[SQUEAKING] [CLICKING] [RUSTLING] [CLICKING]

PROFESSOR: Let us start. So last time, we started talking about the following process. So you consider e plus e minus-- so the electron, the positron they scatter and then they will annihilate and that creates some other particles. OK?

So b and the b bar-- some final particles can be different kinds of things. So if we draw the Feynman diagram with the time going up, and then the process will be like the following. So the initial particle-- so this is the e minus and this is e plus. And then we have b and the b bar, OK?

So we can-- Yeah. So such a process is a very important discovery machine for new particles, OK? Very important for discovering new particles. So let me just mention two examples here. So in 1974 to 1976 at SLAC-- so SLAC is called Stanford Linear Acceleration Center.

So they have an electron positron collider. And so there they first discovered the tau particle. And this tau particle-- it's mass is the 1.8 GeV.

So when your total energy is greater than twice the 1.8 GeV, then you can, in principle, pair create this tau part-- tau plus and tau minus. And another example is in 1974, again, at the SLAC.

And the e plus, e minus goes to c, c bar. So c is the charm quark here. OK, it's the charm quark here. So what they observed is the bound state of the c, c bar-- call it J/psi. So under the mass of the J/psi, it's about 3.1 GeV. OK. 3.1 GeV.

So this J/psi-- so this particle was discovered at the same time at Brookhaven through the proton-- through the collision of the protons by our own colleague Samuel Ting. And then they both get the Nobel Prize in 1975, I think. OK.

So there was actually a funny story regarding the discovery of this particle. So at Brookhaven, they collide the protons on the fixed target. So that is a much messier machine, because protons contain lots of quarks and gluons and so it's with strong interactions.

So if you collide high enough energy, you can create any particles. But since it's involving strong interactions, and strong interactions have many, many particles. So the so the collider at Brookhaven is very, very messy. OK. So in order to find the new particles, it's very, very difficult.

OK. But they managed to find this new particle at Brookhaven. But Samuel Ting was famously extraordinarily meticulous. OK, so he wanted to be absolutely right, so he was checking and checking. So during one of the-- Yeah, so this is a rumor.

So the rumor said that during one of the checking process-- and somehow the rumor spread that he discovered a new particle. And so the person was doing the experiment at SLAC and called him up. Even the rumor-- the rumor even-- Yeah, so the news leaked. It even leaked the energy scale of the particle they found.

And then the person was doing the register at the SLAC then called the Samuel Ting. He said, oh, I heard you have found a new particle. Samuel Ting said, no, I didn't. Yeah. He didn't want to have the-- Yeah, actually it's not very good, because I'm recorded. I'm recorded. So I think this maybe it's not a good spread rumors.

If it's not recorded, actually, I can tell the story. Yeah. Anyway so accordingly, then the register said then, oh, you didn't find the particle. Then I can discover it. Because they already knew the energy scale. They already had the energy scale. And this is a very, very clean machine.

So electron, positron, when they collide only energy created. So it don't create a lot of junks. And if you add the right energy, you can immediately find the new particle. You don't need to do much work. And so they put the machine at that energy, immediately, the new particle was discovered.

And so they were-- Yeah. So they were awarded the Nobel Prize jointly. Anyway. So this was an important discovery, because before that, people didn't believe quarks exist, OK? People didn't quite believe the quark existed, even though there are many models, et cetera. And there are lots of other evidences.

But this was the first-- yeah, because C-quark is very heavy. And so c and c bar, they bound not very strongly. And so this is the first direct evidence actually quark existed. And so this actually shocked the community.

So this was discovered in October. So they call it the October Revolution. So yeah, it played a very important role for people to accept the existence of quarks. Anyway. So in particular, by looking at the more general process-- so this goes to hadrons.

OK, you just collide them. And then you look at it. It's going to hadrons. You can actually show that the quarks have three colors. OK. You can show quark have three colors. And yeah, so we will see that, OK?

So before doing that, let's first have to do a calculation, OK? So we have to calculate this explicitly. Let me see. I get my page a little bit. Right. OK. Yeah, so first we have to do a calculation. We have to calculate this process, OK?

So after we have calculated this, then it's obvious how this can tell us why the quark have three colors. OK. Yes.

AUDIENCE: So since you only see the bound state of the quarks, how can you know for sure that they are a bound state?

