[SQUEAKING]

[RUSTLING]

[CLICKING]

PROFESSOR: So last time, we more or less finished discussing quantization of the free scalar field theory. And then we showed that the excitations of that theory essentially give rise to relativistic particles. And so we also discussed various other things-- so structure of the Hilbert space, conserved quantities, et cetera.

So at the end, we talked about how to understand the conserved charge at the quantum level. So classically-- so we have a, say, symmetric transformation-- means that if you take phi a to some phi a prime-- phi a prime, normally, we write it in the infinitesimal way-- introduce some parameter a-- some parameter epsilon. And I can also have multiple symmetries.

So for each transformation-- say I have epsilon. So alpha labels different transformations, and then-- so this is the transformation, which can depend on phi a. Also depends on its derivatives, et cetera.

And then if you have such a symmetric transformation we discussed before, then that generates a conserved current, and that is labeled by alpha. Alpha just denotes different conserved current. And then yeah, so it is conserved.

So this is the classical statement. Oh, yeah, also Noether's theorem tells us how to write down such a current explicitly. So we can just write it as-- the current can be written explicitly. So k mu is the transformation of the Lagrangian density. Suppose under this transformation the Lagrangian density transform as, say, alpha partial mu, k mu alpha.

So alpha, it doesn't matter. Alpha just label different symmetries-- the index. It doesn't matter whether it's up or down. And then the zeroth component of the charge-- zeroth component of the current when you integrate over all space then give rise to a conserved charge.

So this is conserved charge times the-- yeah. So now, let's discuss-- so when you go to quantum level-- so when you quantize the theory, and this just becomes quantum operator, and the whole current becomes a quantum operator. And then Q is also a quantum operator.

So the quantum level-- and now Q alpha-- because it's conserved, so this becomes time-independent operator. We see explicitly in the case of the, say, Hamiltonian dens-- Hamiltonian and the momentum. So we showed explicitly they actually don't depend on time.

But at the quantum mechanical level, actually, this Q can play a more important role. In fact, it can be viewed as a generator of symmetries. So we can show-- we will show the following statement-- that if you consider so epsilon-- so consider such a quantity-- can you see the commutator of Q with phi a.

So now, let's continue over to here. Maybe just let me write it here. And that generates minus epsilon alpha f alpha a. So the commutator of this Q alpha epsilon alpha with phi a essentially generates this transformation, essentially, generate this infinitesimal transformation.

So that's why we call the Q alpha also generator of symmetries. So this is the statement I'm going to show very soon. So before I do that, do you have any questions? Yes?

AUDIENCE: In the conserved current equation, how can f have an alpha in the upper index when J has an alpha in the lower index, its not meaningful?

PROFESSOR: Yeah. Just alpha-- don't worry about the location of alpha index. Alpha just label a different current and just went-- yeah, here, we don't worry about its notation. I just put it up and down just for convenience. Other questions? Yes?

AUDIENCE: Can you [INAUDIBLE] positions is that a sum over alpha?

PROFESSOR: Sorry?

AUDIENCE: In the proposed, var L is epsilon alpha partial to mu alpha, is that a sum over all alpha?

PROFESSOR: Yeah, that's sum over all alphas. So any repeated indices is sum. So if you only have one-- so for example, if you have one epsilon-- of course, there is no sum, but if you have two different charges, and then this is sum.

Other questions? So we are going to show this statement. And so this statement is very easy to show in the general case. So let me call this equation star.

So let's first consider the case-- first, let's consider the sub case that the-- let's first suppose f a alpha-- just this transformation of phi a does not contain the time derivative of phi a. And also the zeroth component of the k is equal to 0.

Let's consider a first spec-- a more limited case. So this more limited case actually covers many, many examples. It actually covers a majority of examples.

For example, if you look at the translation symmetry, which gives rise to the stress tensor-- so this will-- so this applies to the case I mu-- T mu i. So this corresponding to the spatial-- so this is the conserved current corresponding to the spatial. So this is the current for spatial translations.

When we do spatial translations, then of course, this f given by the spatial derivatives, and then it does not involve time derivative. And in that case, the k also does not involve in-- k 0 component's 0. Yeah, just I try to remind myself of what you worked in your pset.

And so for this example, i is the counterpart-- is the alpha here. So for each direction, you have a translation, and so this-- so the alpha-- of course, i is the alpha here and the mu is the J mu here. And also this applies to the case which-- for all internal symmetries.

By internal symmetries, we mean the transformations don't involve space-time coordinates. For example, you did in your pset the complex scalar field which you can just rotate by phase. And that's called the internal transformation because that transformation does not involve spacetime coordinates.

