MARKUS KLUTE: Welcome back to Special Relativity, 8.20. In this section, we're going to talk about proper velocity. We have seen already concepts of proper time and proper length as the time and the space as seen in the object's own reference frame. So now we want to try to find something similar for velocities, as we have seen the Lorentz transformation applied to velocities of a quite difficult form.

As a reminder, velocity is given as a change in space of a change in time. We have seen the Lorentz transformation of a velocity x in a reference frame which is boosted in the same direction x. And you see that this new velocity x prime is given by new x minus v. So there's a velocity addition going on, which is corrected then by this factor 1 minus uxv over c squared.

You've also seen that even so, the boost is in x direction. There's also modification of the velocity in y direction and in z direction. OK? So you see that basically, there is this-- the velocity itself is corrected with this new factor. Note here that there is a special case in which the direct-- the velocity in x direction is equal to 0.

Think about this object being in its own rest frame, again, where the velocity in the booth direction is 0. You see that both equations simplify for ux prime that would be simply equal to minus v, where uy prime would be uy prime and uv prime would be uv.

So let's try to get at it. Let's try to express velocities in terms of the proper time, at the time as it ticks in the object reference frame. So we have seen that the time is given by gamma times the time in the rest frame or time in the proper-- times gamma times the proper time.

Note here that we have two different gamma factors to play with. One is a gamma factor of the Lorentz transformation. And this gamma here is the gamma using the speed of the object in a specific reference frame. So this is the gamma which is a gamma factor which is a gamma of v of the velocity or the speed of the object.

And now we can just simply define proper velocity if I use this vector eta here, which is a four vector, which is the derivative of the spatial component with the proper time. And when we do this, you find this relatively simple solution of gamma times c for the 0's component, gamma times ux, gamma times uy, and gamma times uz for the last component.

So the question now is, we defined this new velocity of an object where the time of the object ticks in its own reference frame using this property as proper time. OK? If we now try a Lorentz transformation on this, we can simply apply the matrix for Lorentz transformation on this four vector and find the solution here.

You can see that this is consistent by doing this with the original components in this proper time as well. You see that those actually are a consistent answer. It makes a lot of sense.