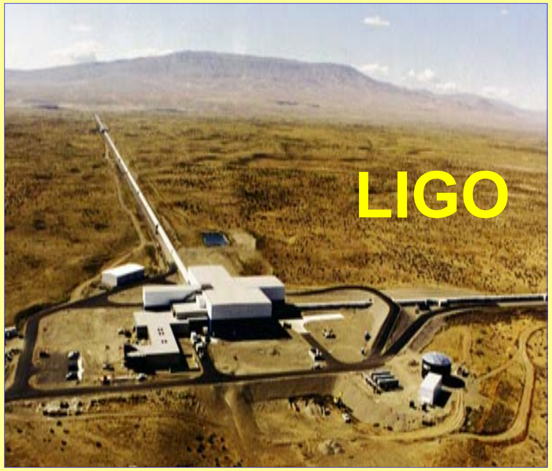
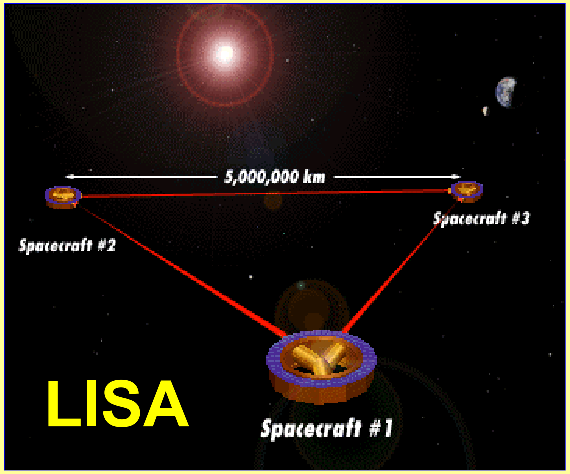
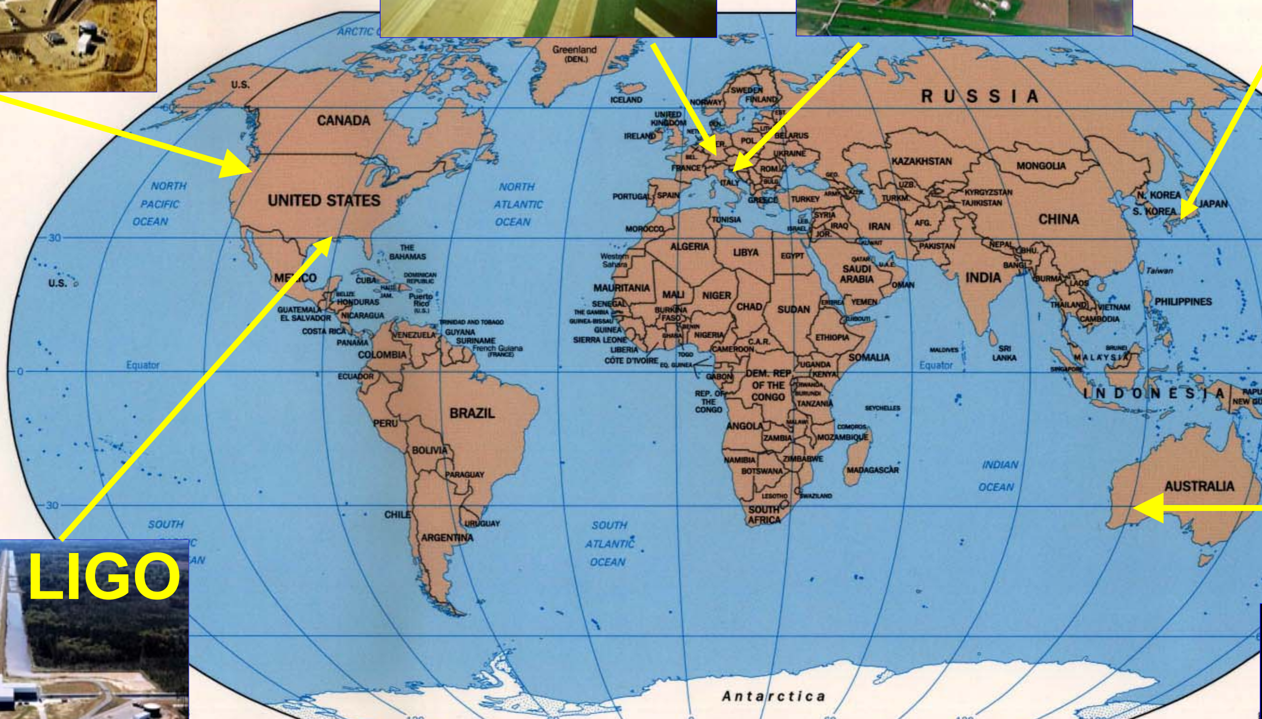
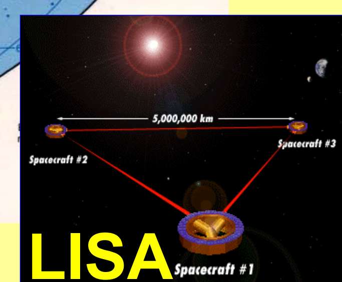
Detection of Gravitational Waves with Interferometers





Global network of detectors





- **Detection confidence**
- **Source polarization**
- **Sky location**

Science goals: Detection of gravitational waves

- Tests of general relativity
  - Waves ➔ direct evidence for time-dependent metric
  - Black hole signatures ➔ test of strong field gravity
  - Polarization of the waves ➔ spin of graviton
  - Propagation velocity ➔ mass of graviton
- Astrophysical processes
  - Inner dynamics of processes hidden from EM astronomy
  - Cores of supernovae
  - Dynamics of neutron stars ➔ large scale nuclear matter
  - The earliest moments of the Big Bang ➔ Planck epoch
- Astrophysics…

A little bit of GR

- From special relativity, “flat” space-time interval is

$$(d\tau)^2 = -c^2(dt)^2 + (dx)^2 + (dy)^2 + (dz)^2$$
$$= \eta_{\mu\nu}(dx)^\mu(dx)^\nu$$

- From general relativity, curved space-time can be treated as perturbation of flat space-time

$$(ds)^2 = g_{\mu\nu}(dx)^\mu(dx)^\nu$$

where $\underbrace{g_{\mu\nu}}_{\text{space-time curvature}} = \underbrace{\eta_{\mu\nu}}_{\text{flat space-time}} + \underbrace{h_{\mu\nu}}_{\text{metric perturbation}}$, $h_{\mu\nu} << 1$

A little bit more GR

- Space-time interval becomes

$$(d\tau)^2 = -c^2 dt^2 + \left[1 + h(z \pm ct)\right]dx^2 + \left[1 - h(z \pm ct)\right]dy^2 + dz^2$$

- When the gravitational field is weak and in the transverse traceless gauge

TT gauge ➔ coordinates are marked by world lines of freely falling masses

Einstein's equations give a wave equation

$$G_{ij} = 8\pi T_{ij} \quad \Rightarrow \quad \left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)h_{\mu\nu} = 0$$

- Space-time tell matter how to move
  Matter tells space-time how to curve

Gravitational waves

- Time-dependent solution ➔ $h(t) = h_0 \cos(\omega_g t + \phi)$
- *h* is wave-like motion of the space-time itself
  ➔ ripples of space-time curvature
- H is dimensionless
- Waves travel at the speed of light
- Waves push freely floating objects together and apart
  ➔ stretching and squeezing of space transverse to direction of propagation
- Frequency of oscillation is ω_g

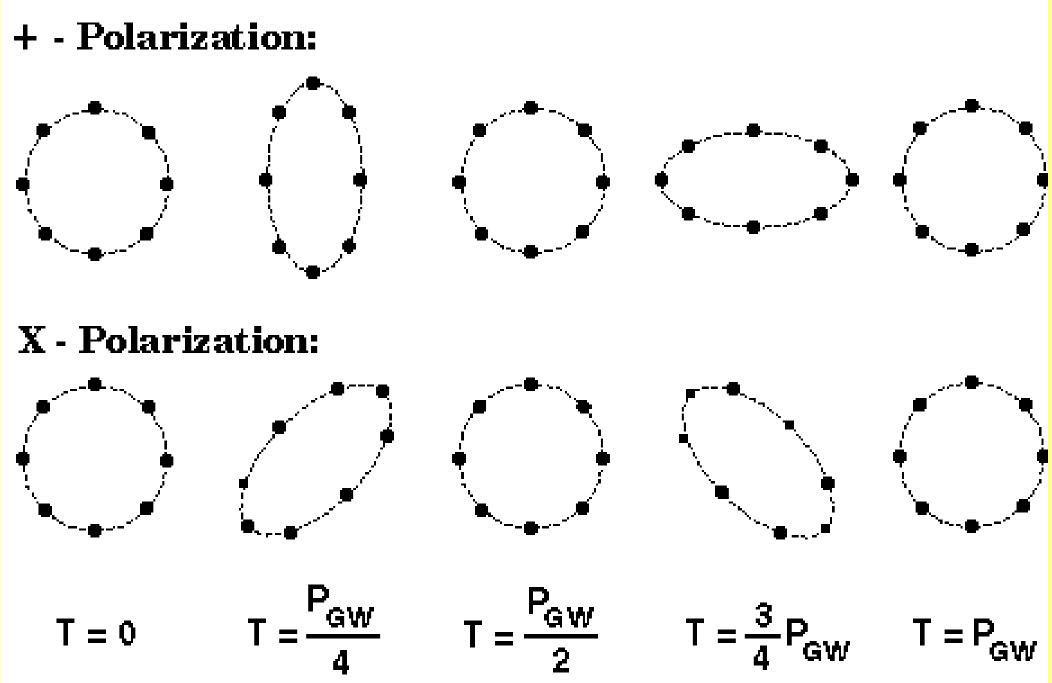
Gravitational waves and GR

- Two polarizations

  ➔

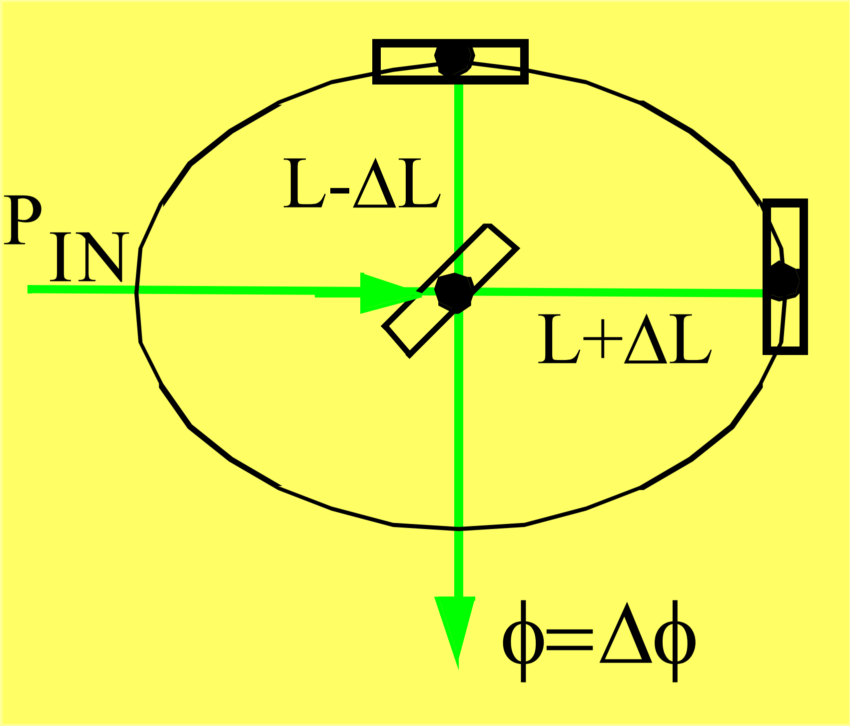
$$h_+ = h_0 \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad h_X = h_0 \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

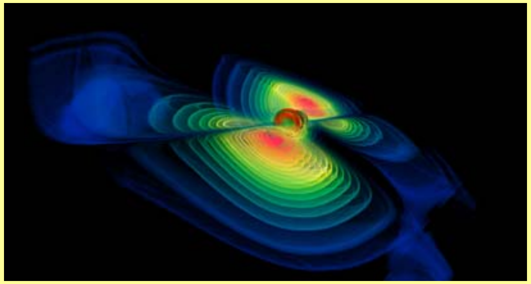
- Interaction with matter



GWs meet Interferometers

- Laser interferometer





$\Delta L = h\,L$

- Suspend mirrors on pendulums ➔ "free" mass

Some properties of gravitational waves

- General relativity predicts transverse space-time distortions propagating at the speed of light
- In TT gauge and weak field approximation, Einstein field equations ➔ wave equation
- Conservation laws
  - Conservation of energy ➔ no monopole radiation
  - Conservation of momentum ➔ no dipole radiation
  - Lowest moment of field ➔ quadrupole (spin 2)
- Radiated by aspherical astrophysical objects
- Radiated by “dark” mass distributions
  ➔ black holes, dark matter

