[SQUEAKING]

[RUSTLING]

[CLICKING]

HONG LIU: OK, great. So now let's talk about QED. And OK. So let's first talk about QED. And we have talked about the Maxwell action. So here is the Lagrangian density.

We have F mu nu minus J mu A mu, OK? And J mu should be a conserved current because for the consistency of the Maxwell equation, OK? Remember-- J mu has to be conserved. So now, the question is what provides this J mu, OK?

So now, let's imagine. Now, let's introduce some other fields. Now, imagine we have some Dirac fermions. We have some fermionic field. Same with the Lagrangian. OK. So this is the Lagrangian for the Dirac fermion.

And then we discussed that for Dirac fermion-- and then there's a conserved current because there's a global symmetry. There's a U1 symmetry corresponding to psi-- goes to exponential i alpha psi with alpha to be a constant. OK?

And then this leads to a Noether current-- J mu, which is conserved. OK? So now, if we want to couple the fermions to the Maxwell fields, then it's natural we identify this J mu with this J mu, OK? Because here we need the conserved current. And here we have a conserved current, OK?

So it's natural to write down the theory. Then this J mu is just replaced by this J mu. That's why I use the same notation. So then that gives us the Lagrangian for QED. We have the Maxwell, which is the free photon. And then we have the free fermion.

But then the fermion and the photon are coupled together through this, OK? So now, this is a cubic term, OK? So now, this is an interaction term, OK? So now, this is an interaction term. And this e can play two roles, OK?

It can be considered as a coupling constant-- essentially, the e, which just determines the coupling strength between the psi and the A, OK? If e is bigger, then of course, the coupling is stronger, OK? And the e can also be considered-- so it plays a dual role. One role is you consider as a coupling. And the second role is you can consider as a unit of charge.

OK? A unit of charge. Because when we talk about the Noether current, this is conserved. You can multiply this by an arbitrary number. This is still conserved, OK? This-- still conserved. And here it just means that the unit of this current is given by e, OK? So the e plays two roles here, OK?

And then of course this e is just our standard electric charge. So if you take the psi to be electron, then this will be standard electric charge for the electron, and then this will be a theory which governs the interaction between the electron and the photon, OK? Any questions on this? Yes?

AUDIENCE: How come different particles still have some multiple of the same charge? You might think that you could set the coupling constant to any arbitrary value.

HONG LIU: Sorry?

AUDIENCE: How come different particles have charges that are multiples and different multiples--

HONG LIU: Yeah, yeah.

AUDIENCE: I think in this framework, you can set the coupling constant to whatever you like.

HONG LIU: That's right. That's right. Yeah, yeah. So in this framework, indeed, this e can be arbitrary. And then different particles can have different charge. For example, if we couple this to electron, this will be e e. And then if we couple this to muon-- so in principle, we have some e tilde, which does not have to be the same as this e.

Indeed. And it's a highly unusual feature. In nature, what we observe-- the charge seems to be quantized. There seems to be multiple of some charge. And that cannot be explained using this framework.

It has to be explained using some other framework, sometimes called the grand unified theory-- can be used to explain that. Yeah. Other questions?

AUDIENCE: So I'm confused. In this middle section, this J mu is the conserved current from the U1--

HONG LIU: Yeah.

AUDIENCE: What was the analogy you were making? That it can do-- you could take it to do the same J mu for the Maxwell and Lagrangian?

HONG LIU: Yeah. Just the natural implication.

AUDIENCE: But this Lagrangian doesn't have this U1 symmetry for this fermionic field because it doesn't even exist, right?

HONG LIU: Yeah, yeah. Yeah, yeah. So no, I coupled them.

AUDIENCE: OK. So you're just taking inspiration now to add both of them--

HONG LIU: Yeah, yeah, yeah. Really, that's what we do here. We add both of them. Yeah. Yeah, so we have a theory with a photon and fermion. And now, their coupling is through this J. And this J is given by this one. Yeah. OK? Other questions?

OK, good. Good. So we can also slightly rewrite this Lagrangian in a somewhat different way. OK? So QED can also be written in the following way, OK? So we can also rewrite this as 1/4 F mu nu. So this part, we don't change.

But we can actually combine these two terms together in the following way. We write it as i psi bar, and then gamma mu. I introduce a new derivative called D mu minus m psi. And this D mu-- capital D mu psi is equal to partial mu psi minus ie A mu psi, OK?

So this is the same as that because the first term here just gives you this term. And the second term-- this e A mu psi term-- just gives you this e A mu psi bar. Yeah. It just gives you that, OK? And so mathematically, these two are just completely equivalent.

I just slightly rewrote it. But writing this way actually makes one new property of this theory manifest, OK? So this theory-- so we know that this theory has a gauge symmetry. But this theory only has a global symmetry, OK?

This theory only has a global symmetry. This alpha must be a constant, OK? So now, when we combine together, and now this new Lagrangian actually has a generalized gauge symmetry, OK? So L max-- so L QED turns out to be invariant under the following.