PROFESSOR: Yeah, so indeed. So that's why, for many years, if you look at the proton, neutron, and the pion, they're very tightly bound. And so even though there were models about the quarks inside there-- so it's not quite-- yeah, but in the c case, c are very heavy. And so they form a weakly bound state.

And so in the sense that you're looking at the-- yeah, so here, it's-- in a sense, you can look at the C-quark, because it's a very weakly bound state of the c and C-quark. Other questions? Yes.

AUDIENCE: Does weakly bound imply it's usually larger?

PROFESSOR: Yeah. Weakly bound just say you can probe with the-- Yeah, you can essentially-- you can probe the internal structure much easier.

AUDIENCE: If electron collider is much cleaner than Hadron Collider, then why is like the most important machine right now is the Hadron Collider?

PROFESSOR: Yeah. That's good, because electron collider is more difficult, because you have to build a straight line, and because the electron have a much stronger synchrotron radiation if they're moving in the circle. And so that loses a lot of energy. And so that's the electron collider. They should build a strong straight line. But straight line you cannot have very high energy, require very long distance.

And so that's why a lot of people built the proton collider, which is easier. And yeah, it's easier to get higher energy. But the next generation of the accelerator of the LHC, people are talking about building an electron collider again.

Yeah. Good. So now let's try to calculate this. OK. So essentially we need to calculate this diagram. And yeah, this diagram is very easy. We need to calculate the cross section corresponding to this. So let me just label a label. So this is e minus. So this is e plus. So this would be b. And this will be b bar.

So let me label the momentum. So here, let's take p1, r1. And then this would be p2, r2 bar. So tell you this is an antiparticle. And so this one is k1, s1. And this one is k2 s2 bar. OK. And so the amplitude for this will be M. So the cross section.

So now, suppose we consider unpolarized process. As we discussed last time, the cross section -- yeah, so d sigma, d omega should be proportional to the spin. So we should sum over all the final states-- sum over the polarization of the final states, because we don't measure individual polarization.

But we also should average over the polarization of the initial states. So the result, as we discussed at the end of last lecture, is that you need to take one quarter, come from the average over the spin of these two initial particles, and then, just sum of spins of all the particles, OK?

OK? And right. OK. So that's the thing we need to-- so let's just try to first compute the m. So m is given by-- OK, so we just follow our rule. So you follow the fermionic line, OK? So here there's a vertex. So here there's a gamma. So each vertex gives you minus ie gamma mu. OK?

So if you follow this one, then we get v bar r2, p2, minus ie gamma mu and ur1 p1. OK? So this is coming from this fermionic line. And from this fermionic line, we start from here. So this would be the u.

Yeah. Also, we have this propagator. Let me just write down the propagator for the photon. So let's take xi equal to 1 gauge. The Lorentz gauge with xi equal to 1. And then, lets follow this line. So here, we should have a vertex gamma nu.

OK. And then we will have u bar s1 k1 minus ie gamma nu v s2 k2. OK? So this is the full amplitude.

So this is a number. OK, so this is a number, because this is a row vector. This is a matrix. And this is a column vector, so this will be a number. And similarly, this will be a number. And then the mu nu indices are contracted with the propagator of the photon.

So actually, if I want to be precise, there's actually a factor of i here. But this factor of i is not important, because-- and then the q-- so the q is the momentum of the photon here. So the q should be the sum of the initial momentum.

OK, so q should be equal to p1 plus p2. And also this means that the q square is equal to minus s, OK? Remember s is defined to be the minus p1 plus p2 square, OK? So now we can write this amplitude using a shorthand notation.

So if I forget all this momentum dependence, and then this is equal to minus ie square divided by s, OK? If you cancel all the i's, and then we have two terms-- v bar r2 gamma nu and ur1 and the u bar s1 gamma nu vs2.

And now we need to-- then what we need to do is we need to square it and sum over all the spin. And then we plug the resulting expression into the formula we derived for the cross section we derived last time, OK?

So the whole calculation is a little bit tedious. But I think we should at least go through one such calculation, because this is the kind of prime example of QED calculation. So each of us should at least see once in our life, OK? So we will do it explicitly here.

AUDIENCE: The final line needs metric.

PROFESSOR: The final line is a metric. You are right, but I can do that, just put the gamma mu here.

