[SQUEAKING]

[RUSTLING]

[CLICKING]

PROFESSOR: So last time we talked about path integral formulation of quantum mechanics. So let me just quickly remind you the main idea.

OK, so classically, if we specify the initial location, say, if you add t prime x prime, and then the final location, t and x, say, at time t you add x. And then there's a unique trajectory, OK? And there's a unique trajectory. So quantum mechanically you ask a different question. You ask what is the transition amplitude. If you say at t prime you add location x prime, and what's the transition amplitude at time t, you add the location x, OK?

And the answer for this question, now is equal to you just sum of all possible paths between t prime and x prime and tx, OK? So you just sum over all possible paths and with some weights, OK? So more explicitly, and with the weight you sum of all paths. And the weight is given by exponential ih s. So for each path you have an action, and then you just evaluate for each path the action. And then here also the explicit h bar here because the action has the unit of h bar. And yeah, so this would be the mechanical answer, OK?

So this is a formulation of quantum mechanics using path integral. And so we can write this more explicitly, mathematical notation for the summing over path is the following. So we introduce a notation like this, which you integrate over all possible trajectories between xt prime equal to x and xt-- or x prime and xt equal to x. And then you with rates s xt. And xt is a function of xt, which you integrate from t prime to t, alpha t double prime. So 1/2 m x dot square minus px. OK, so this is for the one-dimensional particle.

And also, so all the trajectories, this fixed endpoint at t prime should be equal to x prime, and the time t should be equal to x. And also, this integration is a shorthand notation for limit. So this t xt should be understood as a limit of when you take angles infinity this factor and divide it by 2 pi i delta t. So essentially, you divide your paths by intervals of delta t. And then you integrate over all possible-- yeah, OK? So you separate the path from t prime to t by n segment. So this is the ti, and then the location will be xi. So this is ti and location, and then you integrate the xi over that, OK?

So this is the formulation of path integral for quantum mechanics. Any questions on this? Yes?

AUDIENCE: Can you do this computation in some arbitrary basis that's not necessarily a position basis?

PROFESSOR: Yeah, yeah, in principle, you can also do this in momentum basis, yeah, momentum space. And also, you can actually also generalize-- so yeah, but yeah, we will talk about how you generalize this to other bases. But this is the fundamental formulation. And when you write it in other bases essentially you reduce to this basis because this is the most intuitive way to formulate it. Any other questions? OK, good?

So a simple example it's just a free particle. When you have a free particle, then the s is particularly simple. You just integration of 1/2 mx square, OK? So we described that when s is equal to this, and then this is like a Gaussian integral, OK, because this is quadratic in x. So this is like a Gaussian integral. And so you can reduce-- so in this case, you have a Gaussian integral. Then schematically you have something like this.

So when s is of this form, you can write it in this general form. You can write the path integral in this general form. And the k, in this case, will be some delta function. Yeah, I wrote down k last time explicitly, and then this is like a Gaussian integral. And in the Gaussian integral we can just directly write down its answer and some constant divided by delta k. So this determinant is defined in the space of functions and C, some constants, OK.

So you may also view this just as a lateral. So this also just follow from this continuum limit, OK, follow from this discrete case, which you take a continuum limit. Yes?

AUDIENCE: So when you evaluate that Gaussian integral, there's an i in front, in the x component.

PROFESSOR: Right.

AUDIENCE: So how does this converge?

PROFESSOR: Yeah, yeah, this is the same as you do the standard Gaussian integral with the i. Yeah, that integral you can also do.

AUDIENCE: Repetition of this divergent it's just--

PROFESSOR: No, yeah, just with a single-- so this integral we can define, right? This integral is-- we know how to do this integral. It's the same as this one, yeah. So yeah, so you have to do a mathematical trick to do this integral, but this integral is defined. We can give a value. Other questions? OK, good.

So this is the path integral for quantum mechanics with one degree of freedom. So now we can just immediately generalize this to more degrees of freedom, OK? And there's nothing really change. When you have more than one degree of freedom, just integrate over-- so if you have, say, three particles, then you just integrate the dx1, dx2, dx3, and then use the same action, OK? So you just straightforward generalize. And then we can generalize to field theory because field theory we just take lambda degrees of freedom to infinity, OK?

So we can now-- yeah, let me just-- so to field theory, so in this case we just need to replace in the different places. So remember, so the dynamic variable for a single particle you have this operator, and then when you go to field theory the counterpart of this is the phi, OK, is the phi, is your field variable. And then so this quantity when you go to field theory, the corresponding quantity is phi x s mt and phi x, say, phi prime x at time t prime, OK?