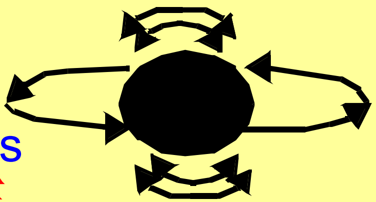
Astrophysics with GWs vs. E&M

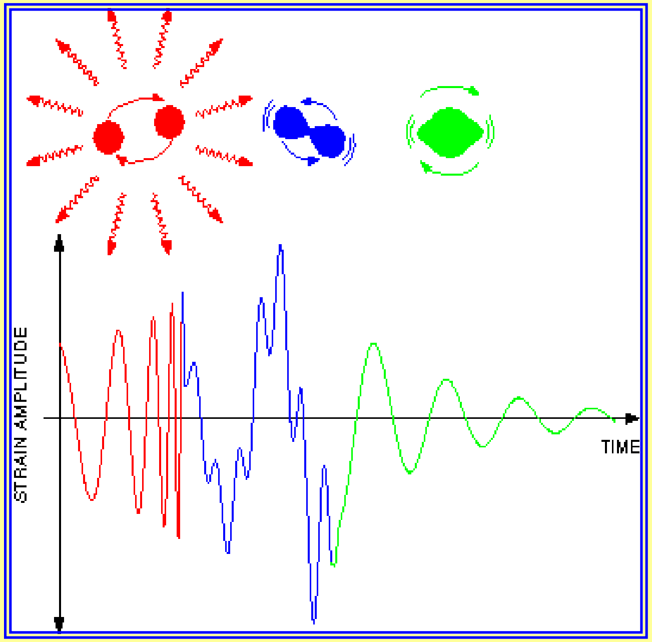
E&M	GW
Space as medium for field	Spacetime itself
Accelerating charge ➔ incoherent superpositions of atoms, molecules	Accelerating aspherical mass ➔ coherent motions of huge masses
Wavelength small compared to sources ➔ images	Wavelength large compared to sources ➔ no spatial resolution
Absorbed, scattered, dispersed by matter	Very small interaction; matter is transparent
10 MHz and up	10 kHz and down
Detectors have small solid angle acceptance	Detectors have large solid angle acceptance

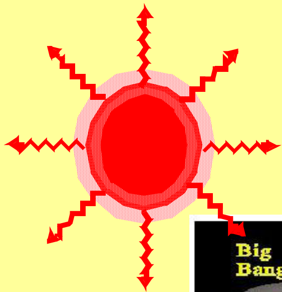
- Very different information, mostly mutually exclusive
- Difficult to predict GW sources based on E&M observations

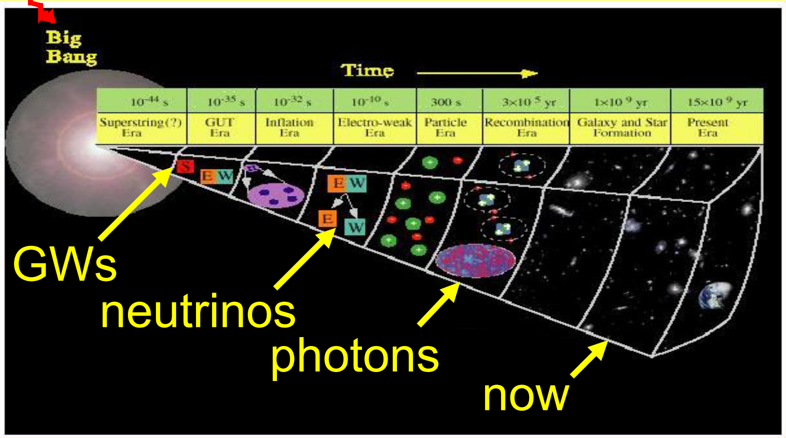
Astrophysical sources of GWs

- Coalescing compact binaries
  - Classes of objects: NS-NS, NS-BH, BH-BH
  - Physics regimes: Inspiral, merger, ringdown
- Periodic sources
  - Spinning neutron stars → ellipticity, precession, r-modes
- Burst events
  - Supernovae → asymmetric collapse
- Stochastic background
  - Primordial Big Bang ($t = 10^{-43}$ sec)
  - Continuum of sources
- The Unexpected









Strength of GWs: e.g. Neutron Star Binary

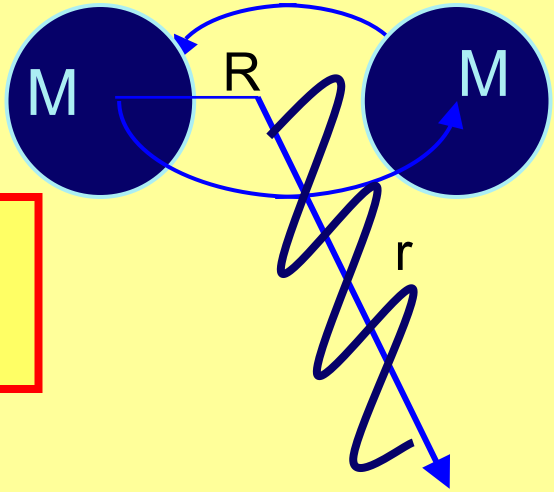
- Gravitational wave amplitude (strain)

$$h_{\mu\nu} = \frac{2G}{c^4 r}\ddot{I}_{\mu\nu} \Rightarrow h \approx \frac{4\pi^2 GMR^2 f_{orb}^2}{c^4 r}$$

- For a binary neutron star pair

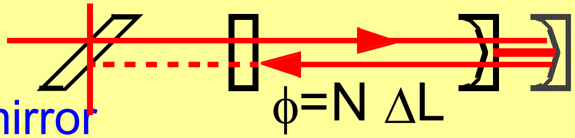
$$M \approx 10^{30}\ \mathrm{kg}$$
$$R \approx 20\ \mathrm{km}$$
$$f \approx 400\ \mathrm{Hz}$$
$$r \approx 10^{23}\ \mathrm{m}$$

$\Rightarrow$ $h \sim 10^{-21}$

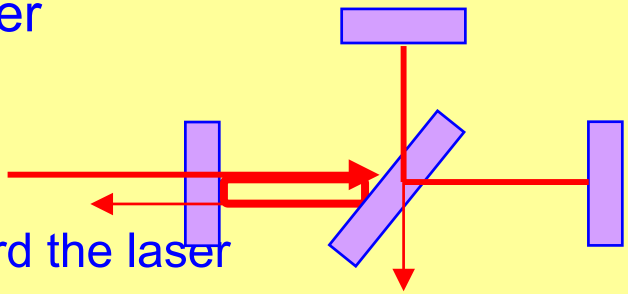


Practical Interferometer

- For more practical lengths (L ~ 1 km) ⇨ "fold" interferometer to increase phase sensitivity
  - $\Delta\phi = 2\,k\,\Delta L \rightarrow N\,(2\,k\,\Delta L)$; $N \sim 100$
  - N ⇨ number of times the photons hit the mirror



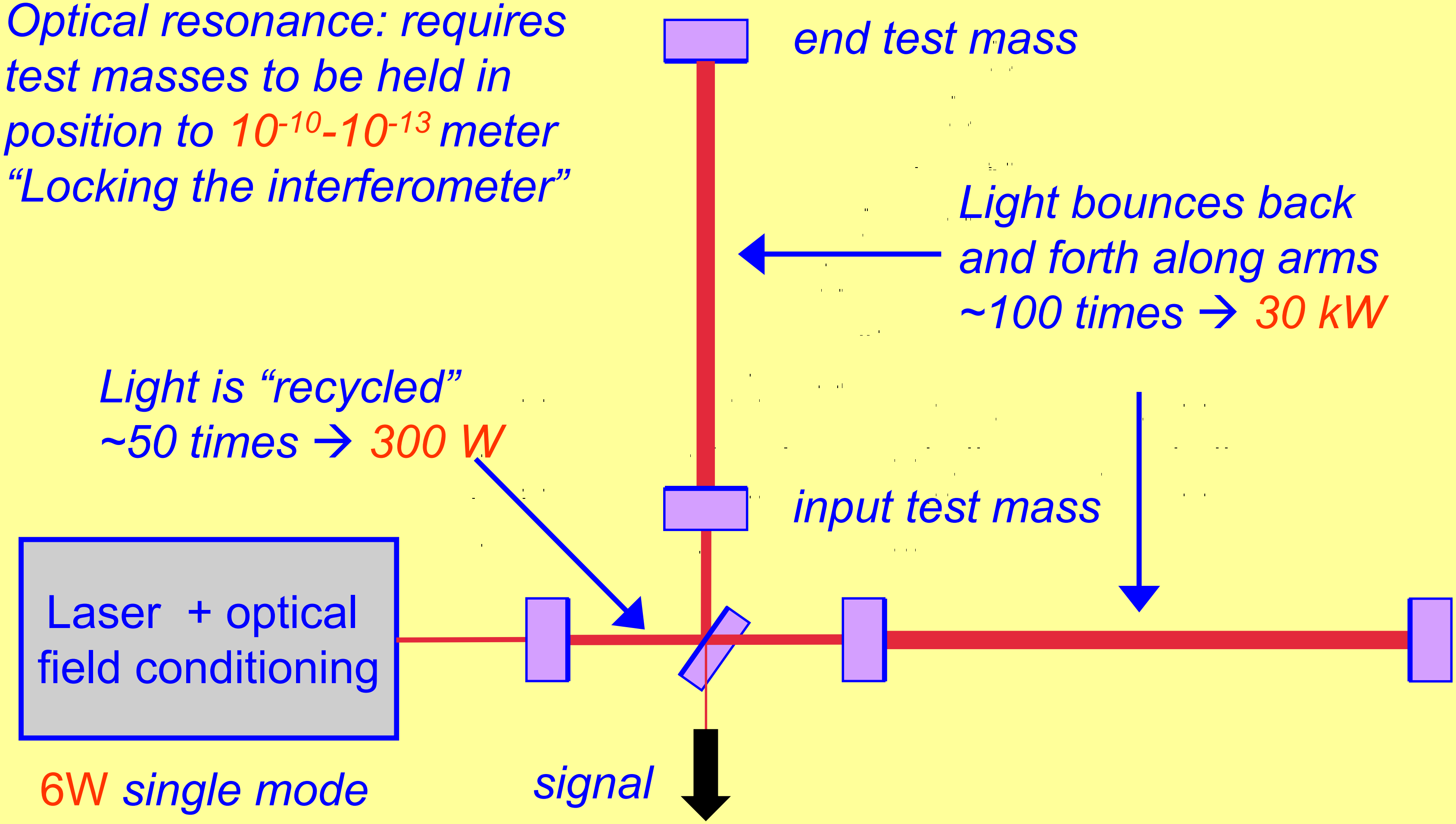
- Light storage devices ⇨ optical cavities
- Dark fringe operation ⇨ lower shot noise
- GW sensitivity ∝ ⌧P ⇨ increase power on beamsplitter
- Power recycling
  - Most of the light is reflected back toward the laser
    ➔ "recycle" light back into interferometer



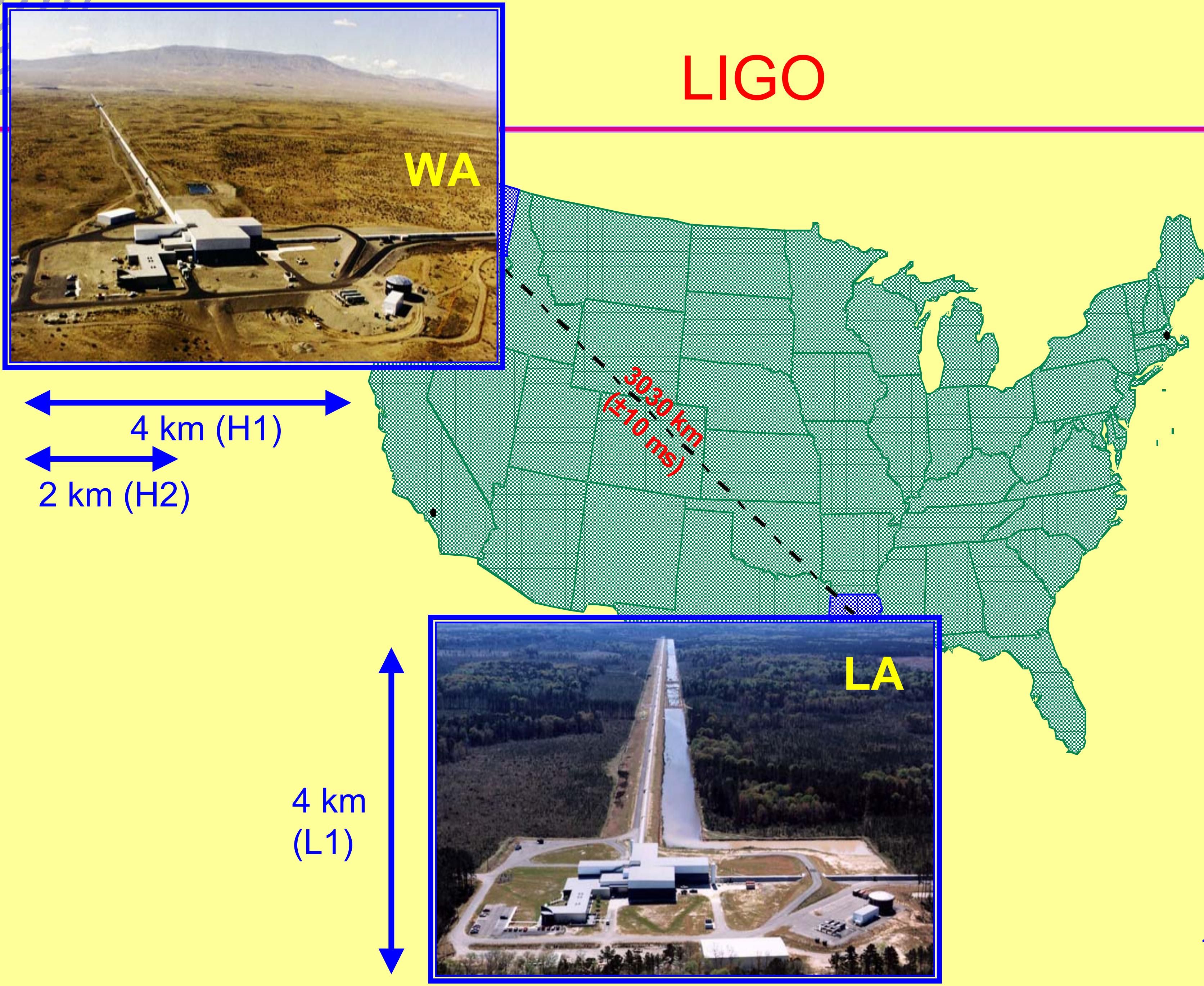
- Price to pay: multiple resonant cavities whose lengths must be controlled to ~ $10^{-8}\,\lambda$