In contrast, when we find the stress tensor, we will do a spacetime translation, and that does involve spacetime. So this corresponding to the spacetime transformations-- spacetime symmetries. And this is-- the phase is the internal symmetries.

So for all internal symmetries, you don't-- the transformation will not contain the time derivative. So in this case, then it's easy because in this case, then we just have the one-- then the zeroth component of J mu. J just given by-- the first term by definition is just the canonical momentum conjugate to the a.

So the first factor, partial L partial partial mu phi a-- when you set mu equal to 0-- so that's just the derivative with respect to the time derivative of phi a, and that, by definition, is just the canonical momentum density conjugate to phi a. And then you have f alpha a-- f a alpha.

Good? So let me just make one comment on this expression. So this expression, of course, classically, you can write whatever way you want. But quantum mechanically, there's a very important subtlety because this typically will involve phi, and then this is momentum, and they don't necessarily commute.

So this is the field theory version of so-called operator ordering ambiguity, which is already in quantum mechanics. In quantum mechanics, when you go from classical-- even it's already in the non-relativistic quantum mechanics. When you go to classical mechanics to quantum mechanics, and then there's an issue how you order operators when you have x and p at the same time. So here, you also have potential operator ordering ambiguity.

You have to pay attention. Often, such kind of ambiguities can be resolved by physical considerations. So we will see some examples in a little bit.

So now, let's just imagine there's some ordering we have chosen, and now let's look at this commutator. So we can just write down the definition. So this is just integrating over all spatial direction epsilon alpha, and then we have pi a f a alpha phi a.

Yeah, let me just-- sorry-- here, let me call phi b, because the phi a-- now, they are differ-- so these two indices are summed. So let me call them b, and then you have phi a here. So now, you can use commutation relation between the momentum density and the phi phi.

So we discussed in general that the-- so we discussed the scalar case. But in general, you have this commutation relation of-- which is delta 3-- say if this is the x, and this is x prime, and then this is x minus x prime, and delta ab.

So each one is only have a canonical conjugate momentum with its own momentum. If they have different fields, then of course yeah. So you have that.

And now, if you use this-- so here is x. Yeah, let me call this prime and this to be x. Sorry, I should label more explicitly x here, and then here would be x. And so here is x, and here will be x prime.

Sorry, I didn't leave myself enough space. Let me just rewrite it. So this gives me d3 x prime epsilon alpha. Then, I will have this pi a, then x prime f alpha a x prime, and then this commutator with-- yeah, not covariant. Sorry, alpha is upstairs. b and then phi x.

And yeah, so we can evaluate this, say, for example, all at t equal to 0. So I suppress the t. And then yeah-- and now, you can just trivially use this commutation relation, and then this just gives you minus i epsilon alpha f alpha a fa alpha x-- evaluate at x.

So that just confirms that expression. So this is not satisfied when you have time translation. So for time translation-- so first, any questions on this in this simple case?

The idea is very simple. Essentially just the first term proportional to the momentum, and the momentum-- a conjugate to this, and then you just take that. Yes?

AUDIENCE: Did we not get a term with the f alpha a and phi because of our restriction?

PROFESSOR: That's right. Yeah, so if there's no partial zero-- if there's no time derivative here, it means there's no momentum here. If there's no momentum here, then this will commute with this one.

Yeah, no matter if it's in here, it always commute. Yeah, a very good question. Yeah, I forgot to mention this point. Other the questions?

OK, good. So now, the special case-- we said if you have time translation-- when you have time translation-- for example, one example is the-- yeah, the Hamiltonian density, H is equal to pi a partial T phi a minus L. So the L here-- for the time translation L is the analog of the k, and the partial T phi A is the F here, and then here is the pi a.

So here you also have time derivative. And so this by itself is also a pi. And then L may also contain time derivative. So L is the k0 here. So in this case, the k0 is non-zero, which is equal to L.

And so in this case, the conserved charge is just H. So in this case, you-- our general argument here don't apply because now, you have pi here also, and you may have a time derivative here also so the story is more complicated.

But this case, we know trivially it's true because it's by definition H acting on any field. It's minus i partial t phi a. And this is exactly the time translation-- transformation of fields. So in this special case, this equation is also satisfied.

Got any questions on this? So this is just follow from the Heisenberg equation itself. And now, you should just understand this equation.

S-- so H now can be considered as a symmetry generator for time translation symmetry, and when we act on the field and generate the time translation. Time translation just infinitesimally just a time derivative on phi. Good?

So these are infinitesimal transformations, but now starting from here, you can actually obtain finite transformations on the field also using Q. So you can generate finite transformations. So let's consider, say, U lambda alpha-- so lambda alpha are just some parameters. Let's consider this quantity.

