

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department
Earth, Atmospheric, and Planetary Sciences Department

Astronomy 8.282J-12.402J

March 8, 2006

Problem Set 5

Due: Friday, March 17. This problem set is not to be turned in due to the upcoming quiz; however, you are responsible for the material covered by the problems.

Reading: Chapters 11 and 12 in Zeilik & Gregory.

Reminder: Quiz #1 will be given on Monday, March 13 during the regular lecture period. The quiz will cover all lecture material up through the lecture on Friday, March 10, and the relevant parts of Chapters 1, 8, 9, and 11 in Zeilik & Gregory. You may prepare and use a single page of notes during the quiz. Be sure to bring a calculator to the exam.

Problem 1

“Derivation of the Stefan-Boltzmann and Wien Radiation Laws”

The Planck blackbody radiation law (1901) describes the electromagnetic power emitted per unit area per unit wavelength from the surface of a “black body” (a surface that absorbs all radiation incident upon it) at a temperature T . There are numerous ways to derive this fundamental radiation law from first principles using statistical and quantum physics. We will not attempt the derivation here; we merely cite the result for the Planck spectrum:

$$B(\lambda) = \frac{2\pi c^2 h}{\lambda^5 [e^{hc/\lambda kT} - 1]}$$

where h is Planck’s constant (6.625×10^{-27} ergs sec), c is the speed of light, k is Boltzmann’s constant (1.380×10^{-16} ergs deg $^{-1}$), λ is the wavelength of the radiation, and T is the temperature in degrees K . The dimensions of $B(\lambda)$ are “energy per unit area, per unit time, per unit wavelength interval”. Very nice plots of the Planck radiation law as a function of frequency (instead of λ) are presented in Figure 8-14 (page 172) of Zeilik & Gregory.

A. Derive the Stefan-Boltzmann law ($F = \sigma T^4$) which relates the total power per unit area emitted by a black body, F , to its temperature, T (see equation 8-40 in Z&G).

Suggested procedure:

i. Integrate the Planck function over all wavelengths. To do this, first extract the constants from the integral in such a way that only a function of the dimensionless variable $x = \lambda kT/hc$ remains inside the integral. The remaining dimensionless integral:

$$\int_0^\infty \frac{dx}{x^5 [e^{1/x} - 1]} = \int_0^\infty \frac{y^3 dy}{[e^y - 1]}$$

can be done analytically and yields: $\pi^4/15$ (you do *not* need to prove this).

- ii. Find the constant σ in terms of fundamental constants of Nature.
 - iii. Evaluate the numerical value of σ (in either cgs or mks units) and compare to the known answer given in the book.
- B. Derive the Wien displacement law which relates the wavelength of the radiation at the peak of the Planck function to the temperature T ($\lambda_{\max}T = 0.29 \text{ cm K}$; see equation 8-39 in Z&G).
- i. Find the maximum of $B(\lambda)$ by differentiating with respect to λ and setting the result equal to zero.
 - ii. If you are on the right track, you will end up with a non-linear equation of the form: $5(1 - e^{-y}) - y = 0$. Solve this with your calculator using a trial and error method, or write a program to solve by Newton's method, or use the "solve" button on some calculators. (The answer is $y \simeq 4.965$). If you do not want to spend time solving this equation, simply verify that the answer is correct.
 - iii. Find the relation between the temperature and the wavelength at the peak of the black body spectrum.
 - iv. Prove that measurements of a given blackbody at an arbitrary (unknown) distance at two different wavelengths, e.g., $B(\lambda_1)$ and $B(\lambda_2)$ are sufficient to uniquely determine the temperature, T , of that blackbody.
 - v. The surface of the Sun acts like a fairly good black body (even though it hardly looks black). If the surface temperature of the Sun is $5800K$, find the peak wavelength of the black body radiation.

Problem 2

"Short Answer Questions on Magnitudes"

Zeilik & Gregory problems 3, 4, 8 a&c, and 10, Chapter 11, page 233.