So again, the way to think about this is you should view this x as label. And then this is just like as if we have infinite number of positions, OK? You just have infinite number of phi's, which are eigenvalues of this operator at time t, at t prime, and similarly here, OK? And so again, this one, you can write it in the form in terms of the position eigenstate. So this is the position eigenstate in field theory. OK, so this is the position eigenstate in field theory.

And then we can use the same technique we discussed last time. We just split this into many, many infinitesimal pieces by splitting the time interval from t prime to t and split into many pieces. And then just do it over and over. And insert a complete set of state of phi, OK? And now it just-- so h is now, of course, it's the field counterpart of it, just h equal to-- but the details of this does not matter.

Say this is the Hamiltonian. You can just insert this into here. And then you just repeat the same procedure, OK? Just keep in mind, now you just have an infinite number of degrees of freedom, OK? That's the only difference. Your rotation becomes a little bit more complicated. But if you keep a very clear mind, with the correspondence with the single particle case, OK, then everything is the same, OK? Just if you understand how to translate the notation here to the notation here, then everything is just exactly the same, OK? Just everything goes through. So I will not repeat that procedure.

And now you just find this quantity, OK? Then again, we can write this in terms of path integral. So now this can be written as path integral phi. Now you integrate over all possible configurations of phi with the boundary condition phi t prime x equal to phi prime x and phi tx-- you could do phi x. And again, you just have exponential as phi. And s is its action for phi. So now the s is now-- so in this case, just it's the integration of the Lagrangian density, and L is 1/2. Yeah, it's just what we wrote down before, OK, some potential phi, OK?

So for any scalar field theory we just work identically, OK? And again, you just get the integration over the Lagrangian, which is the become full integral of the Lagrangian density, OK? Any questions on this? Good?

So whenever you get confused about-- whenever you get confused about passing the going field theory, then try to translate into the language of a single particle, OK, by doing this kind of replacement. And then you will be able to settle your problem. And then, whenever you get confused about the path integral in this quantum mechanics, then just a reduce it by a finite dimensional integral. And then you should be able to understand, OK? Just always reduce it to the simpler case, and often your confusion can be understood in that simpler case, OK? And yeah.

Good. So now let me explain a little bit what this notation means, OK? So this, say, in words means you integrate over all possible configurations of phi between t and t prime and with the final configuration, the initial configuration to be phi prime and the final configuration to be phi, phi x, and the initial to be phi prime x. So let me just say previously d xt, if you think about it in terms of discrete case corresponding to-- you just take the product of all the i at different times, d xi, OK? So in the continuum limit it's like you just take the product of all possible value of t and then integrate over the value of x at that particular value of t. So that's the meaning of this d xt, which is up to a prefactor corresponding to that.

So similarly, phi, is the same scene. So now remember phi x is the now become the dynamical variable. So this now become sum over, take product over t. Oh, yeah, no, no, no. First, you think of take product of x. So this is a label of x you have. So x are the labels of phi, and you take all possible value of x. And then each of them is just like-- OK, so just like you have so many different variables, each x label one degrees of freedom, OK? And then you have phi t. And now then you use this one. Then you have tx d phi. Is it clear what this equation means?

So this step tells you that you have many, many different degrees of freedom. So this step is just enumerate all possible degrees freedom, which are labeled by x. It's just like your standard integral. If you have five dimensional integral, and then you just have a product of five different variables, OK? Here we have all different possible value of x variables. And that variable is a function of t. And then now we use this equation. And now each of them is like here, OK? And then now you have the product all possible value of t. And then you integrate then all possible values of phi, say, at point x and t, OK? But remember, t always is between t prime and t, OK? Yes?

AUDIENCE: [INAUDIBLE] notation for the first equation, the second equality, what is plugging in--

PROFESSOR: You mean here?

AUDIENCE: The one above it. No, the one above it.

PROFESSOR: Yeah, yeah.

AUDIENCE: So that second quality. So when you relabel [INAUDIBLE]--

PROFESSOR: Yeah, yeah, yeah, here I just write it in the continuum form. So here I label it by ti. So each location ti have d xi in that form. But if I go to continuum limit, n going to infinity limit, essentially at each point t, that have integration of x. And so that's roughly the continuum form of that. And here it's similar, OK? Good.

So once you have learned this trick to the reduction, and I think you will be able to settle all your confusions about these definitions, OK? OK?