So still, you take A mu. Goes to A mu plus that partial mu lambda x. And then now, you transform psi also by a local transformation. You transform it by exponential i. So now, this e is different, OK? So now, you can transform psi by, actually, a local phase.

Actually now, it can depend-- this arbitrary function lambda. So you can show that actually, this QED is invariant under this transformation, OK? So if I call this A mu prime-- if I call this psi prime x-- OK? So you can check yourself.

With this transformation, this D mu psi-- so this D mu prime, psi prime-- OK? So D mu prime is obtained by this D mu. You replace the A mu by A mu prime. And then here you can show that this is equal to exponential i e lambda x D mu psi, OK?

So it turns out, in this combination, so when you make such a local transformation on psi, when you take a derivative, then you get an extra term, OK? You get the extra term from taking the derivative on this lambda. But when you have this combination with A mu-- but A mu also transforms with additional lambda term-- partial mu lambda term.

And then these two terms just cancel, OK? So in the end, this D prime mu-- D prime, mu prime-- actually transforms in a very simple way, OK? It transforms in a very simple way. Yes?

AUDIENCE: Is D mu prime any different than just D mu [INAUDIBLE]? Are we changing things out [INAUDIBLE]?

HONG LIU: Yeah, yeah. We replaced the A mu by A mu prime.

AUDIENCE: Oh.

HONG LIU: Yeah, yeah. Yeah. So this tells you-- and then this transforms very simply. And then this transforms very simply. And psi bar-- there's no derivative here. Then this, straightforward, just transforms as the exponential minus e.

And then we conclude. OK? So we conclude that L QED is invariant under this transformation, OK? Indeed, it's invariant. OK. So we call it gauge invariant. So now, this is the generalized gauge symmetry. Now, you also transform the fermions. You also transform the fermions.

OK. Any questions on this? So now, we can turn it around. So let us review our logic. So we first start with the Maxwell theory. So the Maxwell theory-- there's a J here, which you need to provide. And then from the fermionic theory, you have another J.

And then we just combine these two theories by replacing that J naturally from the fermions. But it turns out, when we combine them together, this theory actually has a generalized gauge symmetry. You also now can transform psi locally, OK? You can transform psi locally with this nice structure, OK?

So this theory has this nice structure. And then now, you can transform psi also locally. So we can also turn this around. So we can say-- so if we call this thing to be star-- so if we require the theory of A mu and psi to be invariant under star, and then that uniquely leads to-- maybe not uniquely.

That leads to the interaction of the form J mu A mu, OK? It just leads to this kind of interaction. OK? So we can turn it around. We say, OK. Using a different logic, we want to couple the photon and the psi together. We want to couple A mu and psi together.

And we know that A mu previously has a gauge symmetry. And now, I want to generalize that gauge symmetry to include the fermions. And then I require the full theory to be invariant under this symmetry. And this requirement then requires that the interaction between them must have this form, OK?

Yeah. Let me write more explicitly the interaction of the form. Psi bar gamma mu psi A mu, OK? So that will lead to this kind of interaction. So the reason-- even though this is just rephrase of what we just did, this rephrase is actually powerful.

OK? Here it means that when we impose this gauge symmetry, we actually can deduce the interactions. Deduce the interactions, OK? We can actually deduce the interactions. We can fix the interactions by requiring certain kinds of symmetries. Yes?

AUDIENCE: Is there a natural way to think about what would motivate you to assert this local gauge symmetry?

HONG LIU: Yeah. That's what motivates me. Maxwell theory-- you already have a gauge symmetry. And when I couple the fermion, I just want to generalize it. OK. Yes?

AUDIENCE: I guess, instead of a question, I guess, why is it that we know that it should be a scalar basis? And why couldn't it be anything else?

HONG LIU: Sorry?

AUDIENCE: I guess why should we know-- if you start from the gauge symmetry going to the interactions logic, then why should we know that it should be a scalar function, that it does all those [INAUDIBLE]? Why couldn't it be the [INAUDIBLE]?

HONG LIU: Because we already have this, right? We just want to generalize this. The Maxwell theory-- we already have this. We just need to generalize this to fermion.

And then there's a natural generalization because the fermion is already invariant under the global phase symmetry. We just need to generalize in the way so to make it local. OK. Yes?

AUDIENCE: So in quantum mechanics, when we do a single particle in an electromagnetic field, the gauge transformation on the wave function is the same. Is there a reason why that's--

HONG LIU: Yeah, yeah, yeah. It's the same. Yeah, yeah. There is a reason. Yeah. The reason is that when you go to non relativistic case, this reduces just to one particle-- quantum mechanics. So one particle, quantum mechanics should also have this kind of symmetry. Yeah.

Good. So this is, in fact, not just a reinterpretation of what we are looking at so far. This is actually a very deep dynamical principle, OK? So this is, in fact, a dynamical-- a deep-- let me emphasize that this is a deep, dynamical principle.

So essentially, you have gauge symmetries, or in other words, local symmetries. Can be used to determine interactions, OK? OK. Or in other words, all interactions in nature-- turns out, they are related to some gauge symmetries.

