

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Physics 8.224 Exploring Black Holes
General Relativity and Astrophysics
Spring 2003

ASSIGNMENT: WEEKS 1 AND 2

For the ten days (week #1 and #2)

INTRODUCTION

There are really only two Big Ideas in our text *Exploring Black Holes* (EBH). In the week #2 we study the first Big Idea: In the vicinity of a time-independent structure such as the Earth or a star, neutron star, or black hole, a time-independent equation called a metric relates aging along a segment of worldline to particle displacement in space and time coordinates along that segment.

What does that mean? A free stone wearing a wristwatch moves from event A to nearby event B . Between these two events the wristwatch on the stone records a time lapse d , where d is called the *proper time* or *wristwatch time*. The wristwatch time measures the *aging* of the stone, or the aging of a puppy that rides on the stone. The metric tells us the amount of aging for a given change in coordinates and time between events A and B . For flat spacetime, the metric can be written:

$$\tau^2 = t^2 - s^2$$

where τ is the aging (proper time, wristwatch time), t is the increase in coordinate time and s is the distance between events A and B . Here space and time are measured in the same units, for example years and light-years or meters and meters of light-travel time (EBH page 1-2).

Bookkeeper coordinates. A key feature of general relativity is that you can use any coordinate system you want—it is entirely arbitrary. Typically you choose a coordinate system that simplifies your current project. But you are free to be as screwy as you can; Nature does not care. For example, in flat spacetime we can define

$$u = t - s \quad \text{and} \quad v = t + s$$

A little weird? Sure, but giving the u and v of an event fixes uniquely its location in space and time. In terms of the coordinates u and v , the metric for flat spacetime becomes:

$$\tau^2 = u \cdot v$$

Such an arbitrary set of coordinates we sometimes call **bookkeeper coordinates**.

In Chapter 2 of EBH we meet the famous **Schwarzschild metric**, which describes spacetime external to a spherically symmetric nonrotating center of gravitational attraction.

$$d\tau^2 = 1 - \frac{2M}{r} dt^2 - \frac{dr^2}{1 - \frac{2M}{r}} - r^2 d\phi^2$$

Features of this metric are detailed in Chapter 2 of EBH:

1. As in flat spacetime, time and distance are measured in the same units. In addition, the mass M of the center of attraction is measured in units of distance (EBH page 2-13).
2. For simplicity we assume that the two events occur in a plane through the center of attraction (EBH page 2-15).
3. We use differentials because events A and B have to be close enough together so that r has a unique value and because the Schwarzschild metric treats the arc rd as a straight segment, approximately true only if d is very small (EBH, box on page 2-18).
4. Coordinates r and t are *bookkeeper coordinates*, defined to make the metric look simple. The radius r is the circumference of a circle at that distance from the center divided by 2π (EBH page 2-7). The time t is the time read on clocks far from the center of attraction (EBH page 2-27).

So the metric is the first Big Idea in EBH. The second Big Idea is called the **Principle of Maximal Aging**, which uses the metric to determine how a free stone moves near a center of gravitational attraction. This is the subject of Chapter 3, to be assigned in the fourth week of the class.

1. RECITATION AND EVENING SEMINAR

Recitation is scheduled once a week.

Attendance and citations is required of students. Alums are welcome.

The first scheduled evening seminar is on the first day of week #2.

Speakers at the evening seminar are experts in fields covered in the class. The first speaker will be announced later. Attendance at seminars is required for students.

2. TEXTS

The two texts for this class are:

1. *Exploring Black Holes: Introduction to General Relativity* by Edwin F. Taylor and John Archibald Wheeler, Addison Wesley Longman, 2000, ISBN 0-201-38423-X. Called "EBH" in all assignments. NOTE: You should have the fifth printing; the first number in the very bottom line of the Acknowledgments page (behind the title page) should be a 5.

2. *Black Holes and Time Warps: Einstein's Outrageous Legacy* by Kip Thorne, W.W. Norton, 1994, ISBN 0393312763. Called "Thorne" in all assignments.

3. Homework at the end of week #1

READ Thorne Prologue: A Voyage among the Holes

BEGIN research on one possible end-of-term project (details below)

4. Homework at the end of week #2

READ EBH Chapter 1 Speeding

AND EBH Chapter 2 Curving through page 2-24.

READ Thorne Chapter 1 The Relativity of Space and Time

READ HANDOUT "Coordinates and Proper Time" by Edmund Bertschinger, a pdf file available on the Assignments page of Stellar.

PROJECT SUMMARY: By end of week #2 submit by email a summary of one possible end-of-term project (details below).

5. PREVIEWING END-OF-TERM PROJECTS

The end-of-term project will occupy approximately the last third of the semester. Each project will be carried out by a small group consisting of both MIT students and alums. The deliverables will be (a) a public web page, and (b) a half-hour presentation to the class. The audience will be the class supplemented with experts in the field. Leading up to the class presentations will be a series of deadlines for preliminary outlines and so forth.