AUDIENCE: Do we to keep the i epsilon?

PROFESSOR: Oh, OK. Yeah, also here, because the q square is never equal to 0, so we don't need to care about i epsilon. OK? Good. So now let's do the square. Now let's to the square. So square-- we just square this term and square this term, OK?

So when we square this term-- so we also square this term. So here, we just get the e to the power 4 divided by s square. And when we square this term, we just get another-- yeah, so this is gamma nu gamma mu. So if we square it, then we get another term.

So we just copy this term. Yeah, let me just copy here. ur1-- Yeah, when we square it, we get another copy of this And so let me just write it. Yeah, sorry. So we take the complex conjugate.

So when we take the complex, we need to take the modulus. So that means we need to multiply this, with its own complex conjugate. And the complex conjugate of this is given by u bar r1, then gamma nu and vr2, and then, times this one and its complex conjugate.

And so we have us1 bar gamma nu vs2 and vs2 bar gamma nu us1. OK. So this is the original copy with the nu contract with this nu. And this is the complex conjugate with this nu contract with this nu, OK?

Good? So this looks like a mess. OK, this look like a big mess. And it turns out actually, there's a series of beautiful tricks we can use to calculate this quantity, OK? So naively, you say, oh to calculate this thing, I have to plug-in the individual wave functions for this v and u, et cetera we worked out before.

But fortunately, actually, we don't need to do it, OK? So we can actually go through a series of tricks. And then we don't have to do it. So the first trick-- you say this is a number, OK? Since this is a number, I can put a trace outside of it, because the trace of a number is just a number. OK?

And now you remember the trace have this cyclic properties, OK? Trace have the cyclic property. So I can actually move this to there, OK? So I can move it. So now I can write it as e to the 4 s square. I can just move this to there so that the two come together.

So we have vr2, v bar r2, gamma mu, and ur1. u bar r1 times trace. Similarly, I put this us to the other side, us1, u bar s1, gamma nu.

Sorry, I missed the gamma nu. Sorry, did I-- so I put this-- oh, sorry. Oh, no. Sorry, I just missed that one. u bar, r1, gamma nu. OK.

And similarly, with that, so I have u bar s 1 gamma mu. And then we have vs2, v bar s2, and gamma nu. OK? So I get that. So the reason I do that-- so now, in this form, now this is a matrix.

OK, now this is-- now this is a column. This is a row. And this is a matrix. And this is a column. And this is a row. So this is a matrix times a matrix and times a matrix and times a matrix, OK? And then you take the trace and then you get the number, OK-- similarly with things since here.

And then we need to sum over all the spins, OK? Remember, we need to actually-- when we sum all the spins, it means we sum over all possible with some order, r2, r1, s1, and s2, OK? So now, we can use a trick which we had when we discussed the Dirac spinors.

OK, so now, recall-- now, recall for on-shell spinors means they both satisfy the equation of motion, OK? Those all the eigenspinors, OK? They satisfy the following, sum over r-- if you sum over all the r of the u, urp-- ur bar p-- that give you i, ip slash plus m, OK?

And when you sum over v, you get minus i, minus ik slash plus m. OK? So now, we see we have-- if we sum over spin, then we have these combinations. Now, we can use those formulas. Now, we can use those formulas.

So now we find one quarter when we sum over all the indices, r1, r2, s1, s2, the m square-- so we get e4 divided by 4 s squared.

And then we just apply those formulas into here. OK, so we get trace minus ip2 slash plus m. So m is the mass of the initial particle. And I take the m prime of the mass of the final particle.

So the e plus minus of mass m and b, b bar have mass m prime, OK? So then I have this, then this times, then this gamma mu.

Yeah, so this is still inside this bracket. ip1 slash plus m, gamma nu. And then I have trace ik1 slash plus m prime, gamma mu, and then, minus ik2 slash plus m prime gamma nu. OK?

So I just-- when I do this-- so the spin sum is crucial for this simplification. When I sum over spins, and then without using explicit expression of those u and v, we can write all of them just in terms of p slash and k slash, et cetera. Any questions on this?

And now we can evaluate the trace. OK, so evaluate the trace. Now we need to use the various formula. I think some of them you have done in your pset. Is that now recall. So this is the place, all those exercises, they become useful.

