

MASSACHUSETTS INSTITUTE OF TECHNOLOGY  
Physics 8.224 Exploring Black Holes  
General Relativity and Astrophysics  
Spring 2003

**ASSIGNMENT Week 7**

**INTRODUCTION**

This week we study the spinning black hole, one of the simplest, most interesting, and—you will agree—weirdest structures in the Universe. This study employs almost every skill developed during this course, including manipulation of metrics, derivations of energy and angular momentum, and different measures of time and space. These lead to predictions of some startling physical effects:

- spacetime swept around the spinning black hole
- different speeds of light moving in different directions as measured by the bookkeeper
- a plunging stone with zero angular momentum that nevertheless spirals inward
- an *ergosphere* region outside the horizon in which nothing can stand still
- the possibility that a stone inside the ergosphere can have negative energy-at-infinity
- extraction of energy from a spinning black hole for use by a distant engineer

Thorne, in Chapter 7, describes the ten-year Golden Age of black hole studies between 1965 and 1975, a time dominated by mentors of startlingly different styles: Wheeler, Zel'dovich, and Sciama. Independent players included Roy Kerr, whose solution to the field equations for a spinning black hole we study this week, and Subrahmanyan Chandrasekhar, who came roaring back after almost 40 years of mastering other fields to formulate a full mathematical description of black holes.

**READINGS**

*Exploring Black Holes*: Project F, The Spinning Black Hole  
Thorne, Chapter 7, The Golden Age

NOTE: Check the printing number of your copy of *Exploring Black Holes* by looking at the first number in the bottom line of the Acknowledgments page (back of the title page). If that first number is 5, all is OK. If that number is less than five, do NOT read Project F from the EBH text. Instead, download the PDF file of the new Project F from the website <http://www.eftaylor.com/download.html> (which talks incorrectly about the fourth printing).

## RECITATION

The recitation quiz will include at least one of the following questions:

1. A spinning black hole has maximum possible angular momentum. A rocket ship attempts to move radially inward, that is without circulating around the black hole. (a) Describe the direction and relative intensity of rocket blasts radially inward and tangential to this path required as the rocket descends at a constant radial speed. (b) How close to the black hole can the steady descent along a radial line continue? (c) How close to the black hole is it possible for a rocket ship to keep a constant radius, engines blasting, if it allows motion in a tangential direction?
2. Compare and contrast the observations of a ring rider near a maximally spinning black hole with the observations of a shell observer near a Schwarzschild black hole. These observations include results of local experiments and observation of the "fixed" stars overhead.
3. A particle inside the static limit is moving so that it has negative energy-at-infinity, say  $-km$ , where  $k$  is a positive number and  $m$  is the particle mass. How much energy-at-infinity must be added to the particle to move it to a very great distance from the black hole, with final kinetic energy equal to  $m/2$ .
4. Explain the meaning of the saying. "A black hole has no hair." as it applies to the *formation* of a black hole.
5. A radio galaxy may have "lobes" hundreds of thousands of light years from the galaxy itself. What physical process leads to radio emission from these lobes?

## PROBLEM SET

### PROBLEM 1. Limiting values of R

Carry out QUERY 6, page F-6

### PROBLEM 2. Deriving energy-at-infinity and angular momentum

Starting from the Kerr metric in the equatorial plane, use the Euler-Lagrange equations of the notes "How Gravitational Forces Arise from Curvature" to derive expressions for  $E/m$  and  $L/m$ , the energy-at-infinity and angular momentum per unit mass. (Hint: repeat the derivations in equations 21-26 of the notes.) Determine whether  $E/m$  is conserved, and similarly for  $L/m$ . Show that your results reduce to those for the Schwarzschild metric when the black hole spin  $a=0$ .

### PROBLEM 3. Bookkeeper speed in the "final circle"

Carry out QUERY 18, page F-15

### PROBLEM 4. Ring speed according to the bookkeeper

Carry out QUERY 20, page F-17

### PROBLEM 5. Available energy from the monster black hole in our galaxy

Carry out QUERY 31, page F-26

### PROBLEM 6. Energies of a particle

A particle inside the static limit at reduced circumference  $r = 1.5M$  is moving tangentially so that it has negative energy-at-infinity  $-km$ , where  $k$  is a positive number and  $m$  is the particle mass. In the following, *rotating ring* means a ring of zero angular momentum, as described in Section 9 of Project F. How much energy-at-infinity must be added to the particle to move it (a) to rest on a rotating ring at the same radius  $r = 1.5M$ . (b) to rest on a rotating ring at the static limit, (c) to a non-rotating position at a radius  $r$  equal to twice that of the static limit, and (e) to a state of rest an infinite distance away from the black hole?

### PROBLEM 7. Energy extracted in phase 1

Carry out the following modification of QUERY 35, page F-30

The trouble with the analysis of pages F-28 and F-29 is that light emitted exactly at the horizon will take forever to travel to a great distance, just as it would take forever for light to climb to a great distance from the horizon of a non-rotating black hole. Therefore in the following assume that the mass and antimass are stopped on a freely-rotating ring at  $r = 1.5 M$  in Phase 1.

When Phase 1 is completed, how much energy-at-infinity will have been milked off for use at the distant location? Use equation [44] because it gives more accuracy than Figure 9. Express your answer as a decimal fraction or multiple of the mass  $m$ . (Recall that a total mass  $2m$  started originally from a great distance.)

### PROBLEM 8. Energy extracted in phase 2

Carry out the following modification of QUERY 36, page F-30.

Continue with the NEW assumptions given in the revised QUERY 35, above. Just after Phase 2 is completed:

- A What is the energy of the flash moving FORWARD on the ring at  $r = 1.5 M$  as measured by the ring observer? Express your answer in terms of the mass  $m$ .
- B What is the energy of the flash moving BACKWARD on the ring at  $r = 1.5 M$  as measured by the ring observer? Express your answer in terms of the mass  $m$ .
- C What is the energy-at-infinity of the flash moving forward on the ring as computed by the far-away Kerr bookkeeper? Do NOT use equation [49]; it does not apply to this new value of  $r$  for the ring. Express your answer in terms of the mass  $m$ .

### PROBLEM 9. Total energy extracted

Carry out the following modification of QUERY 37, page F-30.

Continue with the assumptions given in NEW QUERIES 35 AND 36, above. (Recall that a total mass  $2m$  started originally from a great distance.) After both Phase I and Phase II have been completed and the forward-moving flash has arrived at the distant location:

- A What is the total useful energy made available to the distant engineers as a result of this entire procedure? Express your answer in terms of the mass  $m$ .
- B How much energy has been extracted from the black hole?