So this applies to all interactions in nature. So this principle applies to all interactions in nature, OK? So here we see, in the electromagnetic interactions, it turns out the same thing happens for the weak interaction.

Same thing happens for the strong interaction. And the same thing happens for gravity. They all can be formulated as a consequence of some local symmetries. And why is that the case? We don't really know. We don't really understand why, somehow, this dynamical principle should be there.

But this is just a fact that all our fundamental interactions-- they can all be understood this way, OK? So we roughly understand, but yeah. But going into that will be a long story. Yes?

AUDIENCE: So people sometimes say that gauge invariance is an ambiguity of the theories--

HONG LIU: Yeah.

AUDIENCE: So you can shift your fields by whatever, and the theory's invariant. How can an ambiguity in your description lead to something very physical, which is being interacted--

HONG LIU: Yeah, yeah, yeah. Exactly. So we emphasized-- so when you say ambiguity, this is what we said earlier, that the local symmetries, or gauge symmetry-- they just tell you your theory that the degrees freedom are redundant. OK?

Some of the degrees freedom is not important. Then you say, this should be pretty artificial, right? They just tell you you have some redundant degrees freedom. We just get rid of them. But it turns out that they actually-- yeah. Yeah. So this is part of the mystery, OK?

This is part of the mystery-- why, somehow, the gauge symmetry leads to the interactions. Yeah. So Yeah yeah. Yeah, it's a long story. Let me just briefly make some comments here, OK?

So now, we more or less understand that all these interactions-- essentially, all these important interactions in nature, those fundamental interactions-- they're all governed by some massless particle, OK? They're all governed by some massless particle.

But somehow, a massless particle-- you actually require to have some kind of redundancy so that you can describe them in a Lorentz covariant way. Without those redundancy, you cannot describe them in the Lorentz covariant way.

So in the sense, that's where that comes from. OK? Yeah. Yeah, but you make the statement which I just said. Precisely, it's actually a long story. Do you have any questions? Yes?

AUDIENCE: Would it have been possible to quantize the Maxwell theory just using the E field and the B field without involving any kind of gauge?

HONG LIU: We don't know how to-- so we don't know how to-- there's no good way to quantize that to give you a nice quantum theory. And we also know that physically, A mu actually plays a role. And in situations like a Aharonov--Bohm effect, the A mu plays an important role, even when E and B is equal to 0.

So A mu is a more fundamental object than E and B. So E and B should be considered as a derived object from A mu. Other questions? Yes?

AUDIENCE: So here it was quite natural to extend to fermions to just have to identify-- the symmetry is already there. You didn't have to engineer it, I guess.

HONG LIU: Yeah.

AUDIENCE: But is there a situation where extending this symmetry to another field causes an issue when you're trying to fix your gauge like you had before-- increase your redundancy, I guess, when you're making the [INAUDIBLE]?

HONG LIU: I'm not sure I understand your question.

AUDIENCE: So here we had the issue in gauge fixing was from--

HONG LIU: No, no. We're not doing any gauge fixing here. We just have a more generalized gauge symmetry. We're not doing any gauge fixing.

AUDIENCE: No, no, no. I'm saying, so that came out of the local gauge symmetry of our A mu's. But I'm saying when you extend that to the psi, for example--

HONG LIU: Yeah.

AUDIENCE: --you get any other issues like we had from earlier or no?

HONG LIU: What do you mean by other issues?

AUDIENCE: So issues when you're fixing your gauge. Or is that just a step that we already did before that we don't have to worry about?

HONG LIU: Oh, oh, oh, oh, oh. You mean when we-- no, no. That won't change much because we did before for the gauge fixing already. It's enough. Yeah. Yeah. OK? So now, let's talk about the other example.

So if you have a scalar field which is charged-- say, complex scalar field-- you can also couple it to the photon, OK? That's what we did for the fermions, OK? So let's consider if you have a complex scalar. So consider you have a complex scalar.

Yeah. Maybe you can also have some interaction terms, OK? So let's imagine we have some complex scalar. And so this is invariant under phi. Again, there's a global symmetry. U1 global symmetry. Alpha equal to constant.

OK? And then we have a conserved current here-- J mu equal to minus i. So again, your Noether current. OK. So now, you can just try to do the same trick, OK? So let's just couple this theory, combine this theory to the Maxwell theory, but identify that J with this J.

So naively-- so the reason I say naively is because this procedure in this case will not work. So now, we can consider the scalar QED, say, with photon couple, and again, with this minus 1/4 F square term. So let me just write the simplified version-- just write F square.

And then you can add this scalar Lagrangian, OK? And now, you add the coupling E. And then you can put A mu equal to J mu, but with J mu given by this. OK? We say A mu given by this, OK? So this is the natural thing to do based on our fermion story.

But now, you can check. But now, you can easily check. OK? So I will not do the exact check here. You should check yourself. So let me call this L prime, OK? L tilde because this is our naive theory.

You can check that this L tilde QED, scalar QED, is not invariant-- gauge invariant, OK? Means that you cannot find the transformation on phi, OK? That this whole thing is gauge invariant. OK? This whole thing is gauge invariant.