So become mu. So here, if you look at the structure of these terms-- so here, there's a p slash. So there's one gamma matrix here. And there's another gamma matrix here. And here, p slash is the gamma matrix here. There's another gamma matrix.

So at the most, we can have the product of the 4 gamma matrices inside the trace. And also, depend on the product, we can have four, three, and two. OK? Three and two. So for this purpose, we need to delete those formulas. What are the product of the four matrices inside the integral?

So this is just given by eta mu nu, eta lambda rho, plus eta mu rho-- eta mu rho eta mu lambda minus eta mu lambda eta mu rho. OK? And also if you have three gamma matrices, do you know the answer?

AUDIENCE: Zero.

PROFESSOR: Yes. Good. And if I have two gamma matrices, do you know what is this?

AUDIENCE: 4 eta mu nu.

PROFESSOR: That's right, it's 4 eta mu mu. OK. So you can just apply those formulas to here. So we'll not do the calculation explicitly. So you can just plug them in. And then, it's mechanical. You can just calculate it.

And then you find 1 over 4 sum over spin m squared is equal to-- let me write down the answer. So even though this answer is slightly long, but I think it's instructive to write it down so that you see it.

So when you plug all this in and do the algebras, and you find that the expression can be written as this, k1 dot p1 k2 dot p2 plus p1 dot k2 and p2 dot k1.

Then minus m square k1 dot k2 minus m square p1 dot p2 m prime square. Sorry. Here it should be m prime square plus 2m square m prime square. OK.

So we discussed before that the amplitude square must be a Lorentz scalar. OK, it must be a Lorentz scalar. And so you see it explicitly, here. So it only involves either m square, m prime square, or the product dot product of the initial and final momentum, OK? And you see all kinds of combination of the initial and final momentum appearing here.

And now, so it's convention-- so as we mentioned before, actually, if you look at all these different dot product between initial and final momentum, actually, because of the momentum conservation-- And actually, there are only two Lorentz invariant, OK?

So it would be convenient to write this or this in terms of those two Lorentz invariant, OK? So that's the two of the stu variables we defined previously. So it's often just to use stu the same time. So we just-- yeah.

So now, if you write them in terms of stu, and then you find, for example-- let me just give you some example. For example, 2k1 p1.

So this can be written as t minus m square minus m prime square, which is the same as 2k2 dot p2, OK? And 2k1 dot p2, then it's actually equal to 2k2 dot p1, then equal to u minus m square minus m prime square.

And then, since related to the initial momentum-- then related to the s equal to s minus 2m square and minus k1 k2 equal to s minus 2m prime square. OK, So you just apply those equations in here. Then, you can write them in terms of stu variables.

And then, you find that one over sum over spin m squared is equal to the following expression, 2e to the power 4 s square t minus m square m prime square square plus u minus m square minus m prime square square plus 2s.

OK. So recall that s plus t plus u equal to 2. So you can eliminate one of them. So you can eliminate one of them. So this concludes the calculation of the scattering amplitude. OK, so that's the expression you get, OK?

And you see that it's all expressed in terms of this stu-- can be expressed nicely in terms of these stu variables. Any questions on this? Good.

So now, let's calculate the cross section. And our goal will be to calculate the total cross section, OK? And so for this purpose, let's do the center of mass frame, OK? So let's go to consider the center of mass frame. Well, this is the simplest.

So remember, in the center of mass frame we-- so first we need to-- yeah, so remember the center of mass frame you get that d sigma, d omega cm is given by-- yeah.

Let me check my formula. Suddenly I'm not sure what I'm looking for. Yeah, so it's given by, say, the M square divided by p cm prime divided by 64 pi square s pcm prime.

So pcm prime is the momentum for the final particles. And pcm is the momentum in the center of mass frame for the initial particles. So now, we also need to find the pcm, et cetera, OK? We also need to express the m, the amplitude in terms of the center of mass square, OK?

So in the center of mass frame, then we have the p1 would be, say, E, pcm. OK, the p2 will be equal to the same E, minus pcm. OK, they have the same E, because they have the same mass.

Similarly, k1 would be E prime, pcm prime. And the k2 would be E prime, minus pcm prime.

So now, from energy conservation, immediately concluded that E must be equal to E prime. So I can forget about E prime, because the total energy will be 2E. And the final energy will also be 2e. So e have to be equal to e prime. And then we can find-- And so E would be equal to one half square root of s, OK?