Problem 3

"Moving Cluster Method - the Hyades Cluster"

Below is a Table of measurements of 8 representative stars in the Hyades cluster. The nine columns in the Table are: the star number, right ascension (α), declination (δ), stellar magnitude, proper motion in the directions of right ascension and declination ($\mu_\alpha \cos \delta$ and μ_δ), respectively, the total proper motion (μ), the angle from the radiant (θ), and the radial velocity (v_r). The units of proper motion are 0.001 seconds of arc (along a great circle) per year. Find the mean distance to the Hyades cluster and its mean space velocity.

Suggested procedure:

- a. Start with the plot of the Hyades cluster given below. The positions of the stars in the table as well as many others are plotted with a small filled circle. Also plotted for each star is a small vector that represents its proper motion. The direction is given by the two components of proper motion ($\mu_\alpha \cos \delta$ and μ_δ) and its length is proportional to the magnitude of the proper motion (μ).

Project enough of the vectors to find an approximate common intersection point (the convergence point or *radiant*). Find the coordinates (α and δ) of the “center of gravity” of the intersecting lines.

b. For stars in the Table numbered 6, 104, and 129 verify that the angular distance to the *radiant* θ is close to the value given in column 8 of the Table. An approximate expression for the angular separation of two points on a sphere is $\Delta\theta \simeq (\Delta\delta^2 + \Delta\alpha^2 \cos^2 \delta)^{1/2}$. Also note that one hour of right ascension is equal to 15 degrees in angle (24 hours equals 360 degrees). For this problem, an accuracy of 1 degree is adequate.

c. Find the distance from the Earth to any three stars in the Table using the expression

$$d = \frac{v_r \tan \theta}{4.74\mu} \text{pc},$$

where v_r is in km/s, μ is the total proper motion in seconds of arc per year, and θ is the angle from the star to the *radiant*. (Use the values given in the Table for v_r , μ , and θ .)

d. Find the space velocity of the three stars in part (c).

e. From the results of parts (c) and (d) find an average distance to the cluster and its average space velocity.

(Read section 19.3, pages 383-385 in Z&G for further details.)

Table: Some data about Hyades stars

Nr.	α	δ	m_v	$\mu_\alpha \cos \delta$	μ_δ	μ	θ	V_{rad}
				(unit 0".001/a)				km/sec
6	3 ^h 50 ^m 15 ^s	17° 10' 47"	5.96	149 ± 2	- 28 ± 2	151	37°.5	31.6 ± 0.6
14	4 8 40	5 23 40	5.71	152 ± 2	+ 10 ± 2	152	32 .5	35.8 ± 2.6
33	4 17 46	14 58 38	5.27	112 ± 2	- 23 ± 2	114	30 .5	36.1 ± 0.8
74	4 25 55	17 10 35	8.2	106 ± 5	- 46 ± 6	115	29 .0	40.5 ± 1.4
104	4 35 22	12 24 44	4.30	103 ± 3	- 11 ± 2	103	26 .0	44.4 ± 2.1
112	4 43 15	11 36 57	5.43	74 ± 4	- 4 ± 3	74	24 .0	38.2 ± 1.4
129	5 0 6	21 31 13	4.70	68 ± 1	- 42 ± 1	79	23 .5	42.5 ± 1.4
131	5 6 37	27 58 7	6.1	62 ± 4	- 70 ± 3	94	26 .5	41.3 ± 0.9

Problem 4

“Binary Orbit 1”

Zeilik & Gregory Problem 3, Chapter 12, page 249.

(Note: An orbital inclination of zero degrees indicates that we are viewing the binary system directly along its angular momentum axis or, in other words, the orbital plane lies in the plane of the sky.)

Problem 5

“Binary Orbit 2”

Zeilik & Gregory Problem 4, Chapter 12, page 249.

Problem 6

“Binary Orbit 3”

Zeilik & Gregory Problem 5, Chapter 12, page 249.

Problem 7

“Binary Orbit 4”

Zeilik & Gregory Problem 8, Chapter 12, page 249.