Power-recycled Interferometer

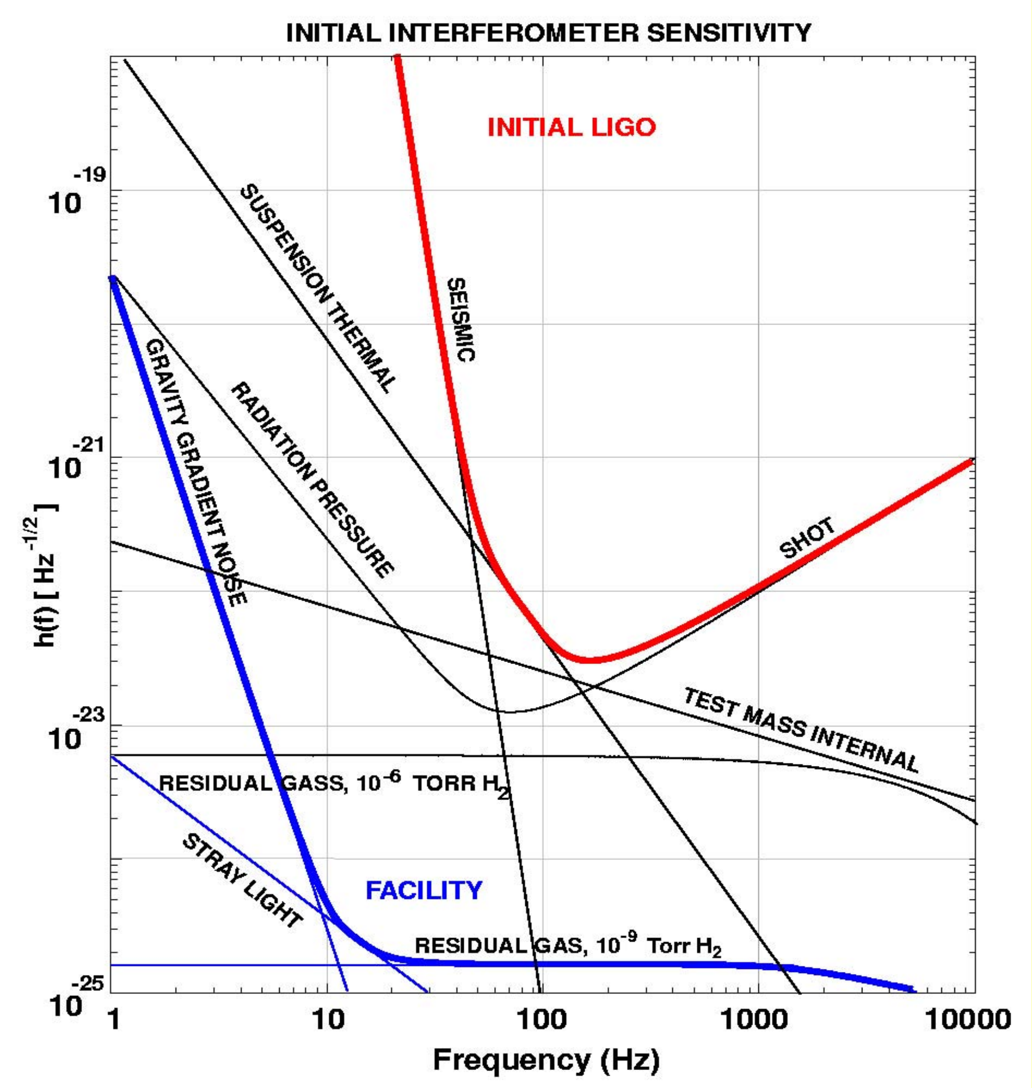
Optical resonance: requires test masses to be held in position to 10^{-10}-10^{-13} meter
"Locking the interferometer"



LIGO



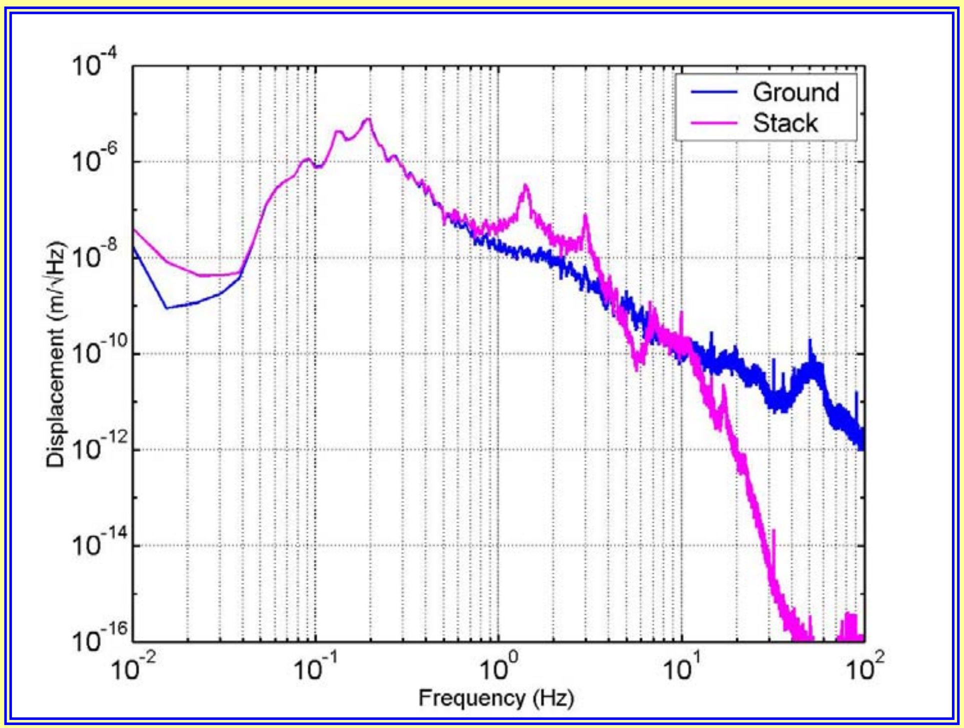
Initial LIGO Sensitivity Goal



- Strain sensitivity $< 3\times10^{-23}$ 1/Hz$^{1/2}$ at 200 Hz
- Displacement Noise
  - Seismic motion
  - Thermal Noise
  - Radiation Pressure
- Sensing Noise
  - Photon Shot Noise
  - Residual Gas
- Facilities limits much lower

Limiting Noise Sources: Seismic Noise

- Motion of the earth few μm rms at low frequencies
- Passive seismic isolation ‘stacks’
  - amplify at mechanical resonances
  - but get f^{-2} isolation per stage above 10 Hz



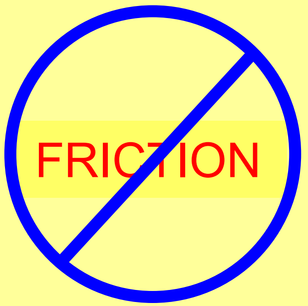
Limiting Noise Sources: Thermal Noise

- Suspended mirror in equilibrium with 293 K heat bath
  ⇨ $k_B T$ of energy per mode
- Fluctuation-dissipation theorem:
  - Dissipative system will experience thermally driven fluctuations of its mechanical modes:

$$\tilde{h}(f) = \frac{\sqrt{k_B T}}{\pi f L}\sqrt{\mathrm{Re}(Z(f))}$$

Z(f) is impedance (loss)

- Low mechanical loss (high Quality factor)
  - Suspension ➔ no bends or ‘kinks’ in pendulum wire
  - Test mass ➔ no material defects in fused silica



Limiting Noise Sources: Quantum Noise

- Shot Noise
  - Uncertainty in number of photons detected ⇨

$$h(f) = \frac{1}{L}\sqrt{\frac{hc\lambda}{8F^2 P_{bs}}}\,\frac{1}{T_{ifo}(\tau_s, f)}$$

  - Higher input power P_{bs} ⇨ need low optical losses
  - (Tunable) interferometer response ➔ T_{ifo} depends on light storage time of GW signal in the interferometer
- Radiation Pressure Noise
  - Photons impart momentum to cavity mirrors Fluctuations in the number of photons ⇨

$$h(f) = \frac{2F}{ML}\sqrt{\frac{2hP_{bs}}{\pi^3 c\lambda}}\,\frac{T_{ifo}(\tau_s, f)}{f^2}$$

  - Lower input power, P_{bs}

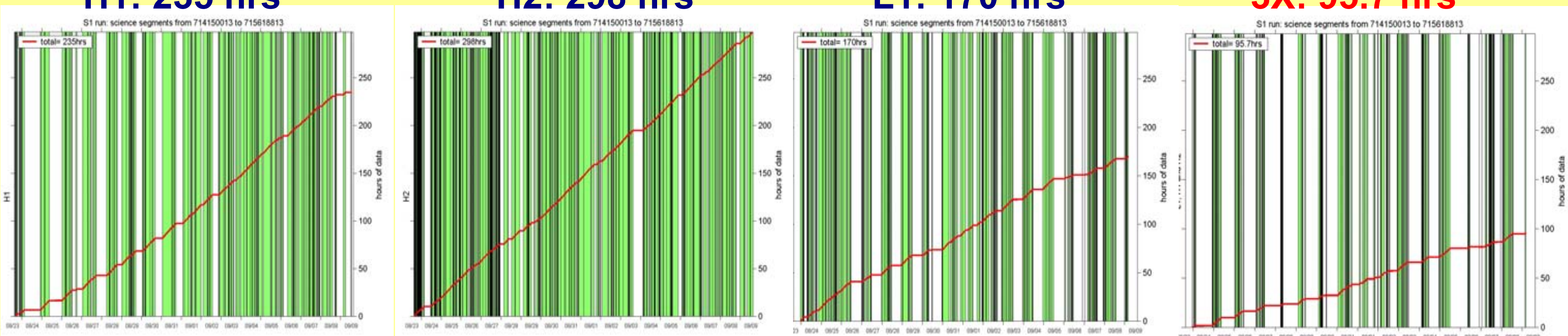
➔ Optimal input power for a chosen (fixed) T_{ifo}

Operations Strategy

- **Interferometer performance**
  - Intersperse commissioning and data taking consistent with obtaining one year of integrated data at $h = 10^{-21}$ by end of 2006
- **Astrophysical searches**
  - Two "upper limit" runs S1 and S2 (at unprecedented early sensitivity) are interleaved with commissioning
    - **S1 Aug-Sep 2002 duration: 2 weeks**
    - **S2 Feb-Apr 2003 duration: 8 weeks**
  - First search run (S3) planned for late 2003 (duration: 6 months)
- **Finish detector integration & design updates...**
  - Engineering "shakedown" runs interspersed as needed
- **Advanced LIGO**

S1 Run Summary

H1: 235 hrs　　H2: 298 hrs　　L1: 170 hrs　　3X: 95.7 hrs

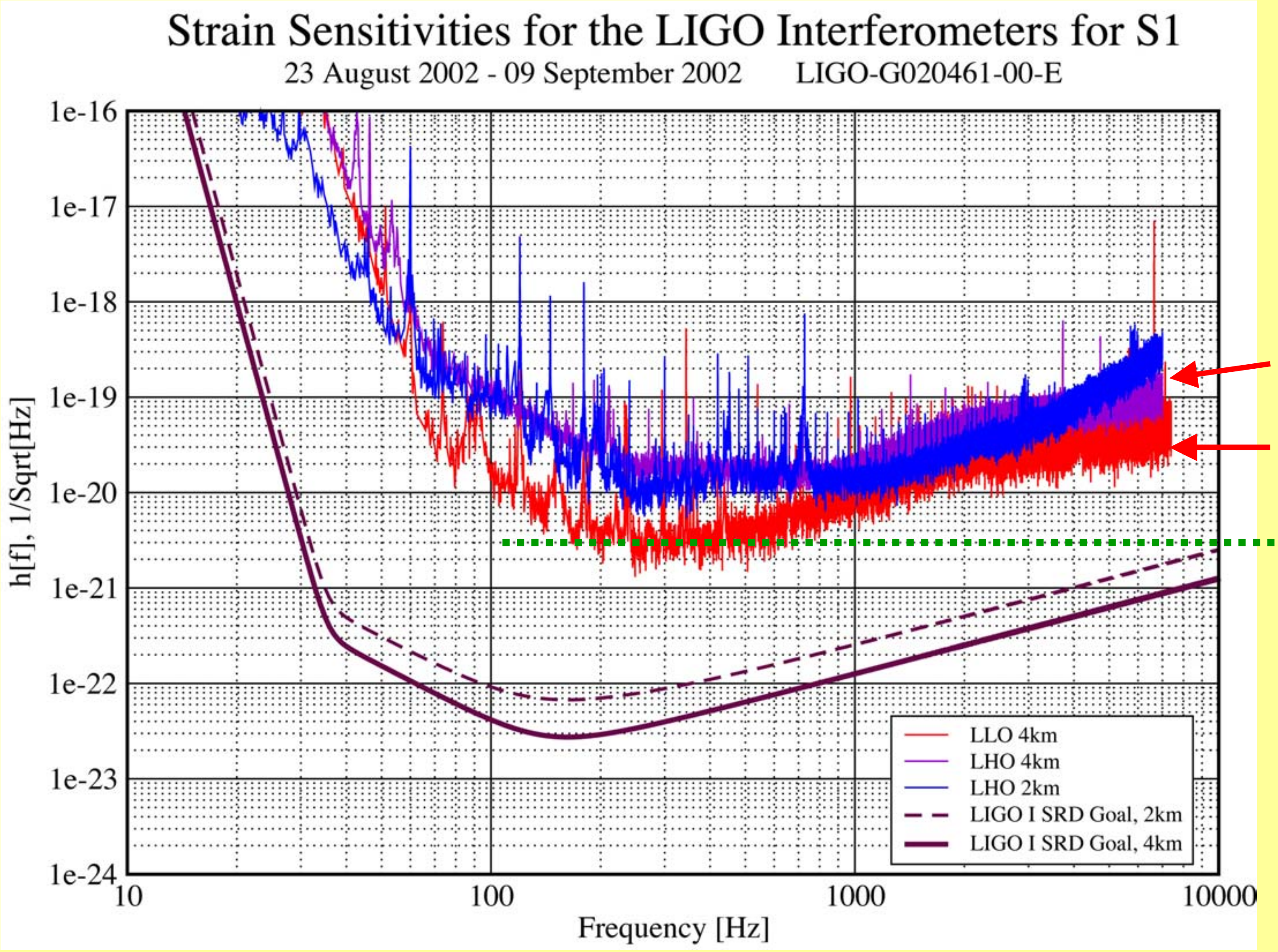


Red lines: integrated up time　　Green bands (w/ black borders): epochs of lock

- August 23 – September 9, 2002: 408 hrs (17 days).
    - H1 (4km): duty cycle 57.6% ; Total Locked time: 235 hrs
    - H2 (2km): duty cycle 73.1% ; Total Locked time: 298 hrs
    - L1 (4km): duty cycle 41.7% ; Total Locked time: 170 hrs
- Double coincidences:
    - L1 && H1 : duty cycle 28.4%; Total coincident time: 116 hrs
    - L1 && H2 : duty cycle 32.1%; Total coincident time: 131 hrs
    - H1 && H2 : duty cycle 46.1%; Total coincident time: 188 hrs