So this is the-- or s is equal to 4E square, because the p1 plus p2 is just 2E. You square it, it becomes 4E square. And then we find the pcm square equal to s divided by 4 minus m square. And the pcm prime square equal to s divided by 4 minus s prime square.

Any questions on this? Yes.

AUDIENCE: We know that [INAUDIBLE] first component of your 4 vector, they have some [INAUDIBLE]. But how do you know that it exactly has to split [INAUDIBLE]?

PROFESSOR: Sorry?

AUDIENCE: How do you know that the final state particles have to have the same energy?

PROFESSOR: Oh.

AUDIENCE: But you know that there's some that have to be--

PROFESSOR: Yeah, because they have the same mass.

AUDIENCE: I see.

PROFESSOR: Yeah. Yeah, because they have momentum opposite to each other in the center of mass frame. And they have the same mass.

AUDIENCE: So you can allow for your final particle to have-- final particles to have different masses and then [INAUDIBLE].

PROFESSOR: Yeah, yeah. Yeah, but in this process, we are considering the-- always the particle and the antiparticle. It's an annihilation process. We always create the particle and the antiparticle together. Other questions?

OK, So you can similarly express the t. OK, you can similarly express the t and u in terms of this center of mass, the momentum, et cetera. and So you can just do that. So let me just write down the final answer. Let me just write down the final answer.

You can also rewrite 1/4 sum spin, m squared equal to, in terms of the center of mass. And then you find it's equal to e to the power 4, 1 plus m squared plus m prime squared divided by E squared plus pcm squared, pcm prime squared cosine theta.

OK, and the theta is the-- so you have the scattering of the initial particle with pcm. And then you have the finite particle with the pcm prime. And so this angle is theta, OK? This angle is theta.

And so the theta comes from when you calculate using the-- involving t and u, OK? Involving t and u. And so now we have everything. So we have the pcm. It's all in terms of s. And then your amplitude is also expressed in terms of either s or E. Yeah, E is also the s. And then we have the theta, OK? Then we have the theta.

So you just plug the whole thing into here. Just plug the whole thing into here. And then you find the total cross-section. You just do d sigma, d omega, center of mass frame, and then sine theta d theta d phi.

OK, nothing here depends on phi. So the phi integral, you can just done trivially. And then you just need to do the theta integral. And then you have this. So again, let me just write down the final answer.

So in the end, you get 4 pi alpha squared divided by s. So the alpha is the fine-structure constant. Yeah, so the alpha is e squared divided by 4 pi, is the fine-structure constant. And this is 4 pi alpha squared, 3s, and then 1 minus m prime squared divided by e squared, 1 minus m squared divided by e squared.

And the whole thing times 1 plus m squared divided by 2e squared. And the 1 plus m prime squared divided by 2e squared. So this is the final answer. And this is the answer you can compare with experiments. So this is also you can compare with experiments.

Any questions? So now, if you look at this answer, it's almost completely symmetric in terms of m and m prime except that for this ratio, the m is downstairs and m prime is upstairs. Otherwise, it's the-- yeah.

So this m prime is upstairs. Of course, it's the-- do I have the-- yeah, yeah. Good. Yeah, so this factor is key. OK, you have to have such kind of factor in the upstairs because of when E, you have to be E to be greater than m prime squared in order to produce the particle.

If your energy is not big enough to produce-- in order to produce a b and b bar, your energy at least-- so each E has to be large enough to be greater than m, because your 2E, total energy, must be greater than 2m prime.

And so you see that when E is smaller than m prime and when E equals to m prime, then this factor becomes 0. And then the cross-section becomes 0. Any questions on this? OK, so now let's consider some specific cases.

So now let's consider e plus, e minus going to muon. So the muon is the next massive lepton after the electron. So in this case, say me -- m square will be m e squared. And the m prime squared will be m mu squared.

And the m mu-- so the muon mass is 207 MeV. Oh, no. No, it's the 100-- actually, I didn't code the number. So the muon mass is 207 times m, the mass of the electron. So does anybody remember the electron mass?

AUDIENCE: [INAUDIBLE]?

PROFESSOR: Hmm?

AUDIENCE: 0.511 MeV.

PROFESSOR: Yeah, that's right. Yeah, yeah, 0.5-- 1? Did you say 1?