Now I exponentiate this Q. So Q is the operator, and lambda is just some numbers-- lambda is some constant parameters. And again, the alpha is summed. Yes?

AUDIENCE: So I understand how this follows from the Heisenberg equation of motion, but here, how would you evaluate this commutator not knowing the form of L mu because you said it can depend on time derivatives as well.

PROFESSOR: Yeah, here, just this general argument don't apply anymore. So in this case, you really have to-- you can check this explicitly-- you can just write down the explicit value-- explicit form of h, and then work it out explicitly. But you don't have to do it because we know this has to be true just by self-consistency because we solved the Heisenberg equa-- yeah, because when we quantize it, essentially, we are solving the Heisenberg equation.

Yeah, but you can check it explicitly yourself that this is true. Good? So here-- so lambda alpha here just some collection of finite transformation parameters.

So this can be used to generate the finite transformations. So if you act-- if u lambda alpha phi-- phi a and u lambda alpha dagger, and then you just get phi a prime. And now, this is a finite transformation-- a finite symmetry transformation of phi a.

And when you expand-- with lambda, alpha is infinitesimal, and then you can just expand this exponential, then the leading term becomes this one-- the leading term becomes this one. Yeah, so when lambda alpha equal to epsilon alpha goes to 0, and this equation star, and then star, star reduces to star as the leading nontrivial order when you expand the exponential.

So this can be done in general just-- so the reason this is true-- without even doing any calculation, you can just imagine you can build this U-- this finite transformation by infinite number of infinitesimal transformations. And the infinite-- so when you build them up, and then you will just generate the finite transformation. So you already worked out the example in your pset-- so you already worked out the example in your pset that-- if you can see the Ht minus I P i x i acting on phi 0 minus i H t plus i P i x i.

And then this just gives you phi t x i-- t x-- just give you phi t x. And so this is-- so here, H t minus P i x i is our lambda alpha Q alpha here. So alpha here runs 0 in i direction.

So in the 0 direction, the t now is just some parameter, and the lambda alpha equal to t and minus x i. And then Q alpha just-- and Q alpha here is equal to-- yeah, t x i. Let me just do t x i. And the Q alpha for here is just H minus P i.

Yeah. Good? So this is just an example of that. This is just an example of that.

And also you can consider Lorentz transformation. So another example is a Lorentz transformation. So let's consider-- so in your pset, you already-- so in the Lorentz transformation, the conserved charge is-- actually, there are six of them. M mu nu going from 0 to 3.

So there are all together six conserved charges associated with Lorentz transformation-- three rotations and three boosts. And then so when you write the finite transformation, and then you will have six parameters. You have six parameters. And omega mu mu-- and because mu mu is antisymmetric, which you worked out in your pset, so this parameter should also be antisymmetric.

And so there are finite angles. Oh, sorry, not finite angles-- finite angles and boosts. So essentially, they give the parameters for the finite rotation angles and the boost.

And so that generates a Lorentzian transformation. That generates a Lorentz transformation. And so for example, you-- say if you act on u lambda on phi x, and u lambda-- you can check dagger on phi x-- what you get is phi prime x.

And phi prime x is also the same as phi lambda 1 on x. So here, lambda-- so here, lambda 1 x is the Lorentz transformation of x mu with parameter determined by-- yeah, just omega mu nu. So this capital lambda is just the corresponding Lorentz transformation. Is this clear? Yes?

AUDIENCE: So you assumed that you can represent, I guess, the operators that perform these transformations like this. But don't you have to think about if it follows in the algebra such that you can represent it like this, or like the group of transformations?

PROFESSOR: Yeah, indeed. So here, it's not-- here, it's not said. I assume. Here, I'm just directly telling you the result.

Just when you build-- yeah, so if you want to generate the-- yeah, I'm just directly-- already directly telling you the answer.

AUDIENCE: So it's the composition of-- infinitesmal transformation?

PROFESSOR: Exactly. If you do the composition of the infinitesimal transformation, you will get that. Yeah, the reason I write down this explicitly-- immediately, because you should have already seen this in quantum mechanics because that's how you do the angular momentum.

In quantum mechanics, when you do the angular momentum, the way-- it's the same thing. Yeah, here, we just generalize it to general symmetries. Yes?