And so for example, so it's not gauge invariant. When we can check explicitly, if you just copy what we do here, this phi x given by ie lambda x phi x-- so this won't work, OK? So it's not invariant. OK. Yes?

AUDIENCE: Sorry. Can you just leave phi the same when you do--

HONG LIU: No, you cannot leave phi the same. Yeah, that's a very good question. So you cannot leave phi the same. So naively, you can ask yourself-- you say this is supposed to be conserved. And we previously said if we have A mu to a conserved current, this is gauge invariant.

But this is conserved only when you use the equation of motion of this theory. But now, when you couple these two to this theory, then the equation of motion for phi changed. And then so yeah. So it no longer works, OK? And so just naively, do this.

Naively, this is not guaranteed to work. It turns out in the fermion case, it just worked because of the existence of this symmetry, because of the existence of the symmetry. And it turns out, this theory-- so now, this theory will not be gauge invariant.

And now, this would be bad, OK? It will be bad because you can no longer find any local symmetry. This theory is invariant, OK? And this is one where the generalization does not work. You cannot find anything. You can also not find others.

But this is bad because we said we need gauge symmetry to get rid of unphysical degrees freedom in A mu because A mu should only have two polarizations, not to have four. So we need those gauge symmetries to get rid of the unphysical degrees freedom in A mu.

But if this gauge symmetry is broken by coupling to the scalar field, then this theory cannot be consistent, OK? So this theory is bad. OK. This theory is bad. And it turns out, we can just generalize this principle, OK? We say let's just impose gauge symmetry, OK?

We can just impose gauge symmetry. So the way out-- we can just impose the gauge symmetry. Impose the theory to be invariant under this A mu, goes to A mu plus partial mu lambda. And the phi goes to exponential minus ie lambda x phi, OK?

So we require this to be under that, OK? So to write the theory invariant under this, we can just easily borrow the inside we already obtained from the fermionic case. But in the fermionic case, we were told that this derivative, this capital D, actually transforms nicely under such transformation.

When we make such transformation, whether psi is a fermion or boson, and whether it's a scalar or spinor does not play any role, OK? So we can now just introduce D phi. We can just introduce D phi equal to partial mu phi minus ie A mu phi, OK?

We can similarly introduce D phi like this. And now, D phi then transforms nicely under this kind of transformation, OK? So D phi-- to mu phi, OK? They transform nicely. And since this transforms nicely, then we can easily write down the Lagrangian, OK?

So the L scalar QED then can be written as just minus 1/4 F squared. And now, instead of the standard derivative, I use this derivative. I use this capital D derivative, OK? And then I have m square phi star.

And then I can have phi phi star, OK? And so this will be invariant because those don't matter because those don't involve derivatives, OK? And so yeah. So this is now gauge invariant. OK? So this D mu is called covariant derivative, OK?

Becaues it transforms nicely under this gauge transformation. So it's called covariant derivative. So this capital D mu also has a very deep connection to mathematics. And so this is related to a subject, say, in differential geometry-- a deep connection to differential geometry, to a thing in differential geometry called the connection, OK? Called the connection.

And yeah, sorry. We're not going into that. Yeah. So now, let's compare this theory with that naive theory. So you can check that L SQED is almost that L tilde SQED except this has one more term.

So the difference with the fermion case-- the fermion only has one derivative, OK? And this one does not have a derivative, OK? And here you have two derivatives. And each derivative gives you additional A mu, OK?

So when you expand this, so here there are four terms, OK? There are four terms. And three terms are encoded in here, OK? Three terms are encoded in here and in here. But there's one more term, OK? It turns out this leaves one more term.

OK. So actually, there's a quartic inaction between A mu and phi, OK? So this only introduces cubic interaction. So if you just have this J mu A mu, you only have cubic interaction. You only have 2 phi and 1 A. But here because of this structure, you actually have 2 A.

Also, a term with 2A and 2 phi, OK? And when you add this term, then the whole thing is nice, OK? Good. Any questions? Yes?

AUDIENCE: Can I also add higher order terms like these quartic terms into the fermionic theory?

HONG LIU: Sorry. Say it again.

AUDIENCE: Can I also add quartic terms or some higher order terms into the QED -- fermionic theory?

HONG LIU: No, no, no. You cannot add this term because this term by itself is not gauge invariant. Fermionic term is already gauge invariant by itself-- because there's no derivative here. And so the structure is simpler. Yes?

AUDIENCE: So I guess to add to that question, can you carefully engineer higher order terms such that they're--

HONG LIU: Yeah, yeah. Yeah, you can engineer more complicated terms, but not this kind of term. Yeah.

AUDIENCE: This is required for--

HONG LIU: Yeah, yeah. This is required for gauge invariance. Yeah. Yeah. For both fermion and the scalar case, you can write down more complicated terms which are gauge invariant. And the one we wrote down so far is just the simplest one. Yeah. Simplest ones. Yes?

AUDIENCE: So which one should of these should I interpret as the electric current? J mu there or I guess J mu plus the A mu psi psi psi?