AUDIENCE: Yes.

PROFESSOR: Yeah, yeah. Yeah, it's 511 KeV. Yeah, 0.511 MeV. And the muon is the 207 times the mass of the electron. So if you want to create the muon, then the initial total energy-- so the square root s, which is 2E, has to be greater than the 2 m mu.

So that means that the E has to be greater than m mu. And then this is much, much larger than the electron mass. So this is about 200 times larger than m E. So that means actually that, for all purposes, that you can just neglect the electron mass. So this ratio is tiny. It's tiny. So we can treat essentially the electron as massless.

AUDIENCE: [INAUDIBLE] the denominator, and we added those to the [INAUDIBLE].

PROFESSOR: Yeah, just not well defined, yeah.

AUDIENCE: [INAUDIBLE] makes sense that it would.

PROFESSOR: No, you don't even-- yeah, in this case, you don't even go to that-- so in the case that the-- yeah. Let me see. Let me think a little bit. Yeah. Right.

So yeah, for this case, of course it's not well defined. But in the opposite case, let's imagine if you have the two very massive particles to collide to create a lighter particle. And then in that case-- yeah, actually, I don't remember the answer from my top of my head. Yeah, it's a good question. I will try to find it afterwards.

Any other questions? OK, good. So in this case, so when the electron mass is much larger than-- when the muon mass is much larger than the electron mass, then we can set essentially m e equal to 0.

And in this case, the formula becomes simpler. In this case, it becomes simpler. The downstairs is just equal to 1. And this factor also becomes 1. This factor also becomes 1. So then there are two interesting regimes.

There are two interesting regimes. So the first is you just near the threshold. OK, just near the threshold means that the pcm prime is much, much smaller than m mu.

So in this regime, then the E would be approximately just m mu. And then the s would be approximately just for m mu squared. So now, if you just look at that expression-- yeah, so let's look at that expression.

Then, you find the-- yeah, then you find that the differential cross-section is equal to alpha squared over 8 m mu squared and then pcm prime divided by m mu plus higher-order corrections.

OK, so you expand in pcm divided by m mu-- pcm prime divided by m mu. And the total cross-section is just given by pi alpha squared, 2 mu squared and pcm prime divided by m mu.

OK, so here, you see explicitly that when you reach the threshold, the threshold corresponding to this quantity goes to 0. So when this quantity goes to 0, then you, at the cross-section, go to 0.

So another regime of interest is the ultra-relativistic regime. So ultra-relativistic regime. So in this case, we can take E to be much, much both than m e and m mu, so also much greater than m mu. So of course, it's already much greater than m e.

So in this case, then you can, just here-- you can just essentially take the both mass to be 0 in this case. And then you find, in this case, that d sigma, d omega cm just given by alpha squared divided by 4s-- squared theta. OK. And then you can find the total cross-section.

So you can find the total cross-section given by 4 pi alpha squared divided by 3s. So now, this final formula is very simple. So you could have written down this formula without doing any calculation, of course up to the prefactor, just on a dimensional grounds.

So in the ultra-relativistic regime, then essentially you can neglect both the mass of the initial particle and the final product. And then in this whole process, if the initial particle is massless and the final particle is massless, then for this whole process, the only scale is, what? The only scale is s, OK? That's the only scale.

So the only scale is s. And then that means-- and this should have dimension area. And then that means that there should be 1 over s. And then alpha squared just comes from because we have two vertex from a two-level process. Good?

Good. So if you draw the plot-- so often, if we draw the total cross-section times s-- so s times the total cross-section as a function of the square root of s, then the threshold is at 2 mu, 2 m mu.

And then here it goes to 0, like a square root. And then you're going to, say, approach your constant. This is 4 pi alpha squared divided by 3. So you have a curve like this.

So now let's consider using this process to show that actually the quarks have three colors. The quarks have three colors. So for this purpose, let's consider e plus plus e minus goes to hadrons. So this seemingly is a very complicated process.

Seemingly, this is a very complicated process. It's because the-- so suppose if you have e plus, you create a quark and an antiquark pair. But you remember, the quarks they are confined, OK? So we don't observe individual quarks. It's because the QCD is strongly interacting. And so the quarks are confined inside the proton, pions, et cetera.

And so in this process, even though it originally created a quark, but each individual quark and antiquark, they cannot survive on their own. So their presence will polarize the vacuum, create many other particles.