AUDIENCE: So it was some similar question, but just like-- so this is like a general procedure if you have a symmetry to get the representation on your Hilbert space, just exponentiate it like this? But like in QM, we usually just postulate that there is a unitary operator, and then demand that the group law holds. So don't we need to show that those things gives-- that this preserves the group law, or is that not--

PROFESSOR: Yeah, so this is guaranteed. So yeah, I didn't go into that, but this is guaranteed. Essentially, that's how you-- yeah, this is how you build the algebra, because the algebra is also-- the finite transformation is also built up by infinitesimal transformations. And so essentially guaranteed-- yeah, so there is a theory of groups and the algebra behind this.

Yes, I didn't go into that. So some of you may already seen a little bit of this when you talk about angular momentum. So in quantum mechanics, so it's exactly the same story there. Yes?

AUDIENCE: So I know that rotations correspond to angular momentum being conserved, but what is the intuition for the thing that's preserved in boosts?

PROFESSOR: Yeah, it's a-- yeah, indeed, there's no very intuitive way to think about it. Also it's not that useful. You can define a boost charge. So if you consider all the spatial components, and if you consider all the edge here, then these are the angular momentum operator, and this is your standard angular momentum operator, which you have seen.

And if you look at 0 i, and then that's what sometimes we call boost charge. It generates the boost. But in terms of the conserved number, we don't use it in the essential way-- say, in other contexts.

AUDIENCE: Thank you.

PROFESSOR: Yes?

AUDIENCE: But to that question, you can think of it like a center-of-mass velocity [INAUDIBLE].

PROFESSOR: Yeah, it just does not give you something additional. Just normally, when you deal with physical problems-- say angular momentum, conservation of momentum, conservation of energy, conservation-- it's already enough.

AUDIENCE: [INAUDIBLE].

PROFESSOR: Yeah, you normally don't need to use a boost charge conservation to solve your problem. Yeah, so that's why we don't see this object very often, say when you discuss special relativity or other things. But here, it plays a very important role that in quantum field theory, this is the operator which generates the boost symmetry.

If you want to do a boost in your Hilbert space, that's the operator to do it. Other questions? Good. Yeah, so in your pset-- yeah, another example is the phase rotation.

Let me just very quickly write here. And so another example is the phase rotation if you have a complex scalar field. So there, we show there's a Noether charge corresponding to the phase rotation, there's a Noether current corresponding to rotating the phase to the field. That's a symmetry. And so that will generate the charge Q corresponding to the first two. And then if you have alpha Q-- and then when you act on phi, and then you can just check explicitly that will generate the phase rotation in phi.

And we're generating phase rotation phi. Any questions on this? Good. Yeah, so let's conclude our discussion of the real scalar field. And now, we can move to a new part.

So before we move to new part, your last chance to ask questions about this part. OK, great. So now, let's very quickly talk about-- so let's very quickly talk about complex scalar field.

And you can see the complex scalar field, we get something new. We get the concept of antiparticle. So now, let's consider the following Lagrangian density.

Again, we only consider the quadratic-- the simplest-- only quadratic Lagrangian, but now the phi is complex. Phi is complex.

So complex is also the same as 2 real. So you can decompose phi into its real part and imaginary part. And if you plug them in here, and then you find you just get two separate pieces, and one is the real scalar field for the real part and the one is the free scalar field for the imaginary part, because each one-- yeah, if you write phi equal to a plus ib, and here, it just become a squared plus b squared, here, it's also a plus b squared.

But there's a reason we write-- yeah, so essentially, we just get two copies of our previous theory. We just get two copies of previous theory. So you say, oh, then it's trivial. Why should we just consider this-- we just get two copies of previous theory?

But this advantage-- but there's some concept-- there's something conceptually new when we write in this form. there's advantage to write in this form rather than write as two separate identical scalar field theory. It's precisely in this form-- this phase symmetry is manifest.

If I just write it as two separate real scalar field, then this phase symmetry is not manifest. And so here, in this form, there's a phase symmetry manifest. So phi goes to exponential i alpha phi. And so you see this-- you immediately see that symmetry.

So this-- when you write it in terms of real and imaginary part, this corresponding to a rotation between the real and imaginary part. And yeah-- good? And the equation of motion is the same because it's just the-- because you can easily find the equation of motion. It's the same.

And yeah, because just two copies of your previous theory. Of course, the equation of motion should be the same. And the Lagrangian density-- the momentum density for phi then becomes partial 0 phi star, and the partial phi star becomes partial 0 phi.

So we treat-- we can treat phi and the phi star as two independent fields. And then the canonical momentum-- say if you take derivative with partial 0 phi, then you get the partial 0 phi star and vice versa. So the complex conjugate momentum like that.

So we can now write down-- again, we can just write down the most general solutions. So the basis of solutions is the same. Same set of solutions as before. We just have u k x equal to minus i omega k t plus i k x, and then you have u k x star. Because it's the same equation, so the basis of the equation, of course, is also the same as before. Yes?

