MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

Physics 8.282 March 8, 2000

Quiz 1

Name Solutions

Last **First** **(please print)**

1. Closed book exam; you may use one page of notes.
2. Be sure to attempt all problems.
3. **Important:** Wherever possible, try to solve the problems using general analytic expressions. Plug in numbers only as a last step.

Problem	Grade	Grader
1	25	
2	25	
3	25	
4	25	
Total	100	

Problem 1 (25 points)

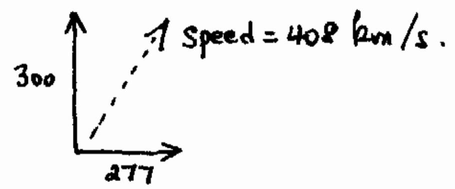
A spectrum is taken of a single star (i.e., one *not* in a binary). Among the observed spectral lines is one from oxygen whose *rest* wavelength is 5007 Å. The Doppler shifted oxygen line from this star is observed to be at a wavelength of 5012 Å. The star is also observed to have a proper motion, μ, of 1 arc second per year (which corresponds to ~1.5×10^{-13} radians per second of time). It is located at a distance of 60 pc from the Earth.

(a -19 pts) What is the 3 dimensional velocity of the star?

(b - 6 pts) What parallax would be measured for this star?

a) $v_r = \frac{\Delta\lambda}{\lambda} c = \frac{5}{5000} c = 300 \text{ km/s}$

$v_T = 1.5 \times 10^{-13} \times D = \dot{\theta} D = \mu D = 277 \text{ km/s}$

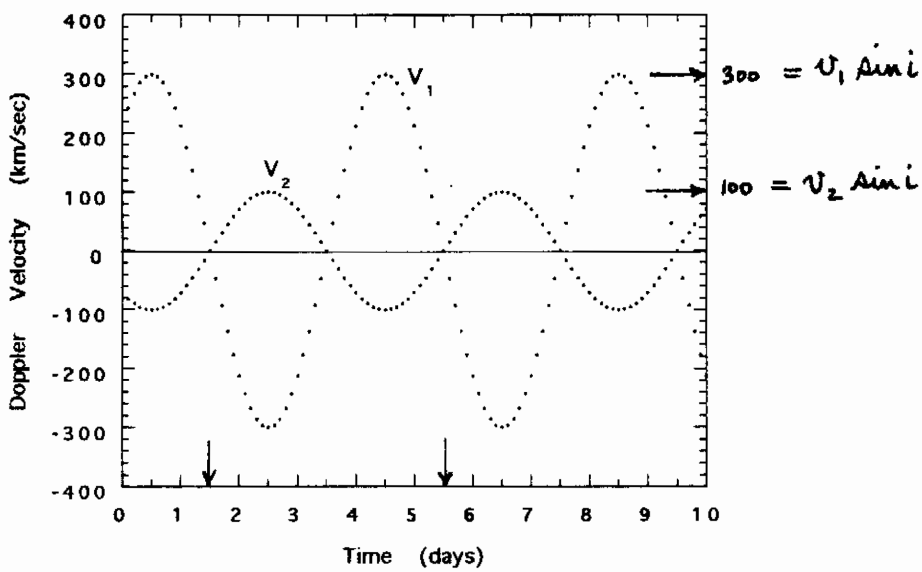


b) $\text{parallax} = \frac{1}{60}''$

Problem 2 (25 points)

The velocity curve for a double-line spectroscopic binary is shown in the sketch. The system is viewed edge on, i.e., with an inclination angle of $i = 90^o$, so that the maximum possible Doppler shifts for this system are observed.

(a - 3 pts) Find the orbital period.
(b - 3 pts) Find the orbital speed of star 1 and star 2.
(c - 3 pts) What is the mass ratio of the two stars?
(d - 16 pts) Find the masses of the two stars, preferably in solar units.



a) $P = 5.5 - 1.5 = 4$ days

b) $v_1 = 300$ km/s $v_2 = 100$ km/s (since $i = 90°$, $\sin i = 1$)

c) $m_1 v_1 = m_2 v_2$ $\frac{m_1}{m_2} = \frac{v_2}{v_1} = \frac{100}{300} = \frac{1}{3}$ So, $m_1 < m_2$

d) $\frac{G m_1 m_2}{(r_1+r_2)^2} = \frac{m_1 v_1^2}{r_1}$ $\frac{G \cancel{m_1} m_2}{r_1 (1+\frac{m_1}{m_2})^2} = \cancel{m_1} v_1^2$

From $\vec{F} = m\vec{a}$

$m_2 = \frac{v_1^2 r_1}{G} \left(1 + \frac{m_1}{m_2}\right)^2$ But $\frac{2\pi r_1}{P} = v_1$

$m_2 = \frac{v_1^2 v_1 P}{2\pi G} \left(1+\frac{m_1}{m_2}\right)^2 = \frac{v_1^3 P \left(1+\frac{1}{3}\right)^2}{2\pi G}$

$m_2 = 3.96 \times 10^{34} g = 19.8 M_\odot$

$m_1 = 1.39 \times 10^{34} g = 6.6 M_\odot$

Problem 3 (25 points)

(a - 9 pts) A candle has a power in the visual band of ~3 Watts. When this candle is placed at a distance of 3 km it has the same apparent brightness as a certain star. Assume that this star has the same luminosity as the Sun in the visual band (~10^{26} Watts). How far away is the star?