So what happens is that you create-- so each of them will create lots of particles along with them. So in the end, it's a complicated product. You observe many hadrons. You observe many hadrons.

And then, of course, in the detector, we don't directly observe those quarks and antiquarks. We just observe those hadrons. So then you say, how do we have any hope to be able to understand this process just by measuring this very complicated product, because this product can have protons, neutrons, pions, can also have other things?

And then this is a very complicated process. But now, the key is the following. But now the key is the following. Indeed, if you look at the question of going to each individual hadron is complicated. But if you can see the so-called inclusive process, which goes to all possible hadrons in this process, then that probability is just given by the total cross-section.

Remember, the total cross-section just matches the total probability to create such particles. And then when they are created, they go through a complicated process. But that process is unitary. Will not-- will be, say, all probability 1 to go to all possible final products.

So if we include the cross-section going into all possible final products, that probability just captures by the total cross-section. So the total cross-section for this process, for the e plus, e minus going to q, q bar, and then just including the total cross-section, so inclusive section for all hadron products, this way, we can actually measure this process indirectly, even though you don't directly measure the quarks.

So now let's calculate the total cross-section. So now let's calculate the total cross-section. And now we also don't have a choice which quark will be produced. So we need to sum over the total cross-section for e plus, e minus goes to, say, different kind of quarks. So i corresponding to different types of quarks.

So each of them, we can just write it down using our formula. So the only difference-- the only difference-- so the same diagram applies. The only difference is that the quark has a different charge from the electron.

So here is i e gamma mu. Here, we should use the charge for the quark. So that would be i qe gamma mu. q would be the quark of the charge compared to the charge of the electron. So then what do we get?

So suppose qi is the charge for the quark qi. And then this gives you-- so this total is going to all hadrons-- to all hadrons. So this will be given by sum over the number of colors. And c is the number of colors for each quark.

And then i will be the flavor. Now the i is the flavor of quark, so whether it's the u quark or d quark or charm quark, et cetera. And c is the number of colors. And then you have qi squared. And then you have sigma e plus, e minus to mu plus, mu minus.

The rest is the same. So for each type of quark, this is exactly the same. You just replace iq-- this vertex by iqi. And then you get the qi squared. Good? Yes?

AUDIENCE: So you talked about how when the quarks hadronize the-- polarized the vacuum.

PROFESSOR: Yeah.

AUDIENCE: [INAUDIBLE] very small [INAUDIBLE] calculate out every possible [INAUDIBLE]?

PROFESSOR: No, that's a very complicated process. We cannot. Yeah, that's a very complicated process. It's a huge field to study that, yeah.

AUDIENCE: But then how do you account for all possible outcomes outputs? You're probably accounting for all possible quarks.

PROFESSOR: No, you just measure. You just measure all possible products, right?

AUDIENCE: But here, you're just summing over the probability of the flavors and the [INAUDIBLE]

PROFESSOR: No, that's the only thing you can produce.

AUDIENCE: And then you're not worrying about the final states that happen from these quarks?

PROFESSOR: No, no, no. No, as I said earlier, we only need to worry about this process. And then this process into hadrons is complicated. But this happens with probability 1, OK? And so if we calculate this probability, then we know the total probability to here. Yeah, yeah, yeah. Yeah, because this happens with probability 1. Yes.

AUDIENCE: [INAUDIBLE]

PROFESSOR: Yeah, yeah, yeah. Indeed, that can happen. But this is the leading process. Yeah, this is the leading process. Yeah, so this is the leading approximation. Other questions?

OK, so now you're just given by this formula because you just multiply the qi squared to the total cross-section of the e plus, e minus to muon. And then you sum all the flavors and times the number of the colors. OK, times the number of colors. OK? Because all different colors have the same charge. All different colors have the same charge.

So now we can define a ratio. So this is the same for everybody. So this is just factorize. So we can define a ratio for the quantity called r-- called r. It's defined by sigma e plus, e minus goes to hadrons divided by sigma e plus, e minus goes to mu plus, mu minus.

And this is just given by sum nc times sum over i flavor and the qi squared. And this i, of course you should only sum over i, which is allowed by your energy, OK?

Those quarks which are too heavy will not be created by your energ-- You are not including the sum. So flavor of quarks allowed by energy, by initial energy. If the quarks are too heavy, then of course you cannot create them.