AUDIENCE: So if the field is an operator, doesn't it-- if it's an observable, doesn't it have to be Hermitian?

PROFESSOR: No, the fields-- it depends. So yeah, indeed if it's something directly observable, it has to be Hermitian, but that can have two observables. For example, I can combine them into a real variable. Yeah, just not necessarily the phi itself have to be directly observable. It's just a field. Yeah, the field itself don't have to always be observable by itself.

AUDIENCE: So then the conjugate momentum also is not--

PROFESSOR: Yeah.

AUDIENCE: The momentum is not our three momentum?

PROFESSOR: Yeah, this is not the momentum. This is just the canonical momentum for the canonical quantization. Yeah, this is not the spacetime momentum.

AUDIENCE: Then is the adjoint of the field operator the same thing as the star?

PROFESSOR: Yeah, here, we are talking about the-- here, we are talking about the classical theory. And then when we go to the quantum, indeed, we will replace the star by dagger. So now, we can just write down the most general solutions to this equation.

Now, I think we can just write down the most general solution to this equation. So we just-- so there's one slight difference from before-- almost the same-- identical as before except with one difference.

u k-- but here, previously, we have a k dagger. We have a k star, say, classically. Let me just first write classically. So previously, we have this. Previously, why we have a k star here when we have a real scalar field?

It's because the field has to be real, and then we have always add to its complex conjugate. But now, the difference is that now phi is a complex, and the phi is complex. And then we no longer have the real condition. That means here, I can choose another arbitrary constant here.

So this is the arbitrary constant. So by convention, let me just put a star here. It doesn't matter. It's just the name you call it. So now, the most general solution-- now, you have two sets.

You have a, which is complex number. Then you have another complex number, which is b. Now, you have two sets of complex numbers. And it makes sense because when you have a complex variable, you double the degrees of freedom, and your integration constant also doubled.

So b k independent of a. So now, you have two sets of variables. So you can get the phi star just by taking the complex conjugate of it-- just by taking the complex conjugate of it.

And then the full set of integration constant just-- you have a k-- you have a k star, you have b k, and you have b k star. So this is the classical story. So now, for quantum, we just do exactly as we did before. You just promoted all those into operating equations.

So the-- so you just call this dagger. Just call this dag-- just view every one of them as operators. And now, this just becomes constant operators. And now, here, you also just call them to be dagger.

And the-- yeah. So now, equal time commutation-- still, we have to impose the equal time commutation relation. So now, we have to impose the equal time commutation relations again. So this all should be simple. So we should have phi, phi dagger.

They can be-- they're considered as independent operators, so they should commute with each other. They're all field variables, and the phi actually commutes with itself. So this is all evaluated at different spatial-- yeah, just to save time, I will not write this expression, but all this should be evaluated at the same time by different spatial locations such as all these equal to 0.

Except the only thing which is non-zero is the phi with its own canonical momentum. So everything else will be 0. The only thing non-zero is phi, which is canonical momentum.

Again, it should be given by i delta 3 x minus x prime. And then the similar thing for the phi dagger. So again, I save effort.

So that's what you should impose. So now, if you plug those things in, then you can find the commutation relation between a and a dagger. Again, you find-- and similarly, with b, you just get two sides of-- two infinite family of harmonic oscillators.

Previously, we have one family, and now you get two families, and all other commutators vanish. So this part is boring. You can almost guess everything without doing any calculations. You essentially guessed everything without doing any calculation.

So now, again, you can write the Hamiltonian in terms of a k and b k. Just will be as you would guess them to be. Just the harmonic oscillator corresponding to a k, harmonic oscillator corresponding to b k. And then the ground state-- again, it's the state annihilated by both of them.

So this is the vacuum state. And now, you can act a k dagger on 0 and b k dagger on 0, et cetera. So now, we have two kinds of particles.

So now, we have two kinds of particles generated by a k dagger and b k dagger now. So now you have two kinds of particles respectively, and both of them have the on-shell condition. Both particles have the same mass.

They satisfy the P squared equal to m squared -- minus m squared. So because they all come from the same equation. So now, the question is, how do we tell them apart?

Now, we have two types of particles. They have the same mass, and we-- yeah, just by definition, they don't have any-- they have same mass. They also have the same spin because they're all spin 0 scalar particles. There's no directions.

And yeah, the question is, how do we tell them apart? So here is this U(1) symmetry becomes useful. Here is this U(1) symmetry becomes useful. So this U(1) so this phase rotation is a symmetry. And mathematically, this is called the U(1) transformation.

