MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

Physics 8.282 April 15, 2002

Quiz 2

Name ______________________________ **(please print)**
Last **First**

1. Work any **8** of the **10** problems - indicate clearly which 8 you want graded.
2. Take two continuous hours to work the problems.
3. All problems are worth 13 points.
4. Closed book exam; you may use two pages of notes.
5. Wherever possible, try to solve the problems using general analytic expressions. Plug in numbers only as a last step.

Time Started: Time Stopped:

Signature ______________________________

Problem	Grade	Grader
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
Total		

USEFUL CONSTANTS

Constant	cgs units		mks units	
c (speed of light)	3×10^{10}	cm/sec	3×10^{8}	m/sec
G (gravitation constant)	7×10^{-8}	dyne-cm^2/g^2	7×10^{-11}	N-m^2/kg^2
k (Boltzmann's constant)	1.4×10^{-16}	erg/K	1.4×10^{-23}	J/K
h (Planck's constant)	6.6×10^{-27}	erg-sec	6.6×10^{-34}	J-sec
m_{proton}	1.6×10^{-24}	g	1.6×10^{-27}	kg
eV (electron Volt)	1.6×10^{-12}	erg	1.6×10^{-19}	J
$M_\odot$ (solar mass)	2×10^{33}	g	2×10^{30}	kg
$L_\odot$ (solar luminosity)	4×10^{33}	erg/sec	4×10^{26}	J/sec
$R_\odot$ (solar radius)	7×10^{10}	cm	7×10^{8}	m
σ (Stefan-Boltzmann cons)	6×10^{-5}	erg/cm^2-sec-K^4	6×10^{-8}	J/m^2-sec-K^4
Å(Angstrom)	10^{-8}	cm	10^{-10}	m
km (kilometer)	10^{5}	cm	10^{3}	m
pc (parsec)	3×10^{18}	cm	3×10^{16}	m
kpc (kiloparsec)	3×10^{21}	cm	3×10^{19}	m
Mpc (megaparsec)	3×10^{24}	cm	3×10^{22}	m
year	3×10^{7}	sec	3×10^{7}	sec
day	86400	sec	86400	sec
AU	1.5×10^{13}	cm	1.5×10^{11}	m
$1'$ (arc minute)	$1/3400$	rad	$1/3400$	rad
$1''$ (arc second)	$1/200,000$	rad	$1/200,000$	rad

Problem 1

Explain *quantitatively* how, for a given astronomical object, the difference in magnitudes at two different wavelengths (e.g., $B - V$) yields a "color" for the object.

Problem 2

Describe the difference in the kind of information that can be obtained from a *spectroscopic* binary vs. a *visual, astrometric* binary. In each case, assume that *both* stars are sufficiently bright for study. With high-quality data, what orbital and stellar parameters can be determined?

Problem 3

Sketch an H–R diagram for a typical globular cluster and another H–R diagram for a typical young open cluster. Point out any interesting features that you can identify on these diagrams. Be as quantitative as possible in labeling the axes of the diagrams. Explain any differences between the two H–R diagrams.

Problem 4

Make a sketch of our Galaxy (top view and side view), including any qualitative structures contained therein and the regions that different types of objects occupy. Mark any size scales you know. If the Sun (more precisely, the local standard of rest) is orbiting the Galactic center at 230 km/sec, estimate the mass of the Galaxy interior to the Sun's orbit (in units of $M_{\odot}$). Choose whatever distance to the Galactic center you happen to know.

Problem 5

Suppose that we are centered on a distribution of a certain class of astrophysical object whose space density, n, falls off as $n = n_0(r_0/r)$, where n_0 is a constant with units of number density (objects per unit volume), r is the distance from us, and r_0 is the distance at which the density is n_0. Find the corresponding $\log(F) - \log(N)$ curve for this population, where F is the flux of the object and N is the number of such objects detected with flux $> F$. For your calculation, assume that all such objects have a fixed luminosity L_0. Useful relation: the volume of a cone of solid angle Ω is given by $\Omega \int r^2 dr$.

Problem 6

For typical spiral galaxies it is found that the rotation curves, $v(R)$, are approximately constant with radial distance, R, from the center of rotation, i.e., $v(R) = v_0 = \text{constant}$. Find $\omega(R)$, the angular velocity, in terms of v_0 and R. Find the Oort A and Oort B coefficients for a galaxy with this type of rotation curve. Express your answer for A and B in terms of $\omega(R)$. Recall that $A \equiv -\frac{1}{2}R(d\omega/dR)$ and $B \equiv A - \omega$.

Problem 7

A radio telescope tuned to the region of the 21–cm line of hydrogen is pointed in the Galactic plane at a galactic longitude l, and the Doppler spectrum of all the hydrogen within the field of view is recorded. Start with the following equation to show how such data collected from a wide range of galactic longitudes can be used to reconstruct the rotation curve of our Galaxy:

$$v_{\text{rad}} = \omega R_0 \sin(l) - \omega_0 R_0 \sin(l),$$

where v_{rad} is the radial velocity of a hydrogen cloud with respect to our local standard of rest, ω is the angular velocity of the hydrogen cloud (which is part of the general Galactic rotation) at a distance R from the Galactic center, and the subscripts "0" refer to the quantities evaluated in the solar neighborhood. Recall that the above equation was derived for certain simplifying assumptions, e.g., that the orbital motion of all Galactic plane constituents is circular. Also, take $\omega_0 R_0 = v_{\text{LSR}}$ and R_0 to be known constants.

Problem 8

A planet has a very hot atmosphere that extends to heights that are *not* necessarily small compared to its radius, R_0. Use the equation of hydrostatic equilibrium to derive the density of the atmosphere as a function of radial distance, r, from the center of the planet. Assume that the atmosphere contains a negligible mass and so does not affect gravity, and that the temperature, T, of the atmosphere is a *constant.* Take the density to be ρ_0 at the planet's surface. Note that gravity *cannot* be assumed to be constant in this problem. Take the gas to obey the ideal gas law $P = \rho kT/\mu$, where μ is the mean weight of the atmospheric gas.

Problem 9

The equation of radiative transport in a star is:

$$\frac{dT}{dr} = -\frac{3\kappa\rho L}{64\pi\sigma T^3 r^2}$$

where T, L, and ρ are the stellar temperature, luminosity, and mass density, all of which are functions of the radial distance, r. The quantity κ is the "opacity" of the stellar material which you may take to be a constant, κ_0. The quantity σ is the Stefan-Boltzmann constant. Use a dimensional analysis of this equation to find how the luminosity of a star depends on its mass, M, and constants of nature. You may use the result derived in lecture that an average temperature in the stellar interior is given by $T \simeq GM\mu/(kR)$, where R is the stellar radius, μ is the mean mass of a gas particle, and k is Boltzmann's constant.

Problem 10

The following nuclear reaction takes place in the Sun as part of the CNO cycle:

$$_6\mathrm{C}^{12} + {}_1\mathrm{H}^1 \rightarrow {}_7\mathrm{N}^{13} + \gamma$$

The atomic mass excesses of $_6\mathrm{C}^{12}$, $_1\mathrm{H}^1$, and $_7\mathrm{N}^{13}$ are 0.00 MeV, 7.29 MeV, and 5.34 MeV, respectively. How much energy is given off in this reaction?
Write down one of the nuclear reactions from the main $p-p$ chain which provides most of the energy input to the Sun.