

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics 8.224 Exploring Black Holes
General Relativity and Astrophysics
Spring 2003

ASSIGNMENT 6

INTRODUCTION

This week we study the motion of light in the fifth and final chapter of *Exploring Black Holes*. After these five chapters, you know how to use the metric and the Principle of Maximal Aging to answer all possible questions about (non-quantum) properties of a black hole and the motion of test particles and light in its vicinity. Next a new set of metrics will inform your investigation of spinning black holes and a powerful model of the Universe. Finally, the projects will empower you and your team to apply accumulated insight and skill to an important topic of your choosing.

The key idea in Chapter 5 is the single effective potential for every light flash, no matter what its energy—one size fits all! Moreover, there is no minimum in this effective potential, so there are no stable orbits for light flashes. There is, however, one unstable orbit, at $r = 3M$. Light can circulate on a knife-edge orbit at this radius, on what is called the *photon sphere*. You will be able to visualize these orbits using the JAVA GRorbits software provided by Slavomir Tuleja of the Slovak Republic. Many thanks to him!

Thorne's Chapter 6 takes us into a long aside about the development of the hydrogen bomb by the Soviets and the Americans. Work on nuclear weapons, both in the Soviet Union and the United States, brought insight into the behavior of materials at extremely high pressures and informed the developing theory of gravitational collapse. Wheeler was finally convinced that gravitational collapse takes place under the right conditions. There remained (and remains to this day) perplexity about what happens at the crunch point, where classical general relativity cannot apply

since the curvature increases without limit. We need quantum gravity. Wheeler speculated that at the center matter might turn into radiation which could tunnel out through the horizon. (Hawking radiation is a faint echo of such tunneling.) In spite of his continuing reservations, Wheeler coined the name *black hole*, which epitomizes the properties of this strange structure and has fueled its popularity.

READINGS

Exploring Black Holes: Chapter 5, Seeing
AND
Thorne, Chapter 6: Implosion to what

PROJECTS

Project teams have been assigned, and each project has a forum on the discussion board.. Go to the discussion board and see which project you are on. This week is the time to organize. You may add topics to your forum and use them as a way to communicate if you wish (instead of or in addition to email, instant message, postal mail, or telephone). Remember that instructors and other class members can read discussion contributions but visitors to the website cannot.

By end of week #6

1. Select a team leader and post his or her name in your project forum on the discussion board.
2. Divide the project into subtopics and assign a team member or members to each subtopic, posting the results on your project forum on the discussion board.

RECITATION

At least one of the following questions will appear on the recitation quiz.

1. Starting from the Schwarzschild metric, derive expressions for bookkeeper speed of light moving (a) radially, and (b) tangentially.
2. For a light flash approaching a black hole but still at a great distance, the impact parameter b can be defined as
3. Compare the (incorrect) Newtonian picture of the trajectory a light flash emitted from the surface of a star with radius inside its critical radius $r = 2M$ (which can be stable according to Newton) to the general relativistic prediction about a light flash emitted in the radial outward direction from a plunger falling inside the horizon.
4. For a long time the so-called "Schwarzschild singularity" at $r = 2M$ was thought to represent a discontinuity in the laws of physics. In 25 words or less, outline the new ideas that led to the current understanding of the horizon.

PROBLEM SET

Playing around with the GRorbits program with the setting $m = 0$ under the Orbits menu can help you develop an intuition for orbits and trajectories of light around a center of gravitational attraction.

1. ORBITS SOFTWARE: RIGHT ANGLE TURN FOR LIGHT

Set the Zoom on X10, the initial radius at $r = 200M$. Find a value for $1/b$ such that the light makes a 90 degree turn as it passes the black hole.

2. ORBITS SOFTWARE: EINSTEIN RING

Set the Zoom on X10, the initial radius at $r = 192M$ (one of the graph circles). A light source is located at this point. Find the value for $1/b$ such that an observer at the opposite side of the black hole at $r = 192M$ will observe an Einstein ring.

3. ENERGY PRODUCTION BY A QUASAR?

This is a modification of Exercise 8, page 5-35 of Chapter 5. Please READ that exercise. But NOTE that the questions below are different from those in the text. And the r -value of the circular orbit of one of the colliding stones is given a numerical value in the present exercise.