So this is normally called U(1) symmetry. U(1) is just a mathematical term for phase rotation-- mathematically, it's a U(1) group. It's a mathematical term for-- so the way we take them apart is by looking at their conserved-- look at their quantum numbers. If we look at two particles, then how do we tell them apart?

We look at their quantum numbers. And they're quantum numbers, so far, they look almost the same because their mass is the same. And by definition, they are scalar particles, and so they have 0-- they're spin 0-- they don't have spin. And so the only other conserved number-- yeah, if you want to talk about quantum number, their quantum number has got to be conserved, because otherwise, if it changes with time, it doesn't matter-- it doesn't make sense to use that number to label a particle.

But we get one more conserved number corresponding to this symmetry. So this U(1) symmetry-- so Noether's theorem-- yeah, from the Noether's theorem, so this is U(1) symmetry-- then tell us there's a conserved charge. So if you work it out-- you work out the loss theorem, then you find the corresponding conserved charge, which you should have already worked out.

So let me just write it classically. So I just write them as phi star. So classically, they have the following form. So at the quantum level, this becomes operators. But here, there's an operator-- but now, here, there's an operator of-- when we go to the quantum level, now there's an operator ordering ambiguity now.

You see, here, the phi don't commute with its own conjugate momentum. And so here, we have phi multiply its conjugate momentum. And if we change the order, and then we will result a delta function. And that will give rise to infinite nu-- yeah.

So at the quantum level, different ordering-- differ by some infinite constant. So you can take phi, and you can-- yeah, if you change the ordering, then you get a delta function. But delta function, again, is evaluated because these two is evaluated at the same x, and then you get the delta function evaluated at the 0.

So essentially, you get infinite constants. But we can fix this ambiguity by requiring your vacuum state-- your lowest energy state should have charge 0, because by definition, when you're at the lowest energy, there's nothing there. It's a vacuum.

By requiring the lowest energy state, have charge 0. So that uniquely fixes whatever infinite-- the ordering. So by requiring this equal to 0, and then you can show that the Q-- so now, you can essentially-- the Q has the following form.

So you can already-- you may be able to guess the answer of a k dagger a k, then minus b k dagger b k. That's what you get. So you plug those expressions in. You plug those expressions at this time derivative. Time derivative gives you pi.

You just plug those expressions in, and again, you find-- because this is conserved, you find all these time dependents cancel, et cetera. And then you get these two expressions up to some constant. And then this condition requires that that constant must be 0.

Yeah, when you write in this form. You see this annihlates the Q because the a is on the right-hand side of a dagger. So this automatically annihilates the vacuum.

So essentially, this is just occupation number for a and this occupation number for b k. So we essentially get Q-- essentially, the occupation number for a k and the occupation number for b k. So occupation number, I just write as N. Yes?

AUDIENCE: I still don't see how this fixes anything, because if you switch them, you still get the same issue--

PROFESSOR: Huh?

AUDIENCE: For this line of argument. If you permute them, you still get the infinite cost, so I'm confused why this is considered a [INAUDIBLE].

PROFESSOR: No, the way you permit them, then you no longer violate the vacuum, so that constant will cancel. So only this form with a is sitting on the right-hand side of a dagger, this will violate the vacuum.

AUDIENCE: Right, but-- OK. I guess because earlier, the line of argument was, if you permute them, you would get a delta function, because you would get the same thing here, right? If you permit them, do you also--

PROFESSOR: No, no, no, no, no, no, no. Here, there is ambiguity. Here, I don't know how I should order them. Whether I should put phi to the right of pi or put pi to the left-- to the right of phi. I'm not sure how should I do.

Here, there's no prescription, and here, there's no prescription. But if I impose this condition, then that fully-- then that tells you when I impose this condition, then Q must be of this form. Of course, when you permute them, it's the same thing. When you permute them, you don't change the operator.

You don't change the operator. Yeah, but if you do, say-- let's do this order. If you do this order, what you get is this expression with an infinite constant. And then this condition tells you somehow you have to change the order here so that constant is 0. Yeah, when you write it in this form-- when you write it in a and b, and that constant has to be 0. Yes?

AUDIENCE: So does the ordering ambiguity have any physical consequences, because besides just things that can be subtracted off?

PROFESSOR: Yeah, it's-- yeah, this is a good question. Which-- it's hard to say. So most of the time, you can fix them by some physical invariant. By phys-- yeah, as here, you can fix it based on physical requirement, and then you don't have to worry about it anymore.

But the ordering ambiguity requires you try to find such a physical requirement. Yeah, because otherwise, what ordering would you use? And the process of finding that requirement by itself is understanding the physics. Yes?