HONG LIU: Yeah, yeah, yeah. So still, so when you interpret the electric current, it's still this J mu. Yeah. Other questions? OK, great.

So now, so let me just make a comment. So in this story, now the global charge before now becomes the-- just to emphasize again-- in the scalar case, again, the global charge for this global current has now become the electric charge coupled to the electro- magnetic fields, OK? Become electric charge coupled to electro- magnetic field, OK?

OK, good. So let's draw the Feynman rules, OK? So from here we can just talk about the Feynman rules. So let's first do the fermionic case. So the rules for fermions is the same as before. The only thing we just need to write down-- the propagators are the same as previously for the Maxwell theory.

Yeah, let's first do the QED. And the propagator for A mu and phi-- it's the same as before, OK? Because the propagator only cares about the quadratic theory. It doesn't care about interactions. Same as before. So we can draw a wavy line corresponding to A mu, OK?

And then the solid line corresponding to, say, psi, OK? Solid line equals 1 and 2 psi. So this is related to A mu propagator. This is related to psi propagator, OK? And now, the interaction between them-- this interaction between them is given by this term.

And then these have the structure. You have one fermion come in. You have two fermions and then coupled to a photon, OK? So this is the same, very similar to this Yukawa coupling. We can that before-- just now, this one becomes a photon, OK? This one becomes a photon.

And this effective vertex is given by minus ie A mu gamma mu, OK? So the minus comes from minus here. The i comes from the i in the action, OK? When you do the passing equal action. And you have minus ie. And then phi, psi, and A mu are taken care of by those lines.

And then you only have a gamma mu, OK? So the vertex here is the gamma mu. But pay attention. This gamma mu is actually a matrix in the spinor space. And so you have to be careful when you contract to the spinor indices because now, these have spinor indices. And it's a matrix, OK?

Good? So this is for the fermion. And oh, yeah. So I should also mention-- for the external legs, so this is for the vertices and the propagator. And if we consider the scattering amplitude-- so for scattering amplitude, we mentioned it before.

For fermion, we need to include the polarization vector. Similarly, for the photon, we also need to include the polarization vector for the external legs, OK? So for external legs, for scattering amplitude-- yeah, if you calculate the green function, it doesn't matter.

You just include the external propagator. But for the scattering amplitude, then we need to include the polarization vector for photons. So for the initial state, supposing the initial state, we have, say, some vector k and alpha. So alpha gives its polarization vector.

And k is the momentum. And then we can denote it as-- so alpha denotes the polarization for the-- and then say we can have k. So this is the momentum index. So this is the momentum arrow, OK? And so this [INAUDIBLE] corresponding to-- OK. OK?

And so this is for the initial state. Then for the final state, if you have a photon in the final state, like this-- and then again, we just have an alpha. And so normally, we throw the momentum arrow to come out if you have a final state.

And then the polarization vector will be just this star, OK? Just the complex conjugate. OK, it's very easy. OK? Yeah. So when you write down the amplitude, you just have to be careful. For external photon legs, you have to include the polarization vector.

So for scalar QED, it's very similar. So here we can introduce a dashed line that's corresponding to a scalar propagator. OK? So corresponding to the scalar propagator, OK? And now, this arrow now has a meaning because the scalar has a charge, OK?

A scalar has a charge. And so this arrow is not the momentum arrow you can do arbitrarily. And so there are various-- and yeah. And so for QED, let's look at the interaction with the photon. And then we have this vertex.

And we also have this vertex with J mu coupled to that, OK? So then that kind of interaction will have the following form. So this J mu coupling is easy. We can schematically write it down.

So again, you have 2 phi and then coupled to a photon, OK? Coupled to a photon, OK? And so A mu will have index. And so we can put the mu here, OK? And this vertex is given by minus i e k plus k prime mu, OK?

Suppose the momentum here is k. And the momentum here is k prime, OK? So yeah, the reason you have this k plus k prime mu is because here there's a derivative acting on phi. And so this derivative acting on this phi gives you a k1, a k.

And this derivative on phi gives you the k prime, OK? And you have the sum of these two terms. So that's why you have this kind of term for the interaction. Yes?

AUDIENCE: Sorry. Going back to the fermion QED for a second--

HONG LIU: Yeah.

AUDIENCE: Do you still also have, for the external legs, for fermions that are usually--

HONG LIU: Yeah, the same [AUDIO OUT]

AUDIENCE: OK.

HONG LIU: Yeah. So fermion-- external legs, exactly the same as before.

AUDIENCE: OK.

HONG LIU: Yeah. And yeah. So we only need to introduce new rules for photons. Yes?

AUDIENCE: Still follow the same rules for starting from one--

HONG LIU: Yeah, yeah, yeah. That rule is exactly the same.

AUDIENCE: If you got a [INAUDIBLE] interaction, [INAUDIBLE] in the right order?

HONG LIU: That's right. That's right. Yeah. And now, you have to just be careful. Some of the vertices are now including matrices. Yeah. Yes?

AUDIENCE: Doesn't photon polarization have to transverse the momentum--

HONG LIU: Hmm?