So now let's list the quarks we know. The process-- I will not do the final one. So we have u quark, d quark, strange quark, charm quark, bottom quark. And then there's a top quark. The top quark is very heavy. And its process will be complicated. So let's not worry about that.

So then the charge for each of them. So this is 2/3. So this is minus 1/3. This is minus 1/3. This is 2/3. This is minus 1/3. And then the mass, twice of the mass. So I'll just give you some rough number.

So u would be about a few MeV. So this is a few MeV. So this, again, is a few MeV. Just a few MeV, OK? So the strange will become-- strange is heavier, twice. It's 190 MeV. And the charm, the threshold is about 3.1 GeV. And the threshold for the bottom quark is about 9.5 GeV-- about 9.5.

So now let's consider the value of R as a function of energy. So then if you consider for s, smaller than the charm, say 3 GeV, in this case, we can just produce uds.

So in this case, we have R equal to nc then times, say this. So there's one 2/3 and two 1/3. So we have 2/3 squared plus twice 1/3 squared-- minus 1/3 squared. So this is giving you nc times 2/3.

And then the others, you cannot create. And for square root s, say between 3 GeV and the 9.5 GeV, the threshold for producing the bottom quark. Then, now you can create the charm quark. So you just add another 1/3. So then now R becomes nc times 10 divided by 9. Just add another 2/3 squared here.

And then, for now, if you have square root of s greater than 9.5 GeV, then now you can create the bottom now. Then you add another 1/3 squared. So this R is equal to nc times 10 divided by 9 plus another 1/3 squared.

So that gives you nc, 11 divided by 9. So now, if you plot the R as a function, say for square root s as measured by experiment, you should see a threshold at 3, another threshold around 10-- yeah, 9.5, say around 10. And then you see something like this.

Yeah, I'm just drawing roughly. You create something-- something. And then you have another threshold. And then you have another threshold. So it turns out the first 9 is precisely at the 2. Yeah, I'd say precisely it's just up to some arrow bar.

And the other one is about, say, a little bit more than 3. Yeah, little bit more than 3. And then your 4 is the one here. 4 is around here, yeah. Anyway, so if you compare with these numbers and you conclude that nc is actually exactly 3. Yeah, nc is 3, to be compatible with this experiment. Any questions on this? Yes?

AUDIENCE: [INAUDIBLE] Does this mean that this is the dominant process for electron, positron?

PROFESSOR: Yeah. Yeah, going to hadrons, yeah. Other questions? OK, good. So this is a very nice, simple-- even though this is a very simple process, actually it has very important physical implications and, in fact, quite profound physical implications.

So we only have a few minutes left. So let's say a few words regarding the next topic. OK, so let's again consider this process of e plus, e minus going to mu-- going to mu. So we have then this diagram. OK?

So suppose this is p1, p2, and k1 and k2. So when we draw this diagram, we take the time as going above. So these are the initial state. These are the final state.

So now, if we view this diagram sideways, say if we view in this direction, if you try to think that at the time going to right direction, say if you try to view it sideways, then in this case, if you view it sideways-- OK? And now this becomes an initial state.

So this is still initial state. And this is still initial state. Let me see. Oh, let me-- oh. One second. Let me see. Yeah, yeah, yeah. Right, right. Sorry, sorry. Yeah, view it sideways, and then this is the initial state. And this is the initial state.

Now let me draw the arrow this way so that because the-- yeah, it doesn't matter. Let's draw the arrow this way so that this is the mu, this is the mu plus. And now let's view this way.

And now this is-- look how you have initial electron. And then you have initial muon, OK? And then this becomes a process, e minus plus mu minus going to e minus and mu minus, because this is the final particle, now going out will be the particle. And this final particle going out will be a particle. And then the final will be the electron and the muon.

And then you get the process, b. So when you view it sideways, then you have a diagram like this. If I draw using our-- again, time going to up-- then this is like that. Now this is e minus, e minus, mu minus, and mu minus.

So now you wonder whether there's some relation between these two processes, whether there's some relation between these processes from you just exchanging initial state to the final state. So it turns out actually there's a very nice relation between them.

If you know one of them, you can try to deduce the other. It can help you deduce the other, which we are running out of time. So we will do it the next time.