AUDIENCE: Should we have done that with the-- when you've got the infinite energy in the previous lecture, because you know how it required the ground state to have 0 energy and then get some different ordering.

PROFESSOR: Right. No, in that case, we don't have ordering ambiguity. So in that case, because we just have pi square itself plus phi squared, and plus phi squared. There's no ordering ambiguity here.

And so whatever you get is whatever you get. And if you get infinite constant, then you get infinite constant. You don't have freedom. Other questions? Yes?

AUDIENCE: So I don't also see how phi and phi dagger-- it's clear that they commute. How do you know that there's no coupling between the fields-- or when there is coupling, how do you represent it in--

PROFESSOR: Oh, this is a definition. You say where this come from?

AUDIENCE: Yeah. Like, why do we know for sure that it's 0?

PROFESSOR: Yeah, this is just the definition of-- this is the same. This may have nothing to do with field theory. Only this equation have to do with field theory.

This have nothing to do with field theory. This is just quantum mechanics that different degrees-- different degrees freedom, they commute with each other. Yeah, just x1 commute with x2 in quantum mechanics. Yeah, phi-- here, just phi and phi dagger just is analog of x1 and x2 there. Other questions?

AUDIENCE: Can every operator commutes do we still have quantum mechanics?

PROFESSOR: No, it's different from that. Here, it just means that the-- here, you should view them as the variables in the configuration space. And just the-- in quantum mechanics, the variables in configuration space, they always commute with each other.

Good. So now, the key thing is that here is the minus sign. So that means when you look at the commutator between Q and a, you get the a. And if you look at the commutator a with b k, you actually get minus b k.

You get-- yeah, let me just do the dagger because this is used to create particle. So if you just calculate the commutator, you just find that. So that means if you look at the states defined by one particle state corresponding to a and one particle state corresponding to b, then both are eigenvectors of Q, but here with eigenvalue 1, but this one is eigenvalue minus 1.

So it means they actually have opposite charge. So k and-- oh, by the way-- so let me explain a little bit of this notation. This k does not mean the magnitude of k.

So this k means the-- because normally, our convention is that the four vector-- say the x mu we always just write it as x. The p mu we just write as p. So here, you should view this as a four vector. It's a four vector.

It just is a shorthand notation. So k and the k bar, then they have opposite charge. Have the same mass, the same spin 0, but opposite charge. So by using this Q, we can now distinguish these two particles.

So normally, because such a particle-- because in this case, which-- they are-- all quantum numbers are the same, except they have opposite charge. We call them particle and antiparticle. So this we call-- create a particle.

So here, we say they create an antiparticle. So here, let me just make a remark without proving it. So in relativistic quantum field theory, you can show that any particle has an antiparticle. In any quantum field-- any relativistic quantum field theory, any particle has an antiparticle.

In the case of the real scalar field, you say we only have one particle. In that case, the antiparticle itself. Good. Do you have any questions? Yes?

AUDIENCE: Can we derive the fact that Q is considered just by the number in the a and the number of b is considered separately? Because I'm having trouble seeing why conservation of Q is a non-trivial statement.

PROFESSOR: Yeah, the conservation of Q, it follows from the symmetry.

AUDIENCE: But couldn't I just-- like, the number of a particles and the number of b particles are kind of separately conserved. So couldn'te I just write off that Q was conserved from the very beginning?

PROFESSOR: Yeah, it's-- no, actually, the number of a particle and number of b particle, they're not a separately conserved. They can actually-- in principle, they can annihilate. Yeah, here, in free they cannot annihilate. But if I can see the slightly more complicated theory-- say, if they are allowed to interact, and then they can actually annihilate and then disappear.

And so in that case-- but this formalism still works. This symmetry is still there. So that's why the symmetry is powerful. Also applies to the case with interactions. Yeah, indeed, what you said is correct in the free theory. Yes?

AUDIENCE: So does this mean particles and antiparticles-- like, the difference between them is conserved, so they work in pairs?

PROFESSOR: Yeah, essentially, their property-- it means their property almost identical except they have opposite charge. Yeah, they have essentially-- just like a real world, we have electron, we have matter, we have anti-matter. They have essentially identical properties. And if the world is made of anti-matter, everything still will behave the same, and yeah-- except just the charge is different.

AUDIENCE: And the difference between the number of antiparticles and particles is conserved?

PROFESSOR: It's conserved. Yeah, it's conserved. Ryan.

AUDIENCE: So I guess this is the electric charge, or is this some other charge?

PROFESSOR: I think some other charge. So in the case we will go to when we talk about electrons-- so electrons involving a more complicated formalism.

