MARKUS KLUTE: Welcome back to 8.20 Special Relativity. In this short section, we want to introduce a new notation, four-vectors. And if you look at previous discussions, this is actually not that new. We have seen that we need to treat time and space in a consistent manner. And you have often applied Lorentz's transformation, for example, to a vector of time and the next component of space. Now you just want to do this with x, y, and z here and not treat the y component and z component as 0.

So as a starting point, you can just simply say, OK, we have this new four-vector. And the 0's component is the time or time times the speed of light. And then the first component, second and third component are the spatial component, x, y, and z.

Now I wrote a vector Xi mew here, with the mew being the upper index. I can also introduce Xi with a lower index. And you see little y and little y is useful. Where the 0's component is not t but minus ct-- but minus ct.

As a reminder for three-vectors, you learned about the dot product, which is just a multiplication of two, three-vectors where all vectors with n components, where you multiply the same component of each vector and add those results together. So the dot product of vector a and vector b is the sum of all indices for ai and bi.

Now for our four-vector, we do the very same thing. We just sum over all four components. And we treat the vectors as a product of the vector with the lower index and the upper index. And you find here then we get minus c squared t squared plus x squared, y squared, and z squared.

More generally, this is for two vectors of the same-- two of the same vectors. More generally for two different vectors, you can write in this way. Or in short, you can define a new notation in which you basically sum over all indices which are equal. So here we have an upper and lower indices together. So you sum over this case here where there's the same index, mew, for both vectors. And one is lower and one is upper.

And we can continue the introduction and just introduce a few tools to work with those vectors. For example, if you wanted to bring the component mew from the bottom to the top, you can do this with multiplying the vector with a matrix. And the matrix here is also called a metric.

And simply what you have to do is multiply the first component with the minus 1 and the rest with 1. You see this here on the diagonal and on other components later on. What this does-- you can check this if you want-- is bringing the index of the vector from a lower to an upper one.

An interesting example is the product of a four-vector with itself. And we have already seen this because we saw this as our invariant interval. Here, the four-vector is the distance in space and time between two events.

So we looked at delta Xi mew times delta Xi mew. And delta Xi mew is the difference between event A and B. And so we have seen this already and calculated the invariant and showed that this squared over a distance of two events is actually invariant in the Lorentz transformation.

But there's other examples for vectors. The first one we'll investigate some more in the next sections to come. It's the energy momentum four-vector, where we place in the first component the energy-- in the 0's component the energy, and then the first, second, and third components the three-vector of the momentum.

But there's others, for example, the four-potential, where in the 0's component, you have the potential-- the electric potential. And then the first, second, and third component, you have this new field A, which is related to the magnetic and electric field. So E and M is not part of this course, but we'll come back to this in the last week and discuss the consequences and ideas a little bit more.

But if you then look at the invariant four-vector, which is a product of the energy momentum vector, you find that the first component, the energy square or minus the energy square over c square plus the three-component vector of the momentum squared. And that's constant, we can just here name this mass or minus mass square times c square.

So if you write this, you find this energy momentum mass relation E squared is equal to p squared c squared, plus m squared c to the fourth power. And if you look at this four particles of 0 momentum, in which case this component here is 0, you find the equation E is equal to mc square.