HYADES STAR CLUSTER

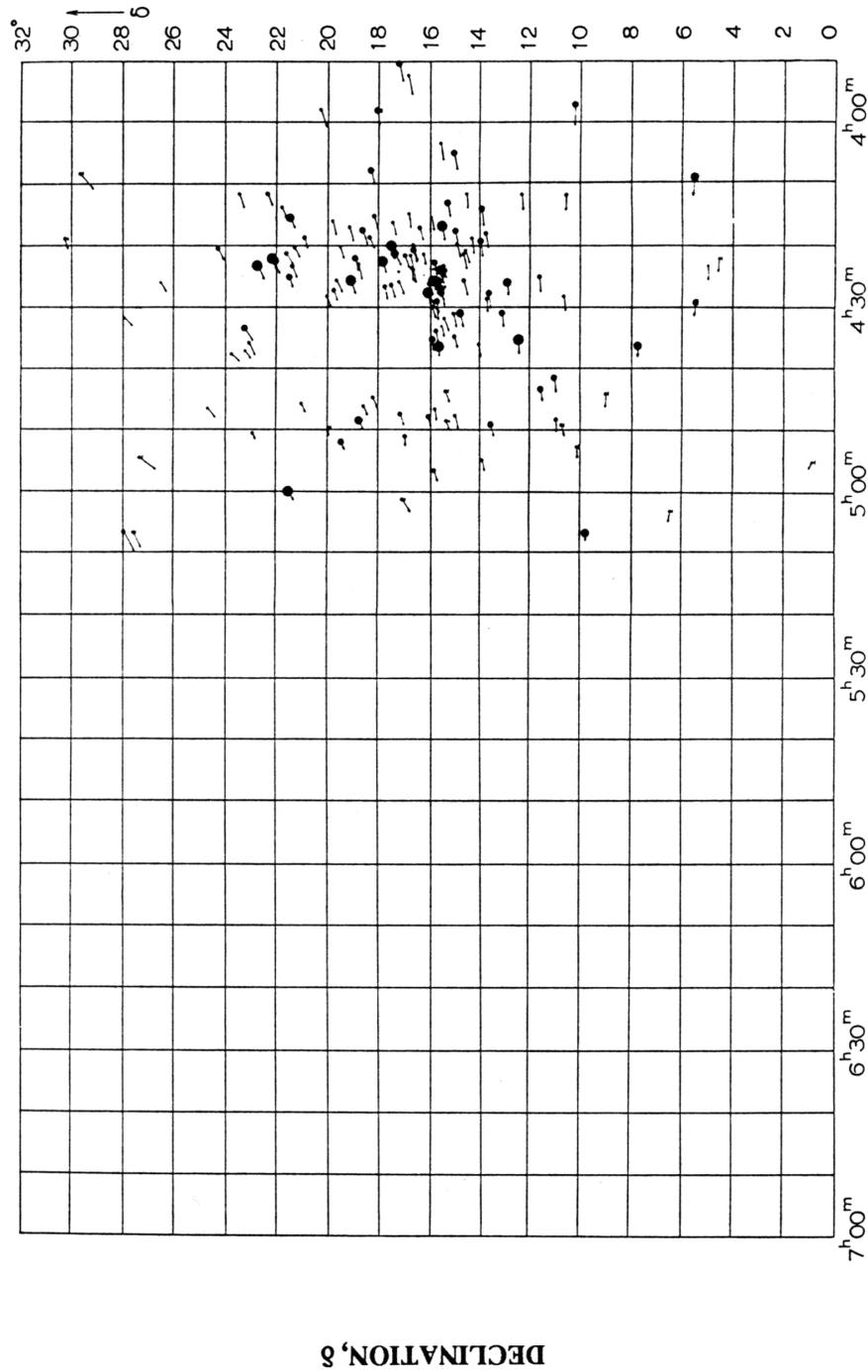


Fig. 67. Proper Motions of the Hyades. After VAN BUUREN, H.: 1952, *Bull. Astron. Inst. Neth.* 11, 392.

RIGHT ASCENSION, α