Triple Coincidence: L1, H1, and H2 : duty cycle 23.4% ; total 95.7 hours

Strain Sensitivities During S1



H2 & H1

L1

$3*10^{-21}$ Hz $^{-1/2}$

@ f ~300 Hz

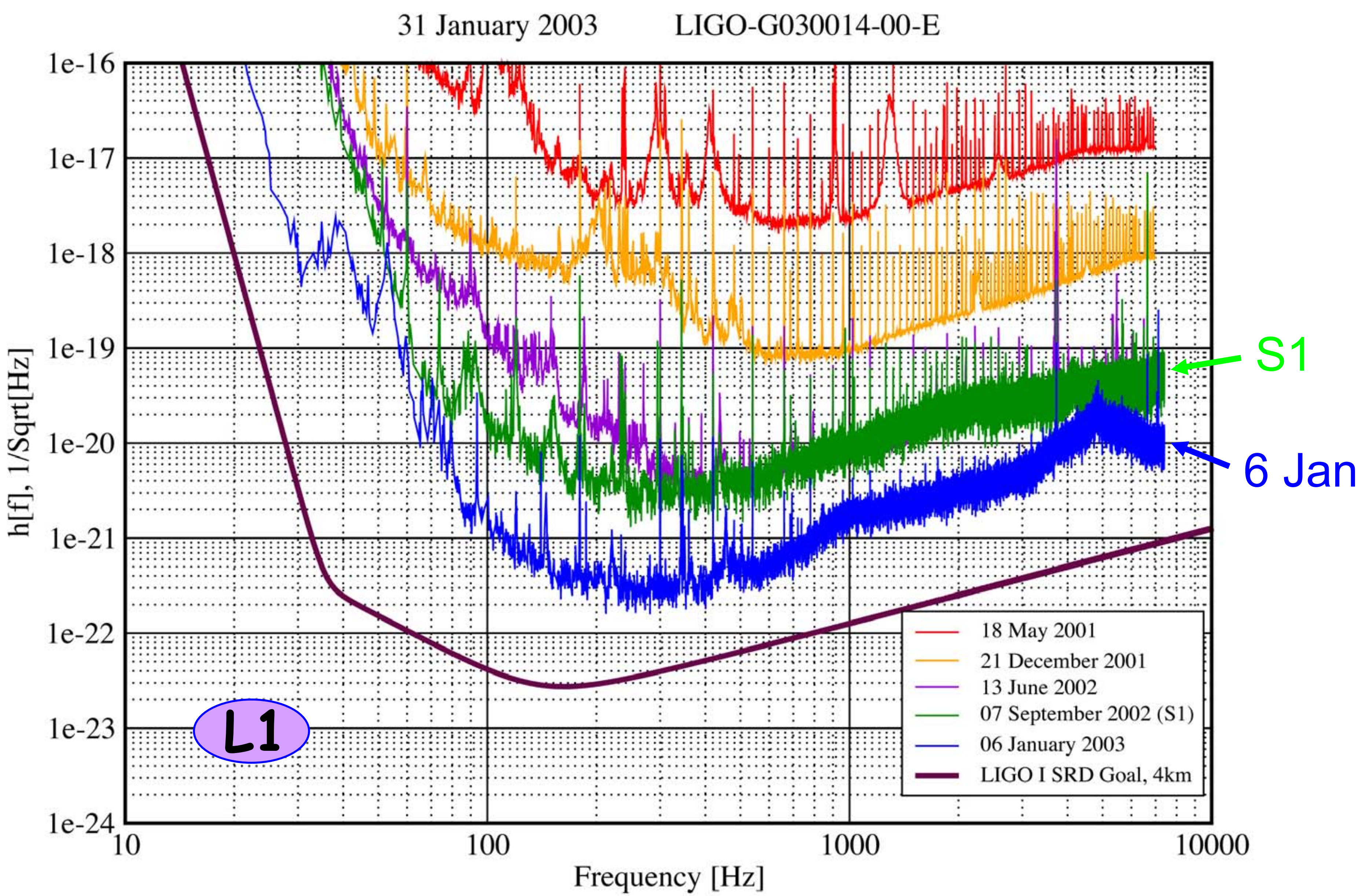
Summary of upper limits for S1

- Upper limits as presented at AAAS meeting Feb 2003
  - Stochastic backgrounds
    - Upper limit $\Omega_0 < 72.4$ (H1- H2 pair)

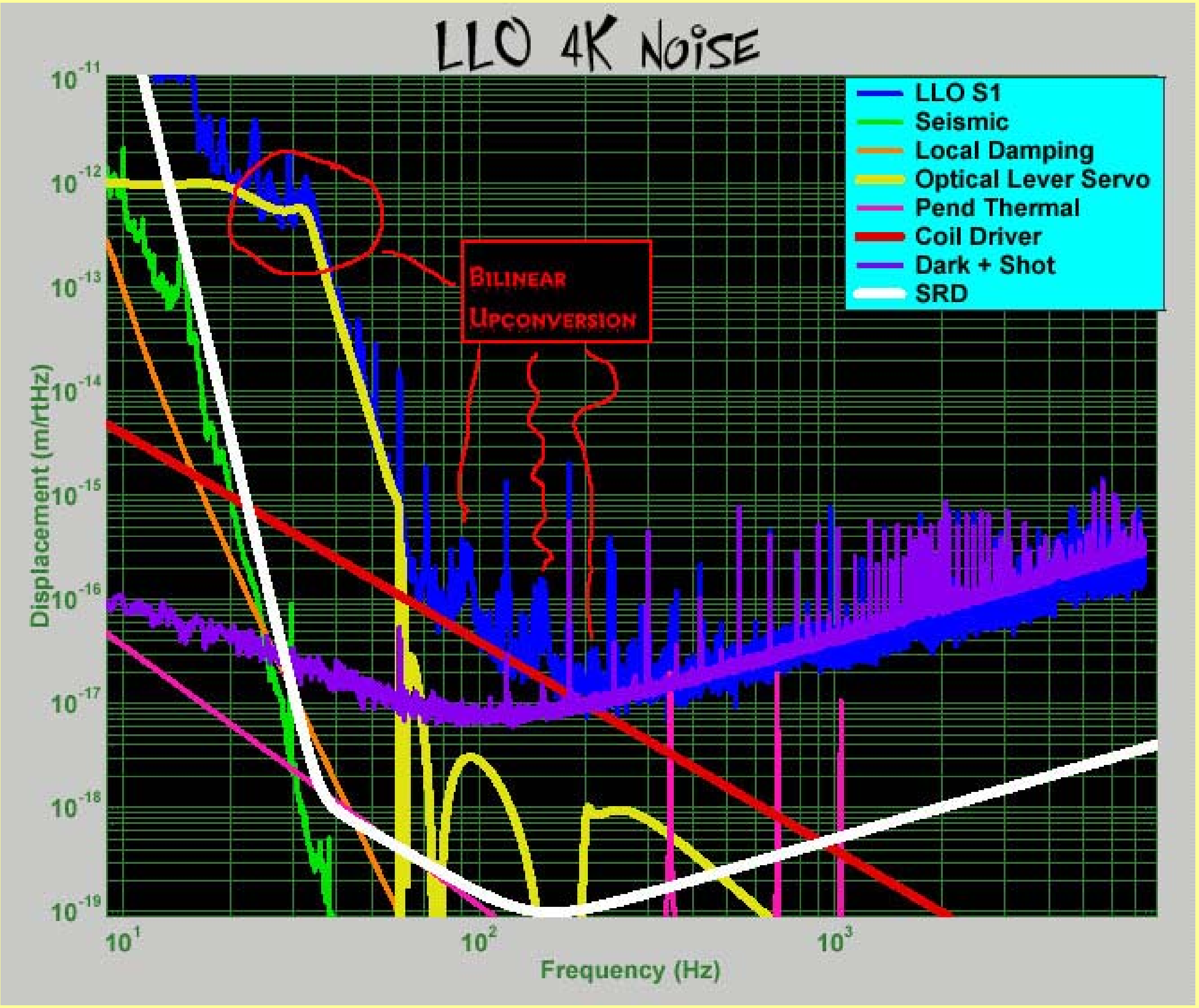
      Limit from Big Bang Nucleosynthesis $< 10^{-4}$

      Standard Inflation Prediction $< 10^{-15}$
  - Neutron star binary inspiral
    - Range of detectability < 200 kpc (1.4 – 1.4 M_{SUN} NS binary with SNR = 8)
    - Coalescence Rate for Milky Way equivalent galaxy < 164 /yr 90% CL
  - Periodic sources PSR J1939+2134 at 1283 Hz
    - GW radiation $h < 2\ 10^{-22}$ 90% CL (expect $h \sim 10^{-27}$ if pulsar spindown entirely due GW emission)
  - Burst sources
    - Upper limit $h < 5\ 10^{-17}$ 90% CL
- S2 is ~10x more sensitive and ~4x longer

Strain Sensitivity coming into S2



Displacement Sensitivity (Science Run 1, Sept. 2002)

