[SQUEAKING]

[RUSTLING]

[CLICKING]

PROFESSOR: Welcome back to 8.20, special relativity. In this section, and also the next time, we talk about electromagnetism.

Electromagnetism is not part of the core of 8.20. We are not requiring electromagnetism as part of the prerequisite. I will not test or include electromagnetism in the final.

But nevertheless, it's interesting to discuss electromagnetic effects in the context of special relativity, as they led to the development of special relativity in the first. After all, the paper which describes the theory of special relativity is about the electrodynamics of moving bodies. And so let's have a look at the natural dynamics of moving bodies.

And so here, we have a source-- a charge which is moving with a velocity V. And moving charges, or currents, create magnetic fields. You can use your right-hand and see particles are moving in this direction here. The electric field lines are curling around you.

Good. So now I have a second charge-- a test charge which is moving with the velocity u. That test charge will experience the magnetic force. The force is equal to the test charge itself, qt, times u cross B, the velocity cross the magnetic field.

So there's clearly a magnetic force acting on this [INAUDIBLE] charge in this reference frame. However, if I now move into a reference frame where the source charge is stationary, a stationary charge is not creating magnetic field, so the magnetic field is 0. Hence the magnetic force on the test charge is 0.

I can, alternatively, look at the reference frame in which the test charge is stationary. And so also here, because u is equal to 0, the magnetic force is 0. So clearly, we do have to treat magnetic and electric fields in sort of a consistent manner as we treated time and space, and energy momentum in a consistent manner.

But we need a concept of electromagnetic fields. And previously in this class, we looked at electromagnetic field light before. The difference here is-- and that's the context of the next section-- is that you want to understand how we create those fields-- how we can create electric and magnetic fields, and how that all works together.

Now the first activity, I want you to consider a cube of length L with n electrons. There's n charges inside, and everything is at rest. And what I want you to figure out is what is the charge and current density [INAUDIBLE] and j0 of this cube?

There's a second step. As you can imagine, I ask you to consider the very same thing from a moving reference frame, S prime, with some velocity u which is the velocity of S prime with respect to S. What is the charge in the current density now in this new reference frame? I'd like you to figure this out.

So now we look at this cube, and the total number of charges is N. The length-- or the volume is l0 cubed. And so the density is N divided by l0 cubed.

The current 0. Nothing is moving. There's no moving charges [INAUDIBLE]. All right, this one is more straightforward. But now in our moving reference frame, the situation changes.

Here, one of the directions-- the direction in which we are moving-- is Lorentz contracted. So we have lx equal to lx 0 or l0, times-- divided by gamma, or times square root 1 minus u squared over c squared.

The volume then of the same cube in the S prime reference frame is l0 cubed divided by gamma. The number of electrons, however, is unchanged, so the charge density is the previous charge density divided by the volume.

And if you compare-- sorry, it's the charge divided by the new volume which is the density times gamma. For the new charge density, we simply have to multiply the current density. For the current density, we have to multiply the charge density times the velocity. And again, we find ro 0 times 0, times gamma.

Good. So if you look at those relations, they look very much like the relationship between the current and the charge density. They look very much like the relations we had between momentum and energy, and time and space. And Lorentz transformation looks very similar.

So we can, motivated by this, write a 4 vector, which is the first component c times the density. And as the first, second, and third component, the current.

And you find that the invariant here is very similar to time and space, energy and momentum given as the density where the charges are addressed. And that's the invariant, and you can calculate this from multiplying the 4 vector [INAUDIBLE] square [INAUDIBLE].

So concept questions. Is the electric charge conserved in the Lorentz transformation, or did we actually change the charge? So we have the charge density here and the current here, but did we actually change the charge on the Lorentz [INAUDIBLE]?

The answer is, no, we did not change them, so the charge is conserved. The charge is invariant in the Lorentz transformation. Is electric current conserved in Lorentz transformation, or invariant on the Lorentz transformation? Should rather use invariant [INAUDIBLE].

And the answer is, no, it's not. So we have seen from the very first example that you start with the current which is 0, and then in the S prime frame, the current is of a certain value. So certainly, the current was seen from two different observers is changed.