In this first assignment you are asked to use the library, books, journals, and the world wide web to research one possible end-of-term project that interests you and to prepare one or two pages summarizing the physics behind the project. We prefer that the report not contain equations, but you may include brief references and links to interesting websites. Of course

your summary will not include technical details of general relativity that you will be learning during the coming semester. Instead it should be a qualitative introduction to the topic, understandable by others who are beginning this class. Some details:

- A. Submit your summary as an attachment to an email to the instructor in any standard electronic form: Word, a pdf file, LaTeX, and so forth. Or you may send him a link to a web page of your own design. Your name should be on your summary.
- B. Your completed summary (or link) will be posted as a contribution to a special discussion forum called Project Previews. The discussion forums can be read by class members and staff but not by anyone else. (In contrast, your web page for the end-of-term final project will be posted on the public portion of the class website.)
- C. You will receive credit for submitting your summary, but the summary itself will not be graded.
- D. Doubtless some topics will be summarized by more than one person. That is OK.
- E. You will NOT be obligated to work on the end-of-term project that you summarize in this homework. Later you will be invited to express a preference for the project you want to work on at the end of term.

6. POSSIBLE END-OF-TERM PROJECTS

Here is an annotated list of possible end-of-term projects. Some of them contain references to EBH and Thorne, but those treatments can be only starting points for your summary and for the final project.

You are encouraged to think of an alternative topic for a project and summarize the physics of that topic instead of one on the list below. There are no limits to these alternative topics in this assignment, but any topic not on the list below will have to be approved by instructor before you work on it as an end-of-semester project. Any alternative project, indeed any project on the following list, will ultimately need to attract enough class members to form a team.

The Search for Black Holes (Thorne Chapter 8, which is by now out of date)

What evidence do astronomers have that black holes exist? How did they form? What effect do they have on their environment? How common are they? If they're black, how can we see them? Half of the 2002 Nobel Prize in Physics was awarded for work that is helping to answer these questions.

The Spinning Black Hole (EBH Project F, Thorne Chapter 7)

How does the spin of a black hole affect the spacetime around it? How does it affect orbits of photons or massive particles? Can the spin energy be tapped? Can spinning black holes be used for travel to other universes? The mathematics of spinning black holes is wicked hard but they are wicked good fun!

Cosmological Models (EBH Project G)

How does the composition of the universe affect its overall evolution and its spatial geometry? Will the universe recollapse or will it expand forever? How can the cosmic microwave background radiation help answer these questions? MIT Professor Alan Guth, who has agreed to be one of our seminar speakers, invented the leading theory of the early universe!

Global Positioning System (EBH Project A)

How does the satellite navigation system depend on understanding general relativity? How are GPS signals encoded to allow precise triangulation? How accurate can GPS be made? How much of our technology relies on GPS? Could GPS ever be used to measure effects besides the gravitational redshift and Doppler effect?

Inside the Horizon of the Black Hole (EBH Project B, Thorne Chapter 13)

If you fall into a black hole and shine a flash light outwards, which reaches the singularity first—you or the light? Since the Schwarzschild bookkeeper coordinates go crazy inside the horizon, are there coordinates that are well-behaved across the horizon? What is the singularity really like?

Advance of Perihelion of Mercury (EBH Project C, Thorne Chapter 2)

Kepler said that orbits around the sun are closed ellipses; Einstein proved him wrong. How much does the direction of perihelion change each orbit? How does this depend on the orbital eccentricity? What happens to the precession rate if the orbit around a black hole is made nearly as small as possible, i.e. a slight elongation of the innermost stable circular orbit?

Light Slowed Near Sun (EBH Project E,)

The Viking spacecraft not only sampled the soil of Mars; they also measured the speed of light in the solar system. What did they find? Why was this measurement called by its inventor “The Fourth Test of General Relativity”? How is this related to time delays in gravitational lenses?

Einstein Rings (EBH Project D, Thorne?)

Why did Einstein say that the image of a star could be a ring instead of a point? Why did he say, incorrectly, that the effect would never be seen? What are gravitational lenses and microlenses and what have they taught us about dark matter, galaxies, and the universe?

Gravitational Radiation (Thorne Chapter 7)

Newtonian gravity assumes action at a distance, in clear violation of the principle of relativity. How does general relativity fix this? Why and how do gravitational waves stretch space and what does that mean? How are they produced and how are LIGO and other instruments preparing to detect them?

Taylor-Hulse Binary

Why did Taylor and Hulse win the Nobel Prize? How did they measure the masses of two neutron stars to better than 1% accuracy? Why is the orbit of their two stars shrinking, how fast is it shrinking, and what will happen when it shrinks to the size of Boston? How did the stars get in this predicament?

Gamma Ray Bursters

How can an exploding star, for a few seconds, appear to emit nearly as much energy as all the other stars in the universe combined? What do we know about these bizarre explosions, and how is MIT's HETE satellite helping us to unravel their nature? What do gamma ray bursters have to do with black holes?

Supermassive Black Holes

What do we know about the masses of real black holes and how do we know it? What are quasars and active galactic nuclei? Does our galaxy have a central supermassive black hole, and are we in danger of falling in? What's the largest black hole known in the Universe and how did it form?

Hawking Radiation (Thorne Chapter 12, EBH boxes on pages 2-4 and 5-27)

What are the laws of black hole thermodynamics? How do they suggest that black holes may not be black, and that by emitting radiation an isolated black hole could evaporate? What is the temperature of the blackbody radiation emitted by a black hole? How long do black holes live before they evaporate?