For you, the radius of the orbiting stone is $r = 6M$, the smallest radius for a stable orbit.

Use special relativity and the results for circular orbits in the exercises of Chapter 4 to analyze the circular orbit

- A** As measured by the shell observer at $r = 6M$, what is the kinetic energy of the stone in its stable circular orbit at this same radius? Measure this kinetic energy as a fraction of the mass m of the stone. *Two warnings:* (1) Kinetic energy is NOT $(1/2)mv^2$ in special relativity. (2) Kinetic energy is only part of total energy in special relativity.
- B** As measured by the shell observer at $r = 6M$, what is the kinetic energy of the stone that falls from rest at a great height? Measure this kinetic energy as a fraction of the mass m of the stone.
- C** Assume that the TOTAL kinetic energy, the sum of the quantities calculated in parts (A) and (B) is converted to a light flash with 100 percent efficiency and this light flash sent outward to a power station remote from the black hole. (Do not worry about conservation of momentum or other such details.) What is the energy of this flash as it arrives at the remote power station? Express this energy as a fraction (or multiple) of the mass m of either original stone.
- D** What fraction of the total rest energy $m + m = 2m$ has been turned into light energy at infinity? (Nuclear reactions on Earth convert less than one percent of the original mass into energy.)
- E** Suppose that each of the two colliding particles, each of mass m , is brought to rest at radius $r = 6M$ after the flash has been emitted in the process described above. Then the resulting mass is released, to fall into the black hole. What is the mass increase of the black hole, in units of m ?

4. THE HORIZON AS A ONE-WAY BARRIER

Carry out exercise 7, page 5-34 of EBH.

5. PLUNGER WINK-OUT TIME

(DOUBLE CREDIT: Credit for this exercise will be twice that of a regular exercise.)

How rapidly does a distant observer see an in-falling beacon light source "go black"?

We call this the *wink-out time*.

This exercise is a *modification* of Exercise 9, starting on page 5-35.

Please READ that exercise.

Assume that the beacon light source falls freely starting from rest at a great distance.

A1 A light signal moving radially outward passes a shell observer at radius $r = 7M$. The shell observer measures this signal to have frequency f_{shell} . What is the frequency f_{far} of this flash detected by the far-away observer when this light flash reaches her? Express your answer as a multiple or decimal fraction of f_{shell} .

A2 A beacon falls radially into the black hole starting from rest at a great distance. This beacon emits a series of flashes at a proper frequency f_0 as observed in its own frame. This beacon passes the shell observer at radius $r = 7M$. Immediately after the beacon passes the shell observer, the shell observer looks inward and detects an outward-moving signal of frequency f_{shell} from the beacon. Using special relativity, find the value of that shell frequency as a decimal fraction (or multiple) of the proper frequency f_0 emitted by the beacon. (Note: In equation [50], page 5-36, the symbol v_{shell} represents a *positive* quantity.)

B Carry out the derivation indicated in part (B) of the exercise in the text and say whether or not you succeeded in deriving equation [52], page 5-36 (honor system).

C Find the radius r_1 that the beacon is passing when it emits a signal whose frequency is observed to be $f_{\text{far}} = 0.9 f_0$ when received by the distant observer.

D Find the radius r_2 the beacon is passing when it emits a signal whose frequency is observed to be $f_{\text{far}} = 0.1 f_0$ when received by the distant observer.

E Using the analysis in the text, find the far-away time, in meters, that it takes for the beacon to fall from r_1 to r_2 as defined in parts (C) and (D). Express your answer as a fraction or multiple of M , the mass of the black hole.

MORE: See next page

- F** Using the analysis in the text, find the far-away time required for the light from the beacon at r_2 to rise to the higher radius r_1 as a multiple of M , the mass of the black hole. You will need a table of integrals.
- G** Find the value of the elapsed far-away time (the *wink-out time*) t_{far} between the arrival of the signal shifted to $0.9 f_0$ and the arrival of the signal shifted to $0.1 f_0$. Our answer to two significant figures is $1500M$, where M is the mass of the black hole. Find the answer to one percent.
- H** What is the wink-out time t_{far} for a beacon descending into a black hole of mass equal to 50 times the mass of our Sun? Express your answer in both meters and seconds