So other than that, other than this additional label x, so other than additional label, yeah, I should call it here label x. So the field path integral go for scalar field is essentially identical to that in quantum mechanics, OK? So just to emphasize this point.

So here actually you can define this general. Doesn't matter what your v is, OK? You can choose whatever v you want. The story does not change. Just like when we do the single particle case, it doesn't matter what this v is, OK? You can choose arbitrary v you want. But if we choose v to be-- when v is for the free theory, then in this case, of course, a particularly simple. And then in this case, the path integral is again Gaussian, OK? Path integral is again Gaussian. And we'll not write it again. So again, that formula and this formula applies, just this case in a more complicated space. k is operating in a more complicated space, OK?

So now you not only have the space of t, but you also have the space of x. But once you generalize to some space of functions, it doesn't matter. This function becomes more complicated, OK? And so conceptually they're the same, conceptually the same. Good?

So now for interacting theory, then essentially then the quantum field theory then reduce to doing this path integral, OK? If you know how to do this path integral, and then essentially you know how to solve the theory, OK? You know how to solve the theory. And so as I said, in the free theory case, it's essentially reduced to a Gaussian integral. Then everything is simple, and we will go back to discuss that in a little bit more detail later. And now let's just think a little bit how to treat the interacting case.

So recall our goal is to compute this object, the vacuum [INAUDIBLE] value of time ordered correlation functions, OK? We want to compute this object, OK? So now we will discuss how we use path integral to compute this object, OK?

So before doing that let's first try to do this in-- so before doing that, let's first discuss how to do this in quantum mechanics. Before doing this in field theory, we can just first understand how we compute the similar object in quantum mechanics. And once we understand how to do that, again, the generalization to field theory will be straightforward, OK? Good?

So first we talk about time ordered correlation function in quantum mechanics, OK? So we will be introducing a number of tricks, and those tricks will then take over to field theory, OK, then will be carried over to field theory. So again, let's just consider this system, OK? Let's consider a system like this. Just visit the Lagrangian of this form, just one particle theory, OK?

And then the analog, then the analog of this object is this object. Let's call it Gn would be, say, so let's call the 0 is the vacuum of the field theory-- of this quantum mechanical system. And then you have time ordered, and then you have xt1 because the dynamical variable here for quantum mechanics just x and x tn at different time. And then 0, OK? So we want to compute such object in quantum mechanics.

OK, so our goal is to develop techniques. So you should imagine all these are operators, OK? All these are operators. So our goal is to develop techniques to calculate this. OK, so before doing that, let's first understand how to do this time ordering using path integral, OK? How do you do the time ordering using path integral. So before I do that, do you have any questions?

OK, so it turns out, actually, path integral is some of the most natural objects to think about such kind of time ordered correlation functions, OK? And we will see in a minute, OK, the path integral is actually the most natural framework to think about this kind of time ordered correlation functions. So before doing that, let's consider one simple example, OK? So firstly, consider--

So let's consider some other time t1 between t prime and t, OK? So we can see that, again, we go back to this problem, OK? We go back to this problem. But we can see there's some time in between t prime and t, OK? So now let's consider this object. So the simplest case of this one, you just take n equal to 1, OK? You just have one of them. Let's consider the simplest case, OK? So let's consider this object.

So this is easy to do. You can just use the same technique we used before. So essentially, most of tricks in quantum mechanics just reduce to one trick. Yeah, I say most of the case, OK? Most of the case, tricks in quantum mechanics will be reduced to one trick. What is that trick, which we already used over and over saying driving the path integral.

AUDIENCE: Identities?

PROFESSOR: Yeah, insert identities. So if you know how to insert identities at the right location at the right time, and essentially you can-- yeah, you know all the tricks in quantum mechanics. And so here we do the same thing. So since these are t1, so we just insert the complete set of a position eigenstate at t1. So let's just do this. So we integrate x1 t1, OK, x1 t1, x hat t1 x prime t prime, OK?

So this integrates to 1, OK? So this integrates to 1. So since this is eigenstate at t1, so this act on this, you just get your eigenvalue. So we just get d x1, x1. So we just get eigenvalue. And then you had xt x1 t1, x1 t1, x prime t prime, OK? But now we know how to do both of them. We just plugging our path integral formula to both of them, OK?

So you can do this explicitly, plug in-- plug in in this expression there, OK? Plug in this expression there. But it's also very intuitive when you know what this looks like, OK? So this is like the following. So previously, we just do all the paths between these two, OK? So now we say now you have this xt, and now we have t1. So suppose this is the time t1. Suppose t goes up, and so this is t prime. So suppose this is the time t1, and this is ti prime x prime, OK?