AUDIENCE: Isn't photon polarization have to transverse the momentum?

HONG LIU: Yeah, yeah, yeah. They do. Yeah, yeah. For the physical photon, yeah. Yeah, but for the physical photon, indeed, yeah.

AUDIENCE: Also, why is it complex?

HONG LIU: Oh. You can choose it to be complex. Yeah. For example, if you choose a spherically-- polarization, then it's a complex vector. Yeah. For the one we wrote down, it's real. But then this complex is the same. Yeah. Yes?

AUDIENCE: [INAUDIBLE] this, but when you have an internal photon like in a diagram, do you sum over all four polarizations or just two?

HONG LIU: Internal photon-- you always just use the photon propagator. And the photon propagator will include everything. Yeah.

AUDIENCE: Thank you.

HONG LIU: Other questions? Yes?

AUDIENCE: For external vertices having matrices-- now, that's just a byproduct of the number of degrees of freedom you have per particle?

HONG LIU: Yeah. Yeah. You mean the gamma mu?

AUDIENCE: Yeah.

HONG LIU: Yeah. It's just because of spinor nature, right? OK, good. So this vertex-- so A mu J mu gave you this vertex, but we also have this vertex. And this vertex is easy. We essentially just have two fermions or two scalars, but this one with two photons, OK?

With two photons. And so these have A mu A mu contracted. So if we write mu here and nu here, then we will have minus ie squared eta mu nu, OK?

OK. Minus ie squared. OK? So minus i for the same reason, and the e squared for the same reason. And this A mu contracted, so you have eta mu nu there, OK? Good?

And depending on what is V here, phi may have some additional interaction. But suppose V is given by phi 4. OK? Suppose V, say, is equal to lambda 4 and phi phi star squared, OK?

And then you will also have interactions like these-- four scalar interactions. OK. Yeah. You have two come in, two come out. OK? And this would be just minus i lambda, OK? Yes?

AUDIENCE: Could you clarify-- the vertex with two scalars and one photon comes from?

HONG LIU: Hmm?

AUDIENCE: Can you explain where the V is a vertex with two scalars?

HONG LIU: Oh, right. Right. It comes from this term. It comes from this term. So this is A mu times J. And the J has two phis here.

AUDIENCE: Oh.

HONG LIU: Right, right, yeah. Other questions? Yes?

AUDIENCE: Would it be possible just to hypothetically, if I wanted to, give the photon mass? I'd just add a mass term to the Lagrangian?

HONG LIU: Yeah, yeah. OK. What's the question?

[LAUGHTER]

AUDIENCE: I'm sorry. Is it possible to add a mass term for the photon in the Lagrangian?

HONG LIU: No, you cannot. No because that violates the gauge symmetry.

AUDIENCE: OK.

HONG LIU: Yeah. Yeah. So the gauge symmetry is what ensures the photon is massless. Yeah, so that's why this is something we don't want to break. Yeah. Other questions?

AUDIENCE: Sorry. This might be a silly question. But if you have the scalar QED and then the fermion QED, can you add them?

HONG LIU: Yeah. You can add them.

AUDIENCE: So then it's the one theory and [INAUDIBLE]? It's combined?

HONG LIU: Yeah. Yeah, yeah. You can combine them just like you have a theory-- the electromagnetic field coupled both to charged scalar and the charged fermions. Yeah, you can certainly combine them. Yeah. The reason I'm separating them is just for convenience.

AUDIENCE: So can we have an interaction term directly between the fermion term and the scalar term?

HONG LIU: Oh, yeah. You can add whatever. You only have to make sure things are gauge invariant. And you can add arbitrary interactions you want. Just, you have to make sure they're invariant under those gauge transformations.

AUDIENCE: So on the other question, can we make another theory that has U1 gauge theory, but with massive particles?

HONG LIU: With massive particles? Massive photons?

AUDIENCE: With the photons. We have something that is massive and coupled to the [INAUDIBLE]. Can we do that?

HONG LIU: No. No. No. You're asking whether the massive photon coupled to fermionic field exists?

AUDIENCE: Yeah.

HONG LIU: OK. Yeah. So this question can be answered in several-- so if you just directly add a mass term-- so let's add a mass term for the photon. OK? So you can immediately check. This is not gauge invariant.

So that's the reason we didn't add such a term to the Maxwell theory because in the Maxwell theory, if you add this term, it will violate the gauge symmetry. The same reason here. So you cannot add such a term, OK? So you cannot add such a term.

So this is ruled out, OK? You cannot add this. But now, you say is it possible somehow, through some other way, that the photon actually gains mass, OK? That's possible. That's called the Higgs mechanism. That's how Higgs got a Nobel Prize.

And that comes from this term. So now, you imagine somehow, this phi now has a vacuum expectation value. And now, phi has a constant part. If phi has a constant path, then this becomes a mass term for the photon, OK?

And then the photon becomes massive. And actually, that's precisely what's happening inside the superconductor. So in every superconductor, something like this happens. OK? And the photon inside the superconductor is massive, OK?

