MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Physics Department and Department of Earth, Atmospheric, & Planetary Sciences

Physics 8.282J – EAPS 12.402J May 22, 2003

Final Exam

Name ______________________________ **(please print)**

Last **First**

1. Work all problems.
2. Closed book exam; you may use two pages of notes.
3. Wherever possible, try to solve the problems using general analytic expressions. Plug in numbers only as a last step.

Problem	Grade	Grader
1		
2		
3		
4		
5		
6		
7		
8		
Total		

USEFUL CONSTANTS

Constant	cgs units		mks units	
c (speed of light)	3×10^{10}	cm/sec	3×10^{8}	m/sec
G (gravitation constant)	7×10^{-8}	dyne-cm^2/g^2	7×10^{-11}	N-m^2/kg^2
k (Boltzmann's constant)	1.4×10^{-16}	erg/K	1.4×10^{-23}	J/K
h (Planck's constant)	6.6×10^{-27}	erg-sec	6.6×10^{-34}	J-sec
m_{proton}	1.6×10^{-24}	g	1.6×10^{-27}	kg
eV (electron Volt)	1.6×10^{-12}	erg	1.6×10^{-19}	J
$M_\odot$ (solar mass)	2×10^{33}	g	2×10^{30}	kg
$L_\odot$ (solar luminosity)	4×10^{33}	erg/sec	4×10^{26}	J/sec
$R_\odot$ (solar radius)	7×10^{10}	cm	7×10^{8}	m
σ (Stefan-Boltzmann cons)	6×10^{-5}	erg/cm^2-sec-K^4	6×10^{-8}	J/m^2-sec-K^4
Å(Angstrom)	10^{-8}	cm	10^{-10}	m
km (kilometer)	10^{5}	cm	10^{3}	m
pc (parsec)	3×10^{18}	cm	3×10^{16}	m
kpc (kiloparsec)	3×10^{21}	cm	3×10^{19}	m
Mpc (megaparsec)	3×10^{24}	cm	3×10^{22}	m
year	3×10^{7}	sec	3×10^{7}	sec
day	86400	sec	86400	sec
AU	1.5×10^{13}	cm	1.5×10^{11}	m
$1'$ (arc minute)	2.9×10^{-4}	rad	2.9×10^{-4}	rad
$1''$ (arc second)	4.9×10^{-6}	rad	4.9×10^{-6}	rad

Problem 1 (12 points)

Jupiter's moon Io has an orbital period of 1.8 days, and an orbital radius of 4.2×10^5 km.

a. Find the mass of Jupiter, starting with $\vec{F} = m\vec{a}$. (Assume that $M_{\rm Io} \ll M_{\rm Jupiter}$.)
b. Jupiter's radius is 1/10 that of the Sun. Use this information and the mass of Jupiter found in part (a) to compare the mean density, $< \rho_J >$, of Jupiter with that of the Sun $< \rho_\odot >$. (A dimensionless ratio is all that is required.)

Problem 2 (13 points)

The figures below show H–R diagrams for the globular cluster, M3, and the open cluster - the Pleiades. (Figures from Chaisson & McMillan's book: "Astronomy Today".)

The figures can be found at the book: "AstronomyToday" by by Eric Chaisson and Steve McMillan.

a. What do these diagrams teach us about how stars evolve? Be as specific as you can.
b. As a star ascends the giant branch, its luminosity is related to the mass of its He core by the approximate relation:

$$L \simeq 2 \times 10^5 \left(\frac{M_{\rm core}}{M_\odot} \right)^6 L_\odot$$

If the luminosity comes entirely from burning H to He, compute how long it takes for the star to evolve from the near the bottom of the giant branch where its core mass is 0.2 $M_\odot$, to the tip of the giant branch when its core mass is 0.45 $M_\odot$? [Hint: Matter that has undergone nuclear burning (and has therefore contributed to the luminosity) is added to the core. Therefore a simple differential equation that describes the evolution of the core mass with time is required.]

Problem 3 (Short Answer Questions – 12 points)

a. An extrasolar Jupiter-like planet is in a circular orbit of radius 5 AU about a solar-type star, which lies at a distance of 5 pc from an observer. When the planet is at its maximum angular separation from the star as seen by the observer, what diameter optical telescope (operating above the Earth's atmosphere) would be needed so the image of the planet would lie in the first dark ring of the diffraction pattern of the stellar image?

b. Use a dimensional analysis of the equation of radiative transport

$$\frac{dT}{dr} = -\frac{3\kappa\rho L}{64\pi\sigma r^2 T^3}$$

to find a proportionality between a star's luminosity and its mass. In this expression σ is a constant, and you may also take κ to be a constant. From other dimensional analyses that we did in lecture, we found that the mean temperature of a star is given by $T \propto M/R$.