So this corresponding to you first do the path integral to some location t1. Integrate over all paths here. And then you multiply x1, and then you do all the path integral here, OK? And then you integrate over x1, OK? You integrate x1, OK? So now if you-- without this x1, OK, without x1, then of course, this is trivially equal to the previous one because you do all possible paths to t1 and all possible paths from t1 to t. And then you iterate over all possible location here. Then it's the same as you integrate from here to there, the arbitrary path, OK? So the only difference is now we multiply by x1, OK, and the value at t1.

And now we can immediately write down in path integral what is this object? So this object is essentially, OK, it's just equal to-- so now I will use a simplified notation, say, here write x prime t prime xt to be that limit, OK? To be that limit. And then again we integrate all possible paths between them, but then we just have x t1 here. OK? You just integrate over all possible location, then just add this time t1. You multiply in the integrand and the value of x at t1. Yeah, so this is essentially just the x1 here, OK? Is this clear?

You can do this explicitly by plugging those formulas and then manipulate it and you will get this. But it's much easier to understand it heuristically, OK, by using a diagram. Good? So this is a very, very suggestive. It tells you, when we insert an operator here, what we do is just we translate the eigenvalue of this operator, OK, plug it in the integrand, OK? Plug in the integrand. So now you can do the same thing. Suppose you have two operators, OK? So now let's look at-- we have two operators, OK?

So you can almost immediately write down, if you try to generalize that, what would you write down the answer? Yes?

AUDIENCE: [INAUDIBLE] s and t1 and s and t2 [INAUDIBLE].

PROFESSOR: Good, so you just d xt, but this is almost correct, not completely correct for one reason, which actually one of you asked before. Yes?

AUDIENCE: Is it because you have to order them?

PROFESSOR: Exactly, this only equal to that only for when t1 is greater than t2. So remember, all these paths, they don't come back in time, OK? The paths, they go forward in time. They don't come back. So here is like you have to insert the two here. Just insert the two sets of complete states. One is here. One is here. But the path integral cannot come back, OK? So the order here have to be the same as the ordering of the path integral, OK, have to be the same ordering of the path integral. So that means this is equal, OK, only for t1 greater than t2, OK?

But the same thing happens, but for t2 greater than t1-- so suppose t2 is greater than t1, if you start this expression, then we can ask what the corresponding the operator form. So what do you think will be the operator form? Yeah, so for t1 greater than t2 we have this equal to that, OK? But now let's ask-- suppose it's opposite, t2 greater than t1. But on the right-hand side, we still have this one. But what should be the left-hand side?

AUDIENCE: X t2 x t1.

PROFESSOR: Exactly, just exchange x2 and t1 because path integral always follows the time order. So that means the order here has always to be time ordered. So that means that the correct formula, which applies for all t1 t2, is just you time order them. You just time order them, OK?

So no matter for what the value of t1 and t2, if you time order them so that the x which is the largest time always sit in the front, then it's always equal to this one, OK? So this now just immediately generalize, OK? So this tells you, because of the path integral, in the path integral the time only goes forward, the time only goes forward, then the time ordering naturally arise, arises in path integrals, OK?

So you can immediately generalize this. So for any t, for any t1 tn between t prime and t, then you always have xt time ordered x hat t1 x tn 0. And this is equal to x prime t prime xt d xt and x t1 x tn exponential is xt, OK? So that's why we say earlier that using path integral-- so our goal is to compute this object. So that's why actually using to compute it is very natural because the time ordering is just very natural from the point of view of path integral. Yes?

AUDIENCE: Should x and x prime [INAUDIBLE]?

PROFESSOR: Hmm?

AUDIENCE: Should x--

PROFESSOR: Oh, sorry, sorry, yeah, yeah, yeah, right. Good. Good? Any questions on this? So this is a key formula, OK? So this is a key formula, OK? Good? Any questions on this? So now we have got the time ordering, OK? Now we want to compute, but here is the vacuum correlation function, OK? So we have to actually go to the vacuum, OK? Here is in such kind of between the position eigenstate, OK? So now we have to see how to do this for the vacuum.