That's what responsible for this London effect, Meisner effect, et cetera, OK? And in the superconductor, that's understood by Phillip Anderson. He was a little bit unhappy because he thought he should have claimed for the Higgs effect because he understood it earlier.

And yeah. But he did understand the mechanism earlier. Yeah, yeah. But Higgs got the Nobel Prize because he predicted the particle. Anyway, that's a long story I'm not going to go into. Yeah.

AUDIENCE: Just a quick question. For the photon propagator, do we give the same factor? Because I don't think we said in the previous lecture that we give the same factor as for the scalar field.

HONG LIU: Sorry. Say it again.

AUDIENCE: For photon propagator--

HONG LIU: Yeah.

AUDIENCE: --which factor do we give?

HONG LIU: Which factor?

AUDIENCE: Yeah.

HONG LIU: You mean-- we just use the photon propagator.

AUDIENCE: Yeah. Because I don't think we did it in this lecture.

HONG LIU: We did. Yeah. Yeah, at the end of last lecture. Yeah, let me just write it down. So the photon propagator depends on this parameter xi. For xi equal to 1, the photon propagator-- if I call it D mu nu, yeah. Anyway, let me just write it down.

For xi equal to 1, this is just given by eta mu nu k squared minus i epsilon. So this is for xi equal to 1. So for xi not equal to 1, so yeah. So this is for xi equal to 1. For xi not equal to 1, then there are some additional terms. Yes?

AUDIENCE: I might be leaving the scope of this class, but the weak force has a carrier that has mass, right?

HONG LIU: Yeah.

AUDIENCE: And it's called a fermion.

HONG LIU: Right.

AUDIENCE: So that is possible to construct.

HONG LIU: Yeah, yeah, yeah. Yeah. Again, this mechanism is what's responsible for the weak interaction in short ranged.

AUDIENCE: I see. Oh.

HONG LIU: So the weak interaction started as massless particle--

AUDIENCE: Got it.

HONG LIU: --but then through the Higgs mechanism and then the analog of A mu for the weak interaction, becomes massive, and then becomes short ranged.

AUDIENCE: Got it. That makes sense.

HONG LIU: Yeah, yeah, yeah. And yeah, yeah. And yeah. Good. Other questions? Yes?

AUDIENCE: In the standard model is the Higgs the only scalar that couples to the photon [INAUDIBLE]?

HONG LIU: So in the standard model, Higgs is the only scalar field. Yeah. But you can have some other composite scalars, like pions. Fundamental scalars-- it's just the Higgs field. Good.

OK, good. So this concludes the discussion of the QED. And now, ready just to study the physical processes in QED, OK? And before doing that, we need to develop a little bit of formalism because when we talk about physical process, then we need to make connection.

Yeah. QED is real life, OK? And then we can make connection with the real experiments. OK? To make connection with the real experiment, we need to develop one more thing. We need to develop one more thing. So we need to learn how to calculate the cross-section, OK?

So let's talk about how to calculate the cross-section. So this is a digression, OK? So first, let's remind you how we define a cross-section in nonrelativistic quantum mechanics, OK? In the scattering theory of nonrelativistic quantum mechanics.

So in nonrelativistic quantum mechanics, you consider there is a target. The target is normally considered to be a point, OK? Here I just make it big so that we can see it. So here is the target.

And then we define our axis-- the scattering axis, which the particle come in. So this is some incident beam of particle. OK, coming into this direction. So say this is a Z-direction, OK?

And moving in this direction to-- and then it will interact with this particle. And then it will scatter, OK? So the particle will scatter. And then we put the detector around this target particle to detect the scattered particle. OK?

So for example, yeah. So you should view this solid angle here. OK? And so this is the phi direction. And so this is the theta direction, OK? So here is the theta direction. And this is some small solid angle, which the detector-- suppose there's a detector here, OK?

Suppose there's a detector here. And then this detector spread the solid angle, D omega, with respect to the target, OK? So this is the standard setup for nonrelativistic-- scattering process in the nonrelativistic quantum mechanics.

So what we measure is dN dt outgoing particle, OK? Scattered particle in the theta and phi direction, OK? So this is theta and phi. This is where the detector locates in the theta phi direction. So this is the number. So this dN dt is the number of particles-- number of scattered particles, number of scattered particles in -- per unit time in the detector.

It means registered by the detector, detected by the detector at the location theta phi, OK? OK? So the quantity we measure is this quantity. We just put the detector there. We just measure particles, OK?

And clearly, so this scene depends on many things, OK? This quantity depends on many things, depends on the interaction between the incident particle and the target, et cetera. But they also depend on many other kinematic factors, OK?

So for example, it will depend on -- this out-- clearly, it will depend on the number of incident particles. OK? So this is the number of incident particles per unit time. If you have more incident particles, of course, you will detect more outgoing particles, OK?

And of course, this will also be proportional to D omega, the solid angle extended by the detector. So if you have a larger detector, of course, you will detect more particles, OK? And then if we divide it by those obvious kinematic factors away, then we will get something more intrinsic to the interactions, OK?

