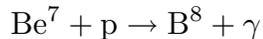


Due 11:04 am MONDAY 10 April 2006

- Reading: Clayton §4.3 through equation (4-62). Hansen & Kawaler §§6.3-5; Bohm-Vitense, Volume 2, Chapters 5 (Eddington-Barbier), 6 (Temperature Stratification) and 10 (Line Formation) as needed.

1. The solar neutrinos observed in the Homestake Mine experiment came from the PP III branch of the proton-proton chain. The limiting reaction is



which is followed almost immediately by the emission of a positron and a neutrino.

- a) Take the central temperature of the Sun to be  $1.5 \times 10^7 \text{K}$ , so that  $kT = 1.293 \text{keV}$ . Calculate by how much (expressed as a percent or as a factor) the central temperature must drop to cause the rate of PP III neutrinos to drop by a factor of 3, which is the observed shortfall. *Hint:* Calculate the derivative of the natural log of the rate with respect to temperature and find the  $dT$  which changes the log of the rate by an appropriate amount.
  - b) In the last problem set we adopted a scaling relation for the energy generation equation of the form  $L = C' M \left(\frac{M}{R^3}\right) T_c^n$ . While our expression for  $\langle \sigma v \rangle$  is clearly not a power law of temperature, it can be locally approximated as such by computing the value of  $d \ln(\langle \sigma v \rangle) / d \ln T$  at any point to give a effective exponent  $n$ . What is the effective  $n$  for part a)? This is similar to Clayton's problem 4-14.
2. In class we derived the functional dependence of photospheric temperature  $T_p$  on mass and radius for fully convective stars. We found an accidental cancellation in the  $R$  exponents, giving an amazingly weak dependence of  $T_p$  on  $R$ , which we offered as an explanation for the nearly vertical "Hyashi track" to which pre-main sequence stars, giants and asymptotic giants all adhere. For the sake of brevity we did not keep track of constants –  $G, k, \xi_1, m_p$  and even  $\mu$ .
    - a) Reconstruct the argument retaining the multiplicative constants, i.e. find an expression for  $T_p$  in terms of  $M$  and  $R$  appropriate to a fully convective star. Recall that we started with an expression for the photospheric pressure,  $P_p = g/\kappa_{H^-}$  where  $g = GM/R^2$  and an approximate expression for the  $H^-$  opacity,  $\kappa_{H^-} = 2.5 \times 10^{-31} (Z/0.02) \rho^{1/2} T^9 \text{cm}^2/\text{gm}$ . We used the fact that the polytropic index for a fully convective star is  $n=1.5$ , and that for such a star  $(\rho_c/\rho_p) = (T_c/T_p)^{3/2}$ . We also used our expression for the central temperature  $T_c$  in terms of  $G, M, R, \mu$  and  $\xi_1$ .
    - b) Evaluate this expression for a star of solar composition with mass  $M_\odot$  and radius  $100R_\odot$ .
  3. The Eddington-Barbier Approximation:
    - a) Show that the mean intensity seen by a distant observer (averaged over the disk of a star) is given by  $2 \int_0^{\pi/2} I_\nu(0, \theta) \cos \theta \sin \theta d\theta$ .
    - b) Show the flux,  $F_\nu$ , emerging from a unit area of a star, is given by the expres-

sion  $\int_0^{\pi/2} I_\nu(0, \theta) \cos \theta \sin \theta d\theta$ .

- c) In class we derived the following approximate expression for the monochromatic specific intensity at the surface of a star:

$$I_\nu(0, \theta) \approx B_\nu[T(\tau^*)] + (\cos \theta - \tau^*) \left. \frac{dB_\nu}{d\tau} \right|_{\tau^*} + \dots$$

This approximation, obtained by expanding  $B_\nu$  in a Taylor series about  $\tau^*$ , is called the Eddington-Barbier approximation. Evaluate the integrals in parts a) and b), find the value of  $\tau^*$  for which the coefficient of  $dB_\nu/d\tau$  is zero, and substitute this value in the first term.

4. Suppose that the same star is eclipsed by a dark companion, with a diameter a factor of  $\sqrt{3}/2$  smaller.
- If the orbital inclination is  $90^\circ$ , for what value of  $\tau^*$  does the coefficient of  $dB_\nu/d\tau$  vanish (after integrating over that part of the star which is visible) at mid-eclipse?
  - Using the crude gray atmosphere presented in class, what is the ratio of the temperature observed in mid-eclipse to the temperature when not in eclipse?
  - What is the ratio of the flux (integrated over frequency) of the eclipsed star to that of the uneclipsed star?