6. Time-dependent perturbation theory & applications to radiation

So far, focused on H independent of t.

To solve:

- Diagonalize H $\qquad H|n\rangle = E_n|n\rangle$
- write $|\psi(t)\rangle = \sum c_n(t)\,|n\rangle$
- $|\psi(t)\rangle = e^{-iHt/\hbar}|\psi(0)\rangle = \sum_n e^{-iE_nt/\hbar}c_n(0)\,|n\rangle.$

In principle, this formalism [describes any closed QM system.]

[can be very complicated in practice — e.g. multi-spin-1/2, many atoms, ...]

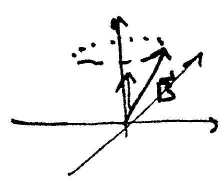
[- does not describe interaction of system with external phenomena]

In many situations, want to isolate a small system described by H_0, describe interactions w/ environment through $V(t)$ (time-dependent)

Examples:

a) spin magnetic resonance

put spin-½ particle ↑ in time-dependent B-field



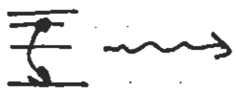
spin precesses around B-field classically...

b) Atom in external EM radiation field: absorbtion / stimulated emission

Phenomena a), b) can be understood by coupling quantum system to a classical EM field (semiclassical approach)

E not conserved since $H(t) = H_0 + V(t)$ is time-dependent.

Also want to consider

c) spontaneous emission 

- For this need to quantize EM field: Quantum field theory.

We will mostly use semiclassical approach, touch on field quantization.

6.1 Time-dependent potentials

Recall the Interaction Picture

$$H = \underbrace{H_0}_{\text{time-independent}} + \underbrace{V(t)}_{\text{time-dependent}}$$

$$|\psi(t)\rangle_I = e^{iH_0 t/\hbar}\, |\psi(t)\rangle_S \qquad \left[\,|\psi(0)\rangle_I = |\psi(0)\rangle_S\,\right]$$

$$A_I = e^{iH_0 t/\hbar}\, A_S\, e^{-iH_0 t/\hbar}$$

[like Heisenberg, but only pull out H_0 dependence]

EOM
$$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle_I = V_I(t) |\psi(t)\rangle_I$$

[V_I as in Schrödinger picture]

$$\frac{dA_I}{dt} = \frac{1}{i\hbar} [A_I, H_0] + (\dot{A})_I$$

$$(\dot{A})_I = e^{\frac{i}{\hbar}H_0 t} \dot{A}_{(s)} e^{-\frac{i}{\hbar}H_0 t}$$

[H_0 as in Heisenberg picture]

[Compare with Heisenberg picture: $i\hbar \frac{\partial}{\partial t} |\psi\rangle_H = 0$, $\frac{dA_H}{dt} = \frac{1}{i\hbar}[A_H, H] + (\dot{A})_H$]

Expand $|\psi(t)\rangle_I$ using basis of ev's of H_0

$$H_0 |n\rangle = E_n |n\rangle$$

$$|\psi(t)\rangle_I = \sum c_n(t) |n\rangle$$

EOM $\Rightarrow$

$$i\hbar \frac{\partial}{\partial t} \langle n|\psi(t)\rangle_I = \sum_m \langle n| \overbrace{V_I(t)}^{e^{\frac{i}{\hbar}E_n t} V_s(t) e^{-\frac{i}{\hbar}E_m t}} |m\rangle\langle m|\psi(t)\rangle_I$$

$$\boxed{i\hbar \dot{c}_n(t) = \sum_m V_{nm}(t) e^{i\omega_{nm} t} c_m(t)}$$

where

$$V_{nm}(t) = \langle n| V_s(t) |m\rangle$$

$$\omega_{nm} = \frac{E_n - E_m}{\hbar} = -\omega_{mn}$$

Coupled 1st order diff. eq.'s describe time evolution.

[exact description]