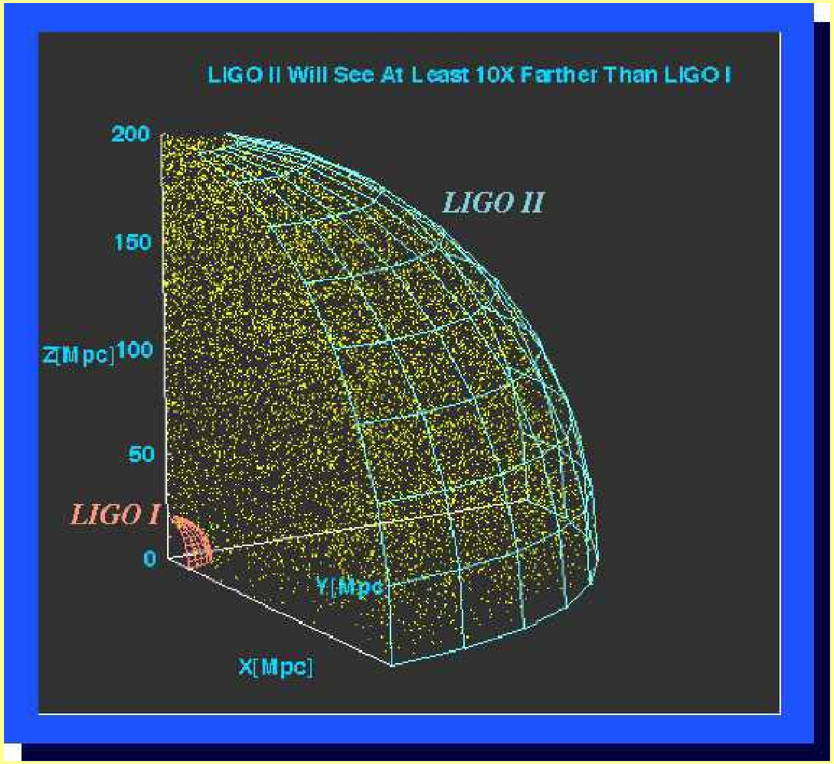


LIGO Science Has Started

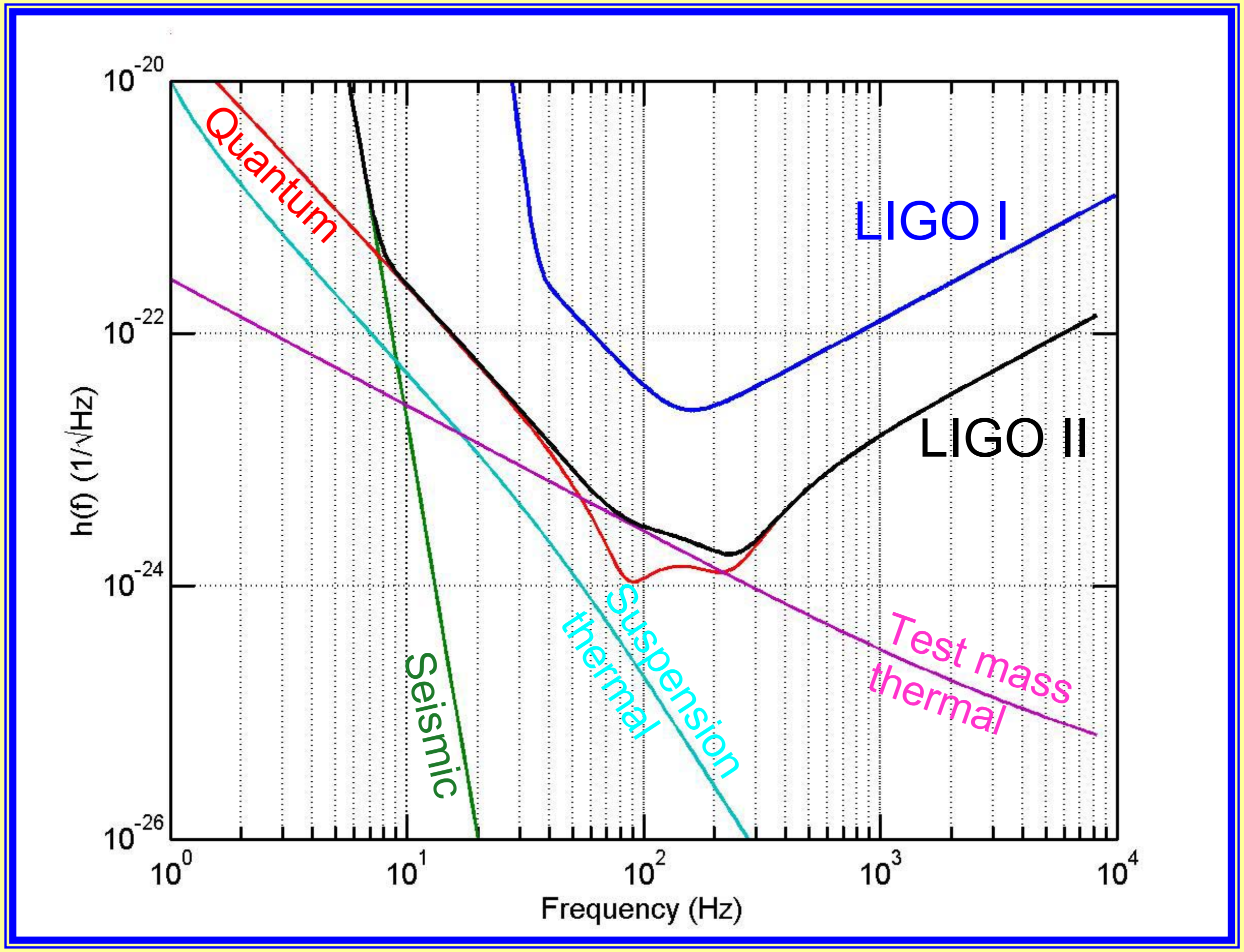
- LIGO has started taking data
- **First science run** (S1) last summer
  - Collaboration has carried out first analysis looking for
    - ✓ Bursts
    - ✓ Compact binary coalescences
    - ✓ Stochastic background
    - ✓ Periodic sources
- **Second science run** (S2) ended last week
  - Sensitivity is ~10x better than S1
  - Duration is ~ 4x longer
    - Bursts → 4x lower rate limit & 10x lower strain limit
    - Inspirals → reach > 1 Mpc -- includes M31 (Andromeda)
    - Stochastic background → limits on $\Omega_{GW} < 10^{-2}$
    - Periodic sources → limits on $h_{max} \sim$ few x 10^{-23} ($\varepsilon \sim$ few x 10^{-6} @ 3.6 kpc)

The next-generation detector Advanced LIGO (aka LIGO II)

- Now being designed by the LIGO Scientific Collaboration
- Goal:
  - Quantum-noise-limited interferometer
  - Factor of ten increase in sensitivity
  - Factor of 1000 in event rate. One day > entire 2-year initial data run
- Schedule:
  - Begin installation: 2006
  - Begin data run: 2008



A Quantum Limited Interferometer



Facility limits

- Gravity gradients
- Residual gas
- (scattered light)

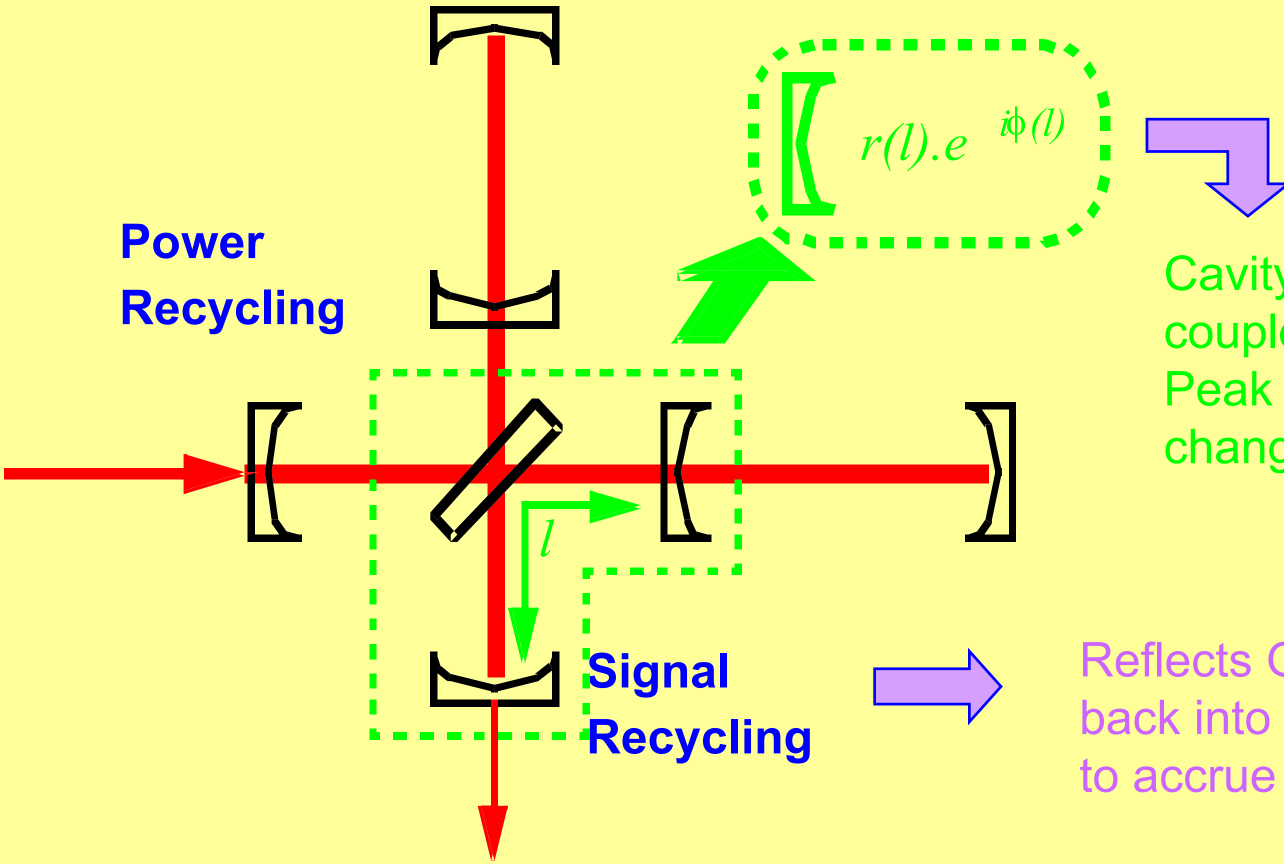
Advanced LIGO

- Seismic noise 40→10 Hz
- Thermal noise 1/15
- Optical noise 1/10

Beyond Adv LIGO

- Thermal noise: cooling of test masses
- Quantum noise: QND

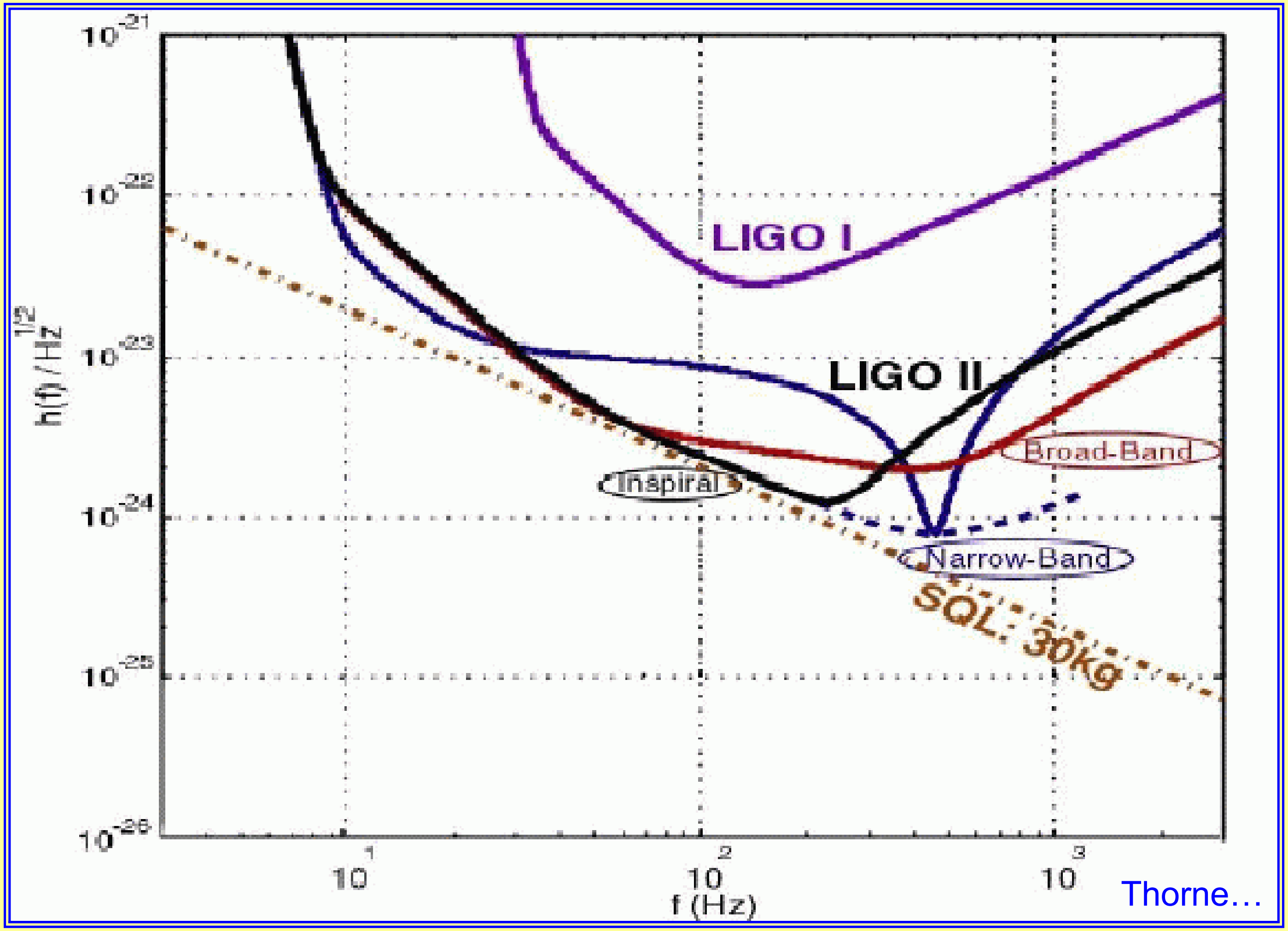
Optimizing the optical response: Signal Tuning



Cavity forms compound output coupler with complex reflectivity. Peak response tuned by changing position of SRM