So we can consider-- we can divide by D omega and divide it by this thing. So this object, dN dt. Then we divide it by D omega per solid angle out and divide it by the incident number of particles per unit time.

So this should give us the probability of a particle to be scattered to theta phi direction, OK? So this is the quantity which captures the effect of the interaction, OK? So this is a more intrinsic quantity.

But experimentally, this is actually not the thing we directly measure in the experiment because in the experiment, it's not easy to count the number of the incident particles, OK? We always consider a beam of particles. We always consider a large number of particles, a beam of particles.

So experimentally, it's more convenient. We often use the incident flux. It's the number of particles per unit time and per unit area, OK? So we just send the beam. And we just need to know the density of the beam, OK?

So that will give us the flux, the number of particles ingoing per unit time and per unit area. So A is the cross-section of the incident beam, OK? So now, this is the quantity. And now, we have to divide-- so instead of dividing by this one, we divide by this one. OK?

So now, this D sigma D omega, which is defined to be dN dt D omega out-- and then you divide it by incident flux, OK? Incident flux per unit time, OK? So that's the object we actually measure, OK?

So some comments on this. So the first comment is that this has dimension of area. So because this thing is dimensionless-- this thing is dimensionless because T canceled, but now, we have divided by dA. So A is going up.

And then this has dimension of area. So the physical meaning of this, which I'm sure you learned in your nonrelativistic quantum mechanics class, is that this gives you the physical meaning of this object. So this gives you the effective area of interaction, OK?

So heuristically, you can imagine the following. So if you imagine the incident particle, this target particle-- they actually have some short range interaction, OK? And then they no longer interact, say, outside some distance. And then heuristically, then this quantity then can be considered as-- give you the area of interaction around this target particle, OK?

Around the target particle. So that's why this D sigma D omega is called the differential cross-section. So the area which is going into the direction of theta phi-- and the sigma total, which is defined to be D sigma, D omega-- you integrate them over all angles, OK?

So this is a function of theta and phi. And now, you integrate over all solid angles. And then that gives you the total cross-section. OK? And to see this effective area of interaction, can be seen in the very simple classical example.

So this definition actually is not quantum mechanical. You can also do it classically, OK? It's not restricted to quantum mechanics. This can be defined for any scattering event, including in classical mechanics. So for example, if you have classical scattering, say, of some bullets, off a billiard ball, and then you find the sigma total-- so suppose you have classical scattering, OK?

And then you find the sigma total. It's just the cross-section. It's just the total area, total surface area. Yeah. No. It's the cross-section area over the billiards, OK? Because classically, when you hit the billiard ball, then you hit the billiard ball, OK?

If you don't hit the billiard ball, then you don't change direction. And then the total cross-section just is the cross-section of the billiard ball, OK? And so when you go to quantum theory, then this is the heuristic way you understand there's some kind of effective cross-section for this target, OK?

And outside that area then you no longer have-- yeah. Heuristically, you no longer have interactions. OK. So now, let me just make two quick remarks in a relativistic theory. So now, we can generalize. So this discussion is in the nonrelativistic context.

So in the relativistic context, we essentially just have D sigma cross-section from initial state to final state, OK? We no longer have-- so this is very specific. We have one particle scattering the other particle. And this particle scatters away, OK?

But in relativistic, you can have two particles. You can create 10,000 particles, OK? And so this language no longer works, OK? So but still, you can define some kind of cross-section of initial state alpha to some final state beta, OK?

And so we will be interested in the situation in which we have two initial particles, OK? Because experimentally, it's convenient just to have two particles scattering. So you have two initial particles. But final state, in principle, can be arbitrary, OK?

And so this object-- since, in the relativistic theory, this should be physically observable, so this should be-- so because this essentially is a measure of the probability of the scattering, so we require this object to be Lorentz invariant.

OK? And also, in the relativistic, we just scatter two particles. There's no notion of what is the target or what is the incident particle. So these two particles should have equal rho. So this should be symmetric in 1 2, OK?

So if you exchange 1 2, this cross-section should be the same. OK? It should be the same. So now, if you consider-- but you can always consider in the rest frame, say, of particle 2, OK? So suppose we consider the rest frame.

Suppose we go into the rest frame of particle 2. And then we roughly have this situation, OK? So this is particle 2. This is the particle 1, which comes to particle 2. And then in this situation, we can define the alpha to beta based on our nonrelativistic considerations.

And then we consider-- so here I use the probability from alpha to beta dP, OK? So this essentially replaces that. OK? Here the final particle only has one particle. So you can define a solid angle. But if we have 10,000 particles here in the beta, of course, I cannot define a solid angle, OK?

So I can just talk about probability from alpha to beta, OK? From alpha to beta, but per unit time. And then again, we divide it by the incident flux of 1. OK? The incident flux of 1, OK? Oh. And let me just call it in.

And 1 means the number of particles of 1, OK? So yeah. So what we will do is that we will first calculate this object in the rest frame of 2. And then we construct the Lorentz invariant version of this, OK? So that's our strategy to define what will be the relativistic generalization of this. OK. So we will discuss this next time.