v is the velocity in which the source is moving.

And similarly for t x2, we find c times t2 minus tau. And that's also equal to x0 plus v times t2. So the distance in time-- we're still in the reference frame as of the source-- is given by c times tau over c minus v. And the distance in space is given by v times c times tau over c minus v.

So the question is not how this observed-- how this is seen by the source but how this is being seen by the observer. So we have to apply Lorentz transformation. So in the s prime frame, which is the observer frame, we find delta t prime is equal to gamma delta t minus v over c squared delta x. And then we just fill in the information as we discussed before.

Tau prime is then gamma times c tau over c minus v times 1 minus v square over c square. And then you make use of delta equal v of over c. And we make use of gamma equals 1 over square root of 1 minus beta square.

And we find then-- this is a little bit of an algebra exercise here-- that the period now is given by 1 plus beta over 1 minus beta square root of that times tau. And the frequency is the inverse. We'll have 1 minus beta over 1 plus beta square root of that [? times ?] the frequency. So we just calculated relativistically how the period and the frequency of a wave is Lorentz transformed.