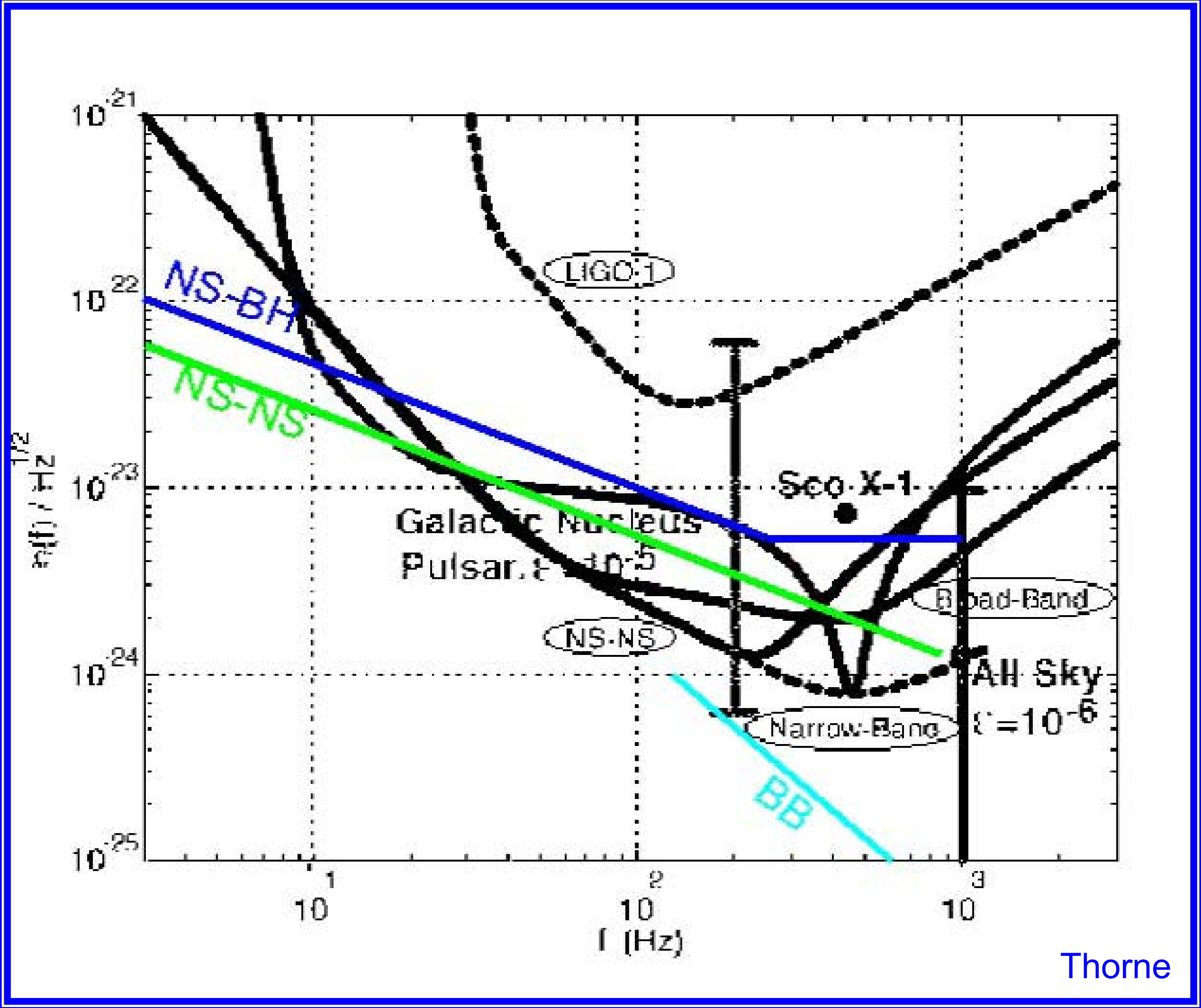
Reflects GW photons back into interferometer to accrue more phase

Advance LIGO Sensitivity: Improved and Tunable



Thorne…

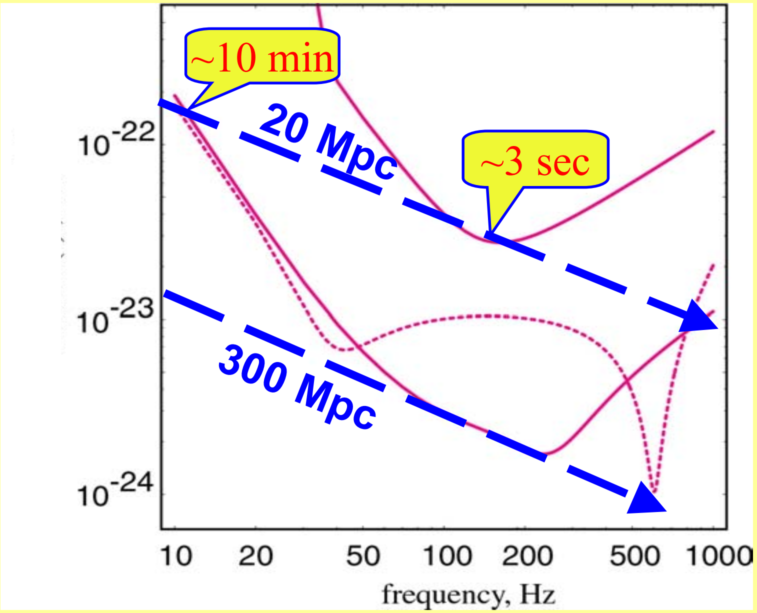
Detection of candidate sources



Thorne

Implications for source detection

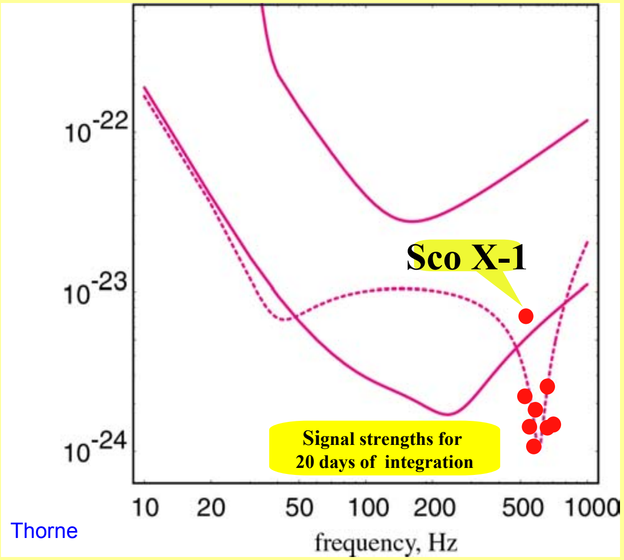
- NS-NS Inpiral
  - Optimized detector response
- NS-BH Merger
  - NS can be tidally disrupted by BH
  - Frequency of onset of tidal disruption depends on its radius and equation of state ⇨ broadband detector



- BH-BH binaries
  - Merger phase ➔ non-linear dynamics of highly curved space time ⇨ broadband detector
- Supernovae
  - Stellar core collapse ➔ neutron star birth
  - If NS born with slow spin period (< 10 msec) hydrodynamic instabilities ⇨ GWs

Source detection

- Spinning neutron stars
  - Galactic pulsars: non-axisymmetry uncertain
  - Low mass X-ray binaries:
    If accretion spin-up balanced by GW spin-down, then X-ray luminosity → GW strength
    Does accretion induce non-axisymmetry?



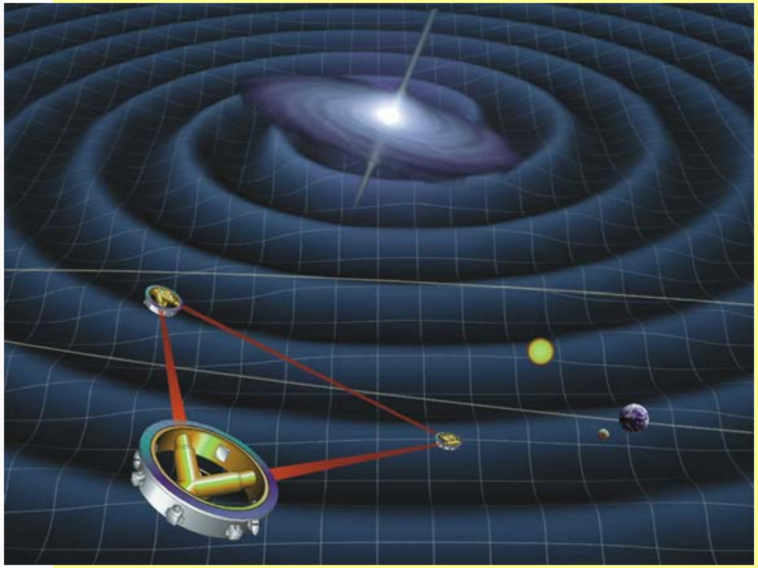
- Stochastic background
  - Can cross-correlate detectors (but antenna separation between WA, LA, Europe ⇨ dead band)
  - $\Omega_{GW}(f \sim 100\ \mathrm{Hz}) = 3 \times 10^{-9}$ (standard inflation ➔ 10^{-15})

    (primordial nucleosynthesis ♎ 10^{-5})

    (exotic string theories ➔ 10^{-5})

GW energy / closure energy

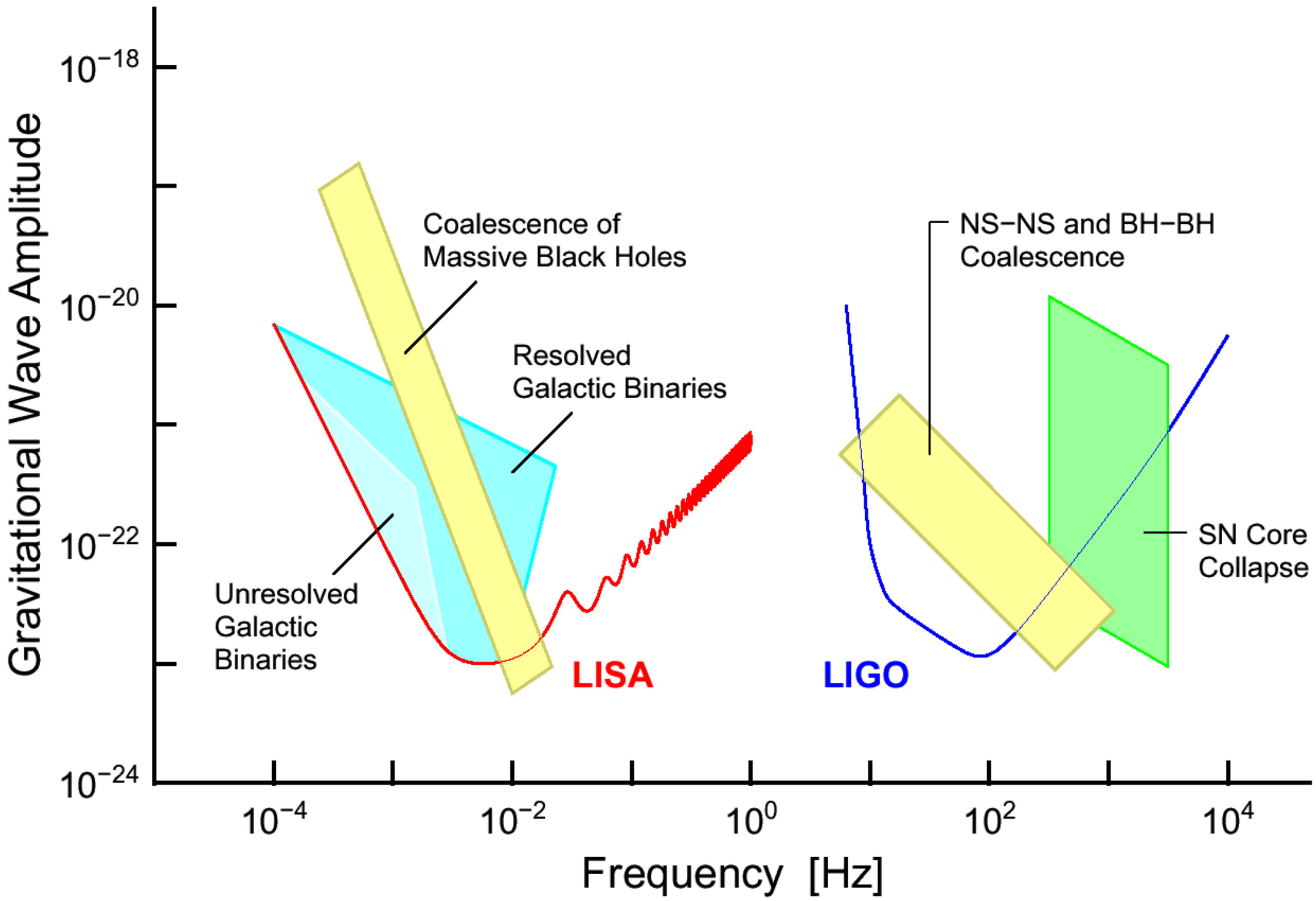
LISA - The Overview



- ***Concept***
  - *3 spacecraft constellation separated by 5 x10^6 km.*
  - *Earth-trailing solar orbit*
  - *Drag-free proof masses*
  - *Interferometry to measure changes in distance between masses caused by gravitational waves*
  - *Partnership between ESA, JPL and GSFC*

- ***Science Goals***
  - *Observe and measure the rate of massive and super-massive black hole mergers to high red shift*
  - *Observe the inspiral and merger of compact stellar objects into massive black holes*
  - *Detect gravitational radiation from compact binary star systems in our galaxy*
  - *Observe gravitational radiation from the early universe*