Problem 4 (Short Answer Questions – 12 points)

a. A cluster of galaxies contains 10^3 individual galaxies. Assume they all have the same mass, $M_{\rm gal}$, and are distributed uniformly over a spherical volume of radius $R_{\rm clus}$. Use the *virial theorem* to estimate the characteristic velocity, $V_{\rm gal}$, of galaxies within the cluster. First, find a general expression for $V_{\rm gal}$ in terms of $M_{\rm gal}$ and $R_{\rm clus}$, then obtain a numerical estimate for $V_{\rm gal}$ if $M_{\rm gal} = 10^{12}\ M_\odot$ and $R_{\rm clus} = 1$ Mpc.

b. Describe what the Eddington luminosity limit is. Specific what the physics and assumptions are that underlie the derivation of this limit.

Problem 5 (Short Answer Questions – 13 points)

a. A white dwarf of radius $R_{\rm WD} = 3000$ km and mass $M_{\rm WD} = 1.4 M_\odot$ collapses to become a neutron star of radius $R_{\rm NS} = 10$ km. Compute, to order of magnitude, the gravitational energy released in the collapse. In what form is most of this energy emitted?

b. A quasar is observed to have a luminosity of 10^{46} ergs $\rm s^{-1}$ (10^{39} Watts). The "central object" in the quasar is a black hole of mass $10^8\ M_\odot$. Find the rate at which mass must be accreted, $\dot{m}$, in order to produce this much luminosity. [Hint: Assume that the accretion disk, from which the radiation is emitted, has an inner radius of $3R_S$, where $R_S = 2GM_{\rm BH}/c^2$.]

Problem 6 (12 points)

The mass density in a model galaxy is given by

$$\rho(R) = \rho_0 \frac{R_0^2}{R^2}$$

where ρ_0 and R_0 are constants equal to 1 $M_\odot$ pc^{-3}, and 5 kpc, respectively, and R is the radial distance from the center.

a. Compute the mass inside a sphere of radius R centered on the galaxy.
b. Use this result to compute the velocity of stars in circular orbits at radius R. Specifically, how does the velocity depend on R?
c. Find the orbital period of stars orbiting at a radial distance of 10 kpc.

[Suggestion: Stick to either cgs or mks units throughout the problem, otherwise it's easy to get confused about units.]

Problem 7 (13 points)

a. Use Newtonian cosmology to show that in the early universe (e.g., $t = 1$ sec – 1000 years), when radiation energy density dominates over matter in governing the dynamics of the expanding universe, the scale factor, a, grows as

$$a \propto \sqrt{t} \quad ,$$

if $\Omega = 1$, i.e., the total "energy" of our test galaxy equals 0. It is useful to know that the energy density in radiation is proportional to T^4 and that, in turn, the temperature, T is proportional to the inverse of the scale factor, i.e., $T \propto 1/a$.

Note: this problem requires a formal derivation. In your work use the following definitions: $a \equiv r/r_0$ and $\dot{a} \equiv \dot{r}/r_0$, where r and $\dot{r}$ describe the distance and velocity of a test galaxy at an arbitrary time, and r_0 is its distance at the current time.

b. What is the Cosmic Microwave Background radiation? How does it originate? What was its approximate temperature when it was formed – and what is it at the current epoch?

Problem 8 (13 points)

The graph below shows a plot of the apparent magnitudes of type Ia supernovae as a function of their redshift z. Assume that type Ia supernovae are "standard candles" and have absolute magnitudes $M = -19.0$.

The graph that pertains to the "acceletation universe" comes from a paper by Saul Perlmutter et al (1998). This is the link to the graphs included in that paper.

a. Choose a representative point on the curve somewhere between $z = 0.05$ and $z = 0.2$, and find the corresponding value of apparent magnitude.
b. Convert the apparent magnitude to a distance.
c. Convert the z value to an approximate recession velocity.
d. Use your velocity–distance pair to estimate the Hubble constant, H_0, (to within a factor of 2).
e. Use the answer to part (d) to estimate the age of the universe.
f. For the case of a "flat", $\Omega_0 = \Omega_m = 1$ universe, state how the scale factor grows with time, t. There is no need to derive anything for this part if you know the answer.

[Note: $z \equiv (\lambda - \lambda_0)/\lambda_0$]