Same flux $\Rightarrow$ $$\frac{3}{(3\,\text{km})^2} = \frac{10^{26}}{d^2}$$

$$d = 10^{13} \times \frac{3}{\sqrt{3}} = 1.7 \times 10^{13}\ \text{km}$$

$$\underline{d = 0.56\ \text{pc}}$$

(b - 10 pts) Two stars have the same surface temperature. Star 1 has a radius that is 2.5 times larger than the radius of star 2. Star 1 is ten times farther away than star 2. What is the difference in apparent magnitude between the two stars?

$$m_1 = -2.5 \log\left[\frac{T_1^4 R_1^2}{d_1^2}\right] + \text{const}$$
$$m_2 = -2.5 \log\left[\frac{T_2^4 R_2^2}{d_2^2}\right] + \text{const}$$

$$m_1 - m_2 = -2.5 \log\left[R_1^2/R_2^2\right] - 2.5 \log\left[d_2^2/d_1^2\right] \qquad (\text{since } T_1 = T_2)$$
$$= -5 \log\left[R_1/R_2\right] + 5 \log\left[d_1/d_2\right]$$
$$= -5 \log 2.5 + 5 \log 10$$
$$= -2 + 5 = 3 \qquad \underline{m_1 - m_2 = 3}$$

(c - 6 pts) A certain red giant has a radius that is 500 times that of the Sun, and a temperature that is 1/2 that of the Sun's temperature. Find its bolometric (total) luminosity in units of the bolometric luminosity of the Sun.

$$L_{bol*} = 4\pi\sigma T_*^4 R_*^2$$
$$L_\odot = 4\pi\sigma T_\odot^4 R_\odot^2$$

$$L_{bol*} = \left(\frac{T_*}{T_\odot}\right)^4 \left(\frac{R_*}{R_\odot}\right)^2 L_\odot = \left(\frac{1}{2}\right)^4 (500)^2 = \underline{1.56 \times 10^4\ L_\odot}$$

Problem 4 (25 points)

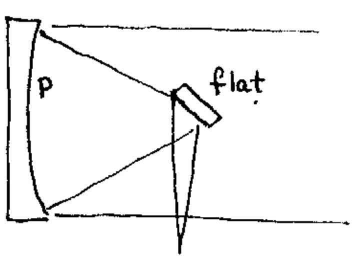
Short answer questions:

(a - 9 pts) A radio interferometer, operating at a wavelength of 1 cm, consists of 100 small dishes, each 1 m in diameter, distributed randomly within a 1 km diameter circle. What is the angular resolution of a single dish? What is the angular resolution of the interferometer array for a source directly overhead?

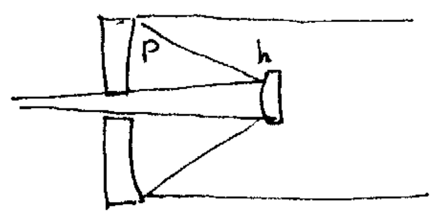
$$\theta_{\text{Single}} \cong \frac{\lambda}{D_1} = \frac{1}{100} = 10^{-2} \text{ radians} \simeq 0.^\circ 6$$

$$\theta_{\text{array}} \simeq \frac{\lambda}{D_{\text{array}}} = \frac{1}{10^5} = 10^{-5} \text{ radians} \simeq 2''$$

(b - 8 pts) Sketch the mirrors for one type of optical telescope and label what shapes their surfaces must have. Sketch a few light rays incident along the optical axis until the point where they reach the focal plane.



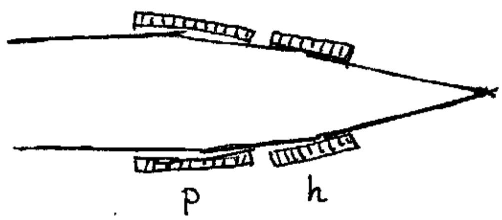
Newtonian



Cassegrain

(c - 8 pts) Briefly describe and sketch how an X-ray telescope works?

Double reflection from a combination paraboloid-hyperboloid, at grazing angles smaller than the critical angle for "total external reflection".



p = paraboloid
h = hyperboloid