LISA and AdLIGO Sensitivities

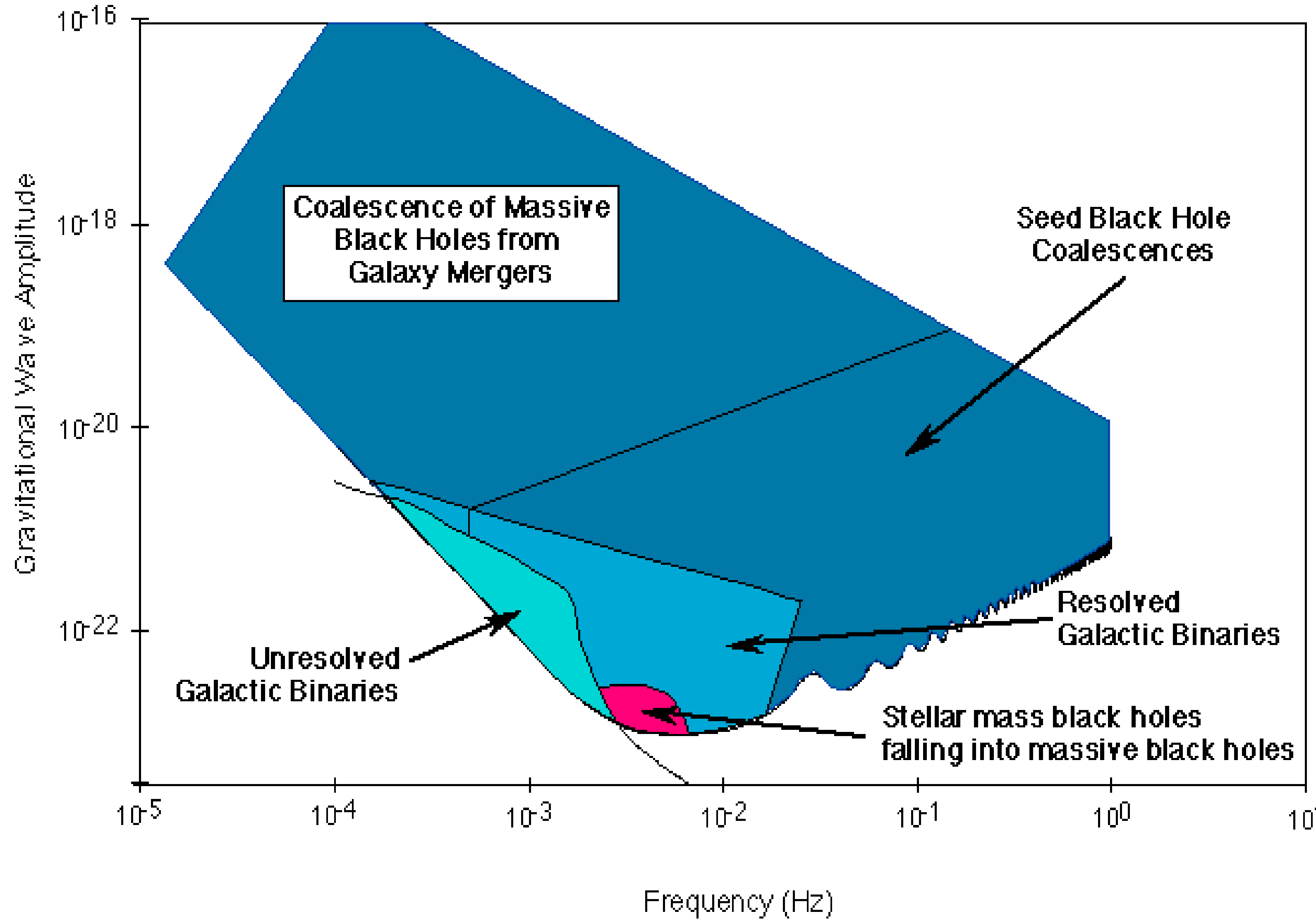


Astrophysical Sources

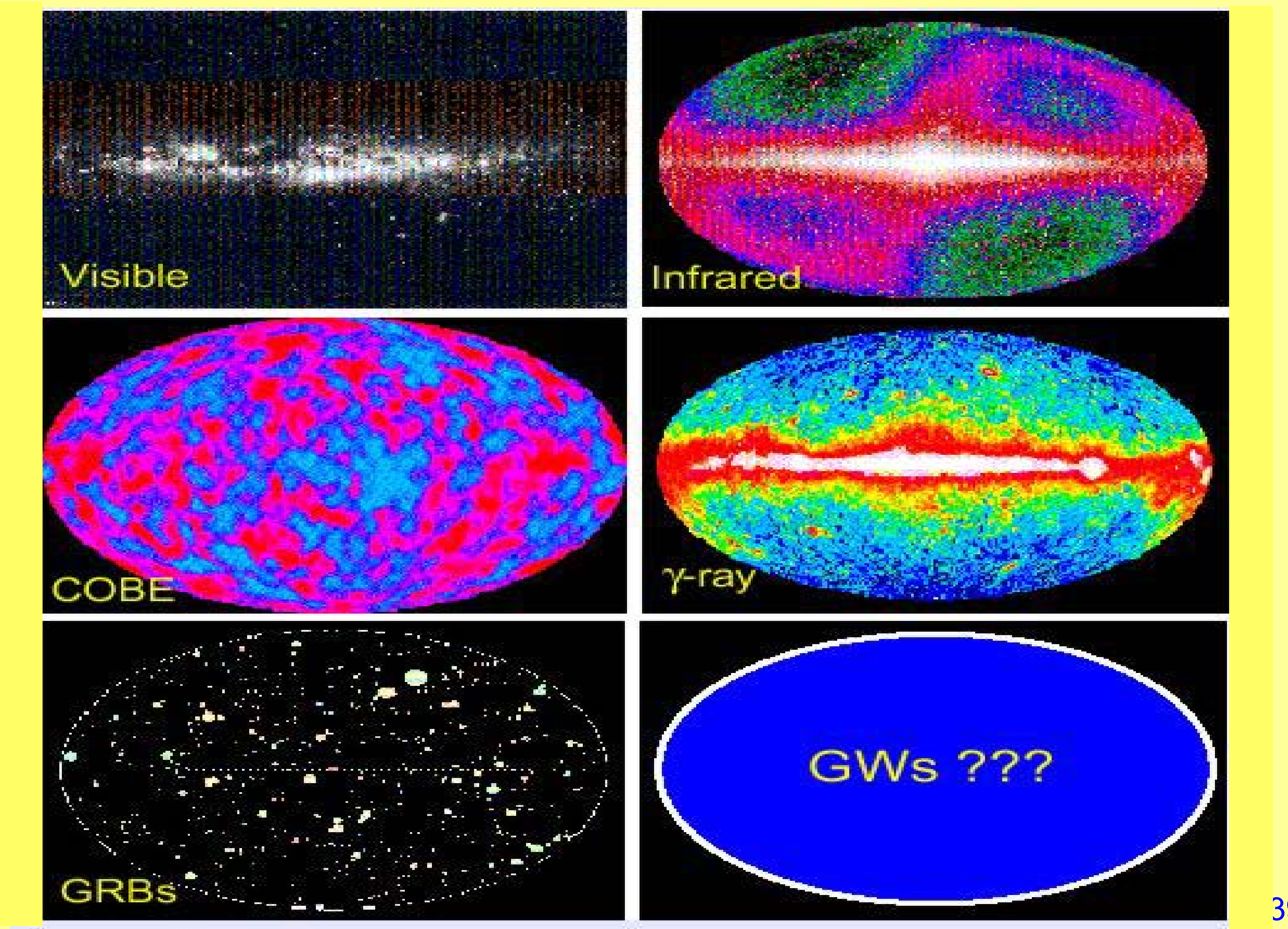
- ***Mapping the gravitational wave sky between 0.1 mHz and 1 Hz will be an exploration of astrophysical systems involving compact objects such as***
  - *Supermassive black holes* (10^5-10^7 $M_{\odot}$)
  - *Intermediate mass black holes* (10^2-10^5 $M_{\odot}$)
  - *Stellar mass black holes* (1-10^2 $M_{\odot}$)
  - *Neutron stars* (~1.4 $M_{\odot}$)
  - *White dwarfs* (O 1 $M_{\odot}$)

  . . . which are rapidly accelerated in non-spherical mass distributions, typically close binary systems
- ***Some of these objects may not radiate***
  Other unexpected objects may exist

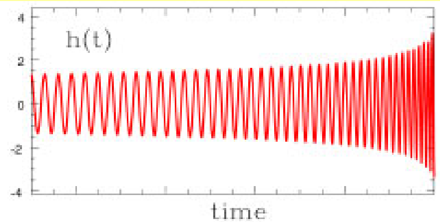
The LISA Spectrum

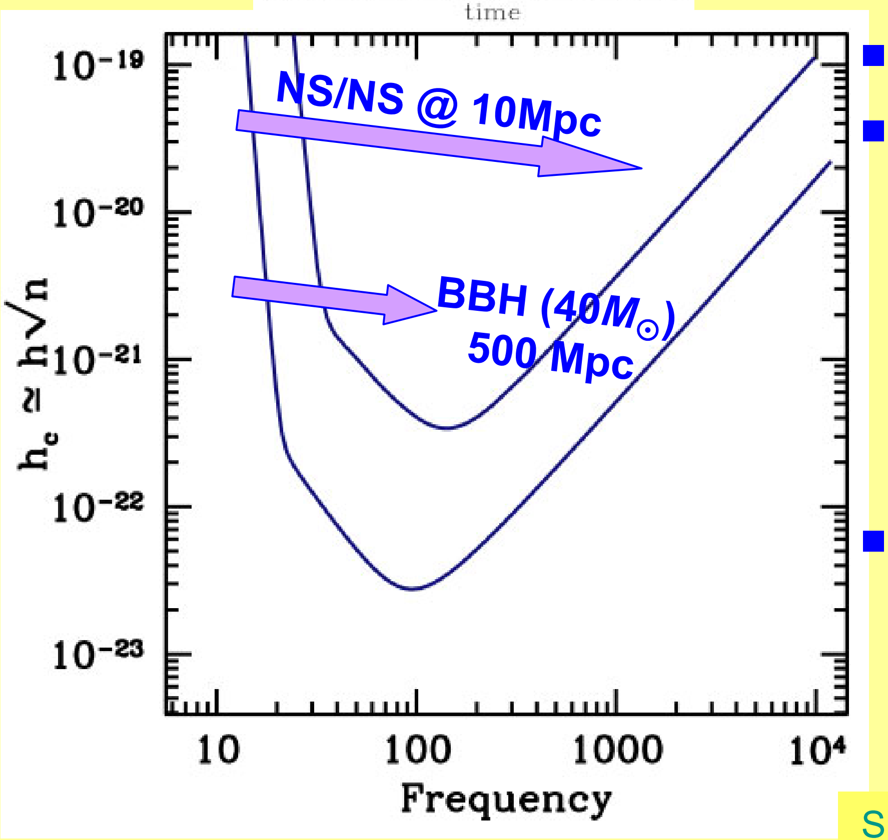


New Instrument, New Field, the Unexpected…



Binary Inspiral Sensitivity





- Waveforms from slow motion (pN) approx.
- Measure:
  - Inspiral rate → "chirp mass"
  - Relativistic effects → component masses
  - Amplitude → luminosity distance
  - Polarization (network) → inclination
  - Timing (network) → sky position
- Rates estimated from
  - Empirical estimates based on observed binaries
    - Sensitive to faint pulsar population
  - Stellar evolution/dynamics models
    - Sensitive to formation channels, stellar winds, supernova kick velocities, etc.

	LIGO	AdLIGO
Range (Mpc)	~20	~200
Event Rate (per year)	$3\times10^{-4} - 3\times10^{-1}$	2 – 1000
Event Rate (per year)	~0.5	~500