We have to introduce particles with spin. And in that case, we also have electron and anti-electron. And in that case, it's-- the story is parallel.

AUDIENCE: So this does not necessarily involve E&M or things reppelling.

PROFESSOR: No. Yeah, but this is the analog of that. So the charge here, they can be some other kind of charge. Does not have to be -- yeah.

AUDIENCE: So how can you get something that's physical that's conserved from a symmetry that's not really physical? Like, this is not like you can translate in space or translate in time. This is-- I don't understand. I don't know how to think about it.

PROFESSOR: Yeah, that's why we call it internal symmetry. So for things that we don't have good intuition about, you just invent the new name for it--

[LAUGHTER]

--and once you get used to this new name, you say, oh, I understand it. Yeah, it is more or less getting used to it. And indeed, at first sight, it's not very intuitive because it's not something you can-- yeah, from your normal experiences, et cetera.

But yeah, this is the-- but this is-- at the fundamental level, it's not that different from energy conservation. It's just some symmetry, and then leads to some conserved quantities. Yes?

AUDIENCE: If we have another conserved charge, like Q and now electric charge and-- which ones do we call, I guess, particle/antiparticle pairs? Does that make sense?

PROFESSOR: Yeah, that's right. They can, in principle, have multiple charges, but the particle and antiparticle, they're always opposite. Yeah, in terms of charges they're always opposite-- whatever charges you have.

AUDIENCE: What if there's two different charges that each one can have, like a set of four things? Is it a different language?

PROFESSOR: Yeah, I think-- yeah, you can have them-- but for particle and antiparticle, they're always opposite. Yeah, they can have multiple charges, but they're always opposite. Other questions?

Good. So this concludes our discussion of the complex field. So we have a few minutes-- then let's do to the next topic.

So hopefully-- so now, let's talk about the concept called the propagators. So again, let's recall in non-relativistic quantum mechanics-- So this position vector plays a key role. So this is the eigenvector of this x, which is the position operator.

So again, I will-- yeah, let me just put a hat here to emphasize this is the operator. And so this is the eigenvector of x. And because this plays a key role-- because we need to use this to define the wave function.

And then the amplitude of the wave function-- the square gives us the probability of a particle at location x. So this is a crucial quantity. So this language-- normally, we talk about the Schrodinger picture. You start with psi x at t equal to 0, and then you try to find-- yeah.

Or let me say, you start at psi x-- say, t prime-- at some t prime, you want to find psi x at some t. So this is the question of quantum mechanics. So if you know the wave function at the initial time and you want to find the wave function at some future time.

So now let's imagine we consider this question-- Heisenberg picture. So in the Schrödinger picture, there's just one eigenvector-- yeah, just one set of eigenvectors for x for different eigenvalues. But for the Heisenberg picture now, your position operator now depends on time.

So the position operator at different times, they are different. So now, if you want to talk about the position eigenvector, now you need the label. So now, the position eigenvector-- it's a t label.

This t does not mean that this vector evolves with time. It's just this is the label. This means that this is the eigenvector of position vector at time t. This just tells you this is the eigenvector at time t.

And now, the question of-- this question of-- this is the question of quantum mechanics-- then can be reformulated into a different equation. So in this story, the question of quantum mechanics becomes the following-- what is the value of this object? Suppose you are in the eigenstate-- you are the eigenvector-- you are in the position eigenvector-- you are in the position eigenstate at time t prime with the value x prime.

And what's the amplitude for you to go to location x at time t? And I remember, this does not involve time evolution. This is just-- because one is labeled in different eigenvector at different time.

So if you know this object, this is called-- this will be normally denoted G. G-- this is called the propagator. This is called the propagator. So if you know the propagator of a quantum system, then you have full knowledge of that system, so you already solved that system.

Why? Because say let's imagine we want to find psi t x wave function at time t, and then by definition, this is given by this. So this is the Heisenberg picture definition. It's the psi overlap with the position vector at time t.

And now, this is-- now, we can insert a complete set of state. I just insert a complete set here. Just this is the identity. And then this object is just G.

So you just get G x t x prime t prime, then psi t prime x prime. So if you know-- so yeah, so this just gave you the wave function at t prime. So now, if you load this-- given the initial wave function, if you know the propagator, you just need to do an integral. Then, you find the wave function here.

So the full knowledge of your quantum mechanical system essentially is encoded in this propagator. It's encoded in this propagator. So yeah, now, we want to talk about what is the analog of this object, say, in relativistic quantum field theory? In quantum field theory, how do we define a propagator, and what is the analog of the position eigenstate, say, in quantum field theory? And we will discuss next time.