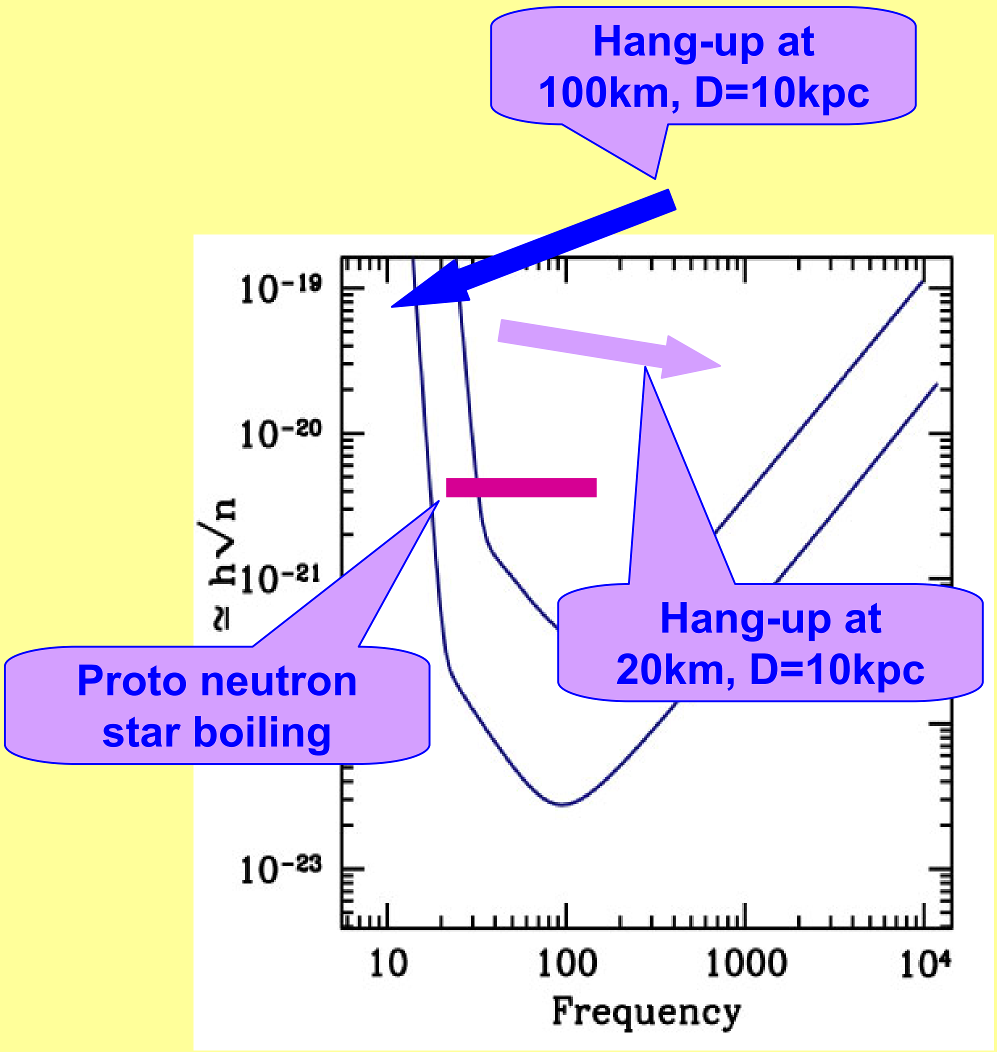
binary neutron stars

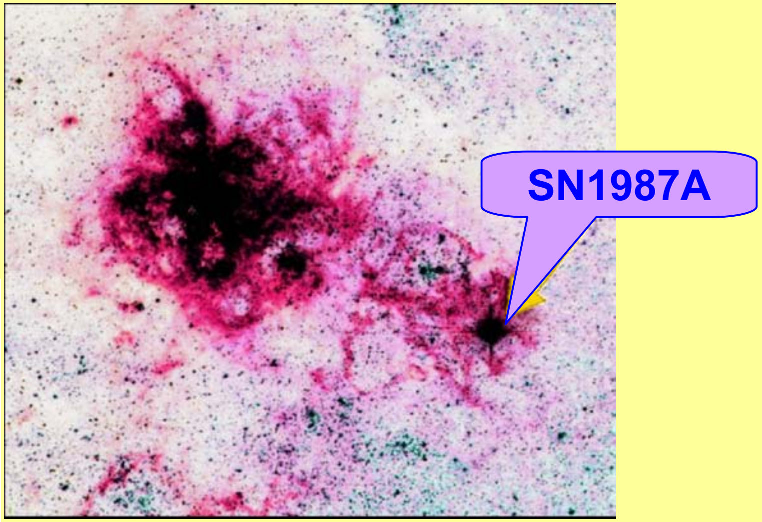
binary black holes

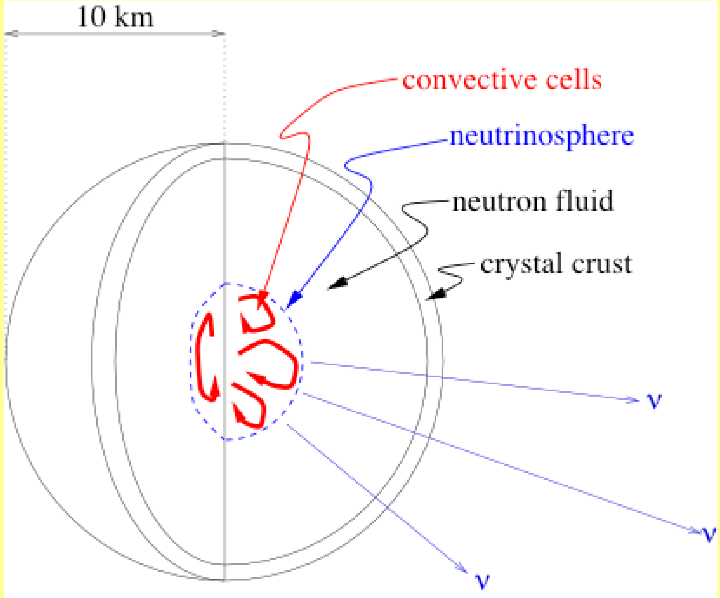
V Kalogera et al, Astrophys J **556** 340 (2000)

S Portegies Zwart, S McMillan, Astrophys J **528** L17 (2000)

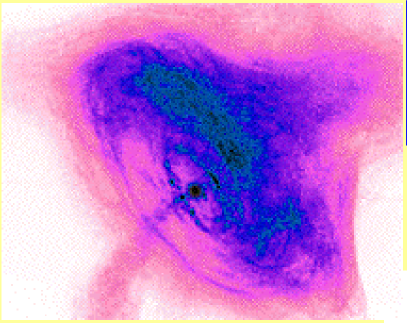
Unmodeled Burst Sources
Supernovae and Core Collapse



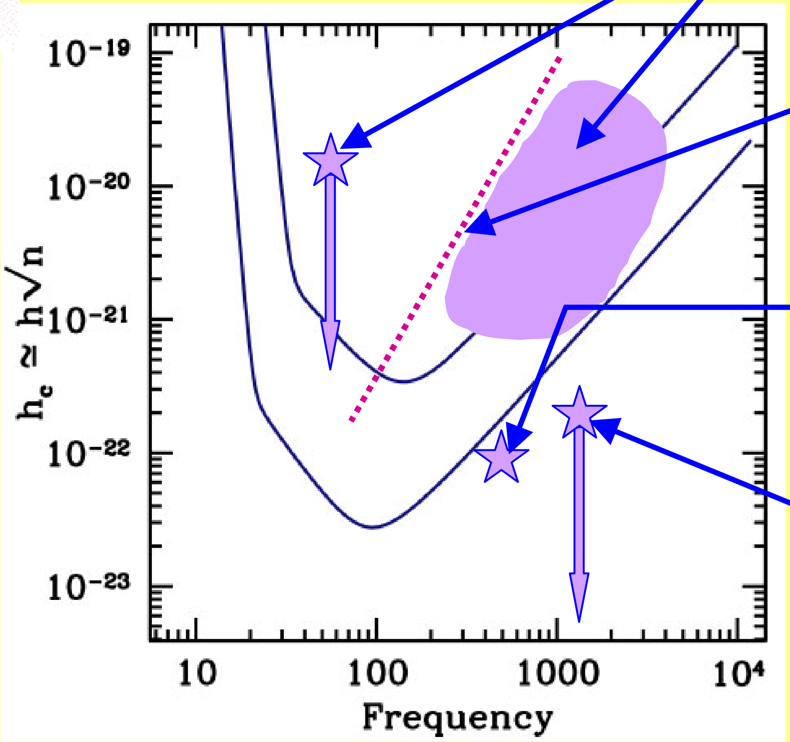




Sensitivity to Pulsars



Crab pulsar limit
(4 month observation)



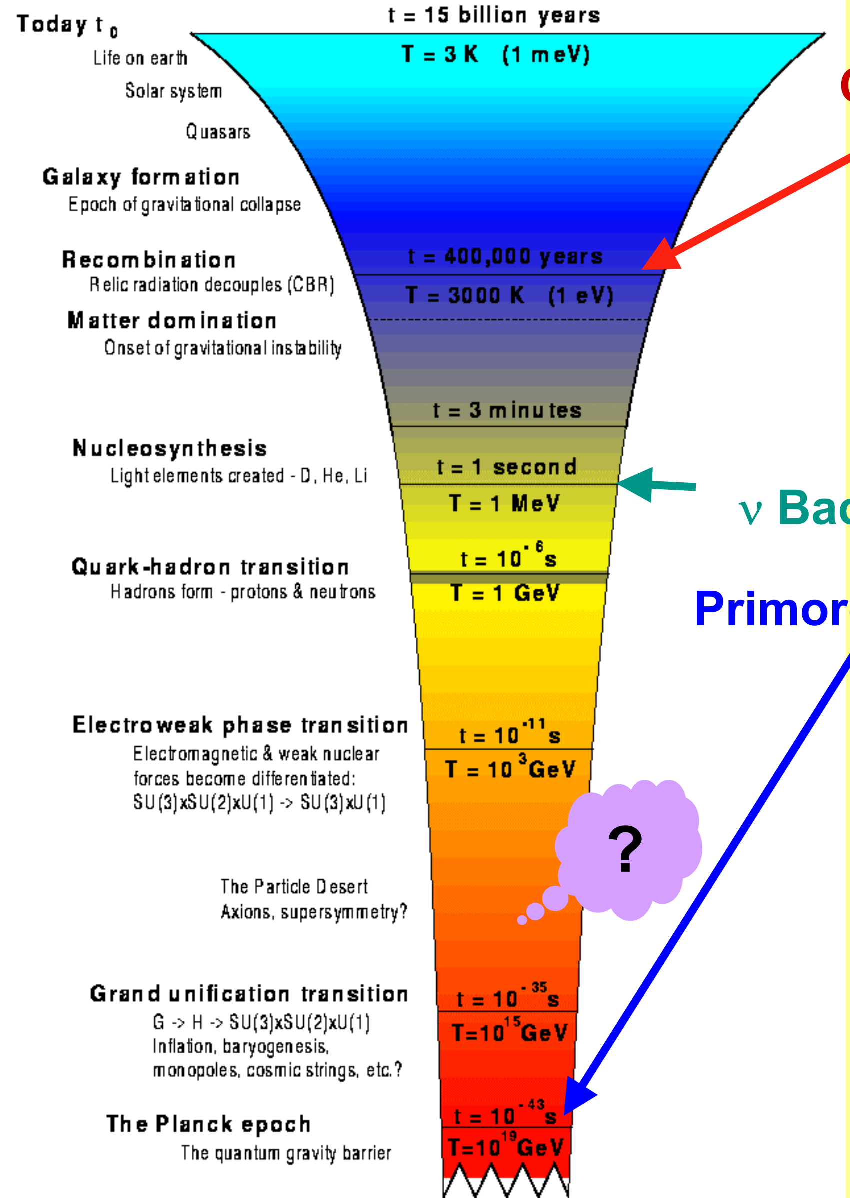
Hypothetical population of young, fast pulsars
(4 months @ 10 kpc)

Crustal strain limit
(4 months @ 10 kpc)

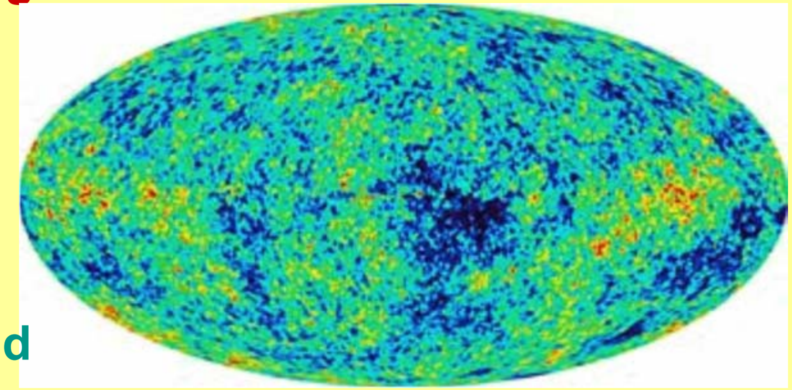
Sco X-1 to X-ray flux
(1 day)

PSR J1939+2132
(4 month observation)

Stochastic Background

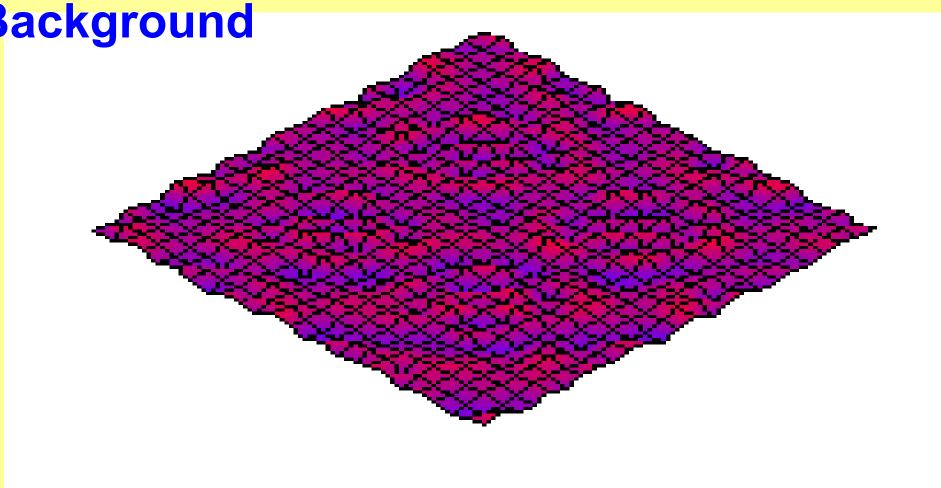


Cosmic Microwave Background

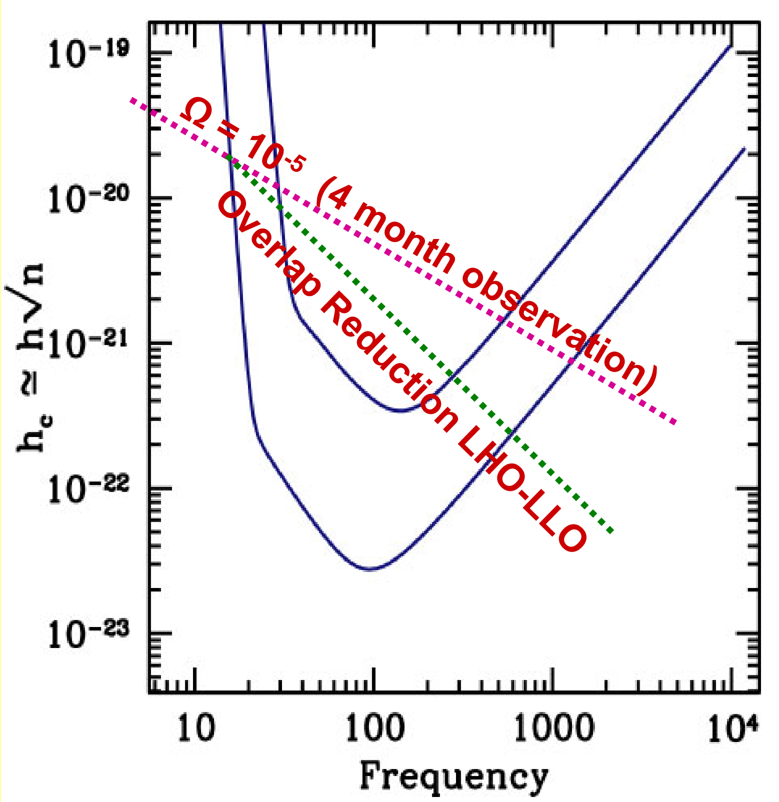


ν Background

Primordial Gravitational-Wave Background



Stochastic Background Sensitivity



- Fraction of energy density in Universe in gravitational waves:

$$\frac{\rho_{\mathrm{GW}}}{\rho_{\mathrm{critical}}} = \int \Omega_{\mathrm{GW}}(f)\, d\ln f$$

- Constraint from nucleosynthesis:

$$\int \Omega_{\mathrm{GW}}(f)\, d\ln f < 10^{-5}$$

- More recent processes may also produce stochastic backgrounds