So now let me just introduce a simplified rotation. So let me call this whole thing X, capital X, OK? I call this whole thing capital X. So now, we want to consider the vacuum correlation function. So the idea is very simple, OK? Right, OK, so we want to consider a vacuum function we want to interested in the Gn, Gn for arbitrary t1 tn belongs to minus infinity to plus infinity, OK? So you want to compute this. OK, and the Gn using [INAUDIBLE] notation we now equal to 0 x 0, OK? So [INAUDIBLE] to 0 for that.

So now again, we insert the complete set of states, OK? Again, we insert a complete set of states. We insert 1 equal to dx xt, which is also equal to dx prime x prime t prime x prime t prime, OK? This t goes to plus infinity, and t prime goes to minus infinity into the here, OK? So insert that into here, OK? And then we find-- so yeah, so let me-- I need a bigger space. So let me just write it down.

So if you do that-- so this trick is general, does not restrict to the vacuum. So in any case, evaluating any state, you can reduce it to the path integral by doing this trick of inserting these guys. So then we have Gn now just equal to. Now we have limit t equals to plus infinity. t prime goes to minus infinity. Then we have dx dx prime. So we insert 1 here and the 1 here, OK, in both places. Here we insert the upper line. Here we insert the lower line, OK? Then we have 0 xt, then xt x x prime t prime, and then x prime t prime, then 0, OK, times, OK?

So now we just have-- yeah, so let me just save some-- so this then become dx dx prime. So these two are simple. So essentially they just become the vacuum wave function, OK? So we have psi 0 star x and psi 0 x prime. And then we have this path integral, which we already know how to do, OK? We already-- then we have this path integral. OK? Then we have that path integral, OK? So yeah, so [INAUDIBLE] equals star, you just insert the star here, OK?

And now with t and t prime to infinity-- [INAUDIBLE] to infinity. And yeah, the reason we take t prime and t go to infinity is obvious because we want to include-- so here the t has to be go between the t prime and the t. So if we want to have arbitrary t, and then we want the t prime to go to minus infinity and t go to plus infinity, OK? So we have used that the x prime t prime 0 is just equal to psi 0 x prime, just the ground state wave function, OK?

So that means that-- so this trick actually works for other states too, OK? You can do the similar trick. And you just essentially get two more integrals to do, and you take that path integral. And then you just integrate it over the initial and the final wave functions, OK? Integrate over initial and final wave functions, OK? So the trick applies to any state, OK? You understand, not just the vacuum correlation functions. Yes?

AUDIENCE: This 0 [INAUDIBLE] because this is still [INAUDIBLE] your [INAUDIBLE] state [INAUDIBLE] energy eigenket? [INAUDIBLE]?

PROFESSOR: Yeah. Sorry, say it again.

AUDIENCE: So since this is still quantum mechanics, I'm saying the 0 [INAUDIBLE] is like an energy eigenket.

PROFESSOR: Yeah, yeah, yeah, yeah, it's a ground state. It's the lowest energy eigenstate, yeah. It doesn't matter. You can also take other states too. It just will be some wave function there.

AUDIENCE: Sure.

PROFESSOR: Will be some function there, yeah. Yes?

AUDIENCE: So [INAUDIBLE] someone who said [INAUDIBLE] evolved and then we'll [INAUDIBLE] here would cancel [INAUDIBLE]--

PROFESSOR: Sorry?

AUDIENCE: --t and t prime?

PROFESSOR: Oh, yeah, yeah, if you not take-- yeah, that's a very good question. So if you don't take-- and so if you take other states, and then in general there may be some dependence on t and t prime, the t in this wave function, indeed, yeah. So you have to take the limit also in the wave function. Yeah, but here we are using the simplification that the ground state is actually time independent, yeah. Good, yeah, that's a good question. Other questions?

OK, good. So this works essentially for any state, but for the ground state, actually, there's a lot of trick which can simplify the problem, which you don't even have to do this two additional integral, OK? You can actually directly-- so for the ground state, for the vacuum, there's actually another trick to get rid of-- to make these two additional integrals are necessary, OK?

So now let me tell you how you do this trick. And this trick is very important also in quantum field theory because if you do a harmonic oscillator, of course, without the wave function. But if I give you a harmonic oscillator, which we don't know how to solve the ground state wave function, and then this will be a nightmare, OK, because then you don't know the wave function. And for quantum field theory, in particular in the interacting case, we don't know the wave function, OK?

So even though this formally gives you the answer, but in practice, it's actually often not convenient to use, OK? But fortunately, for the ground state, this additional trick, which actually you don't need to use the ground state function at all, OK? So now I tell you this additional trick. But this trick only specific to the vacuum, OK? You cannot apply it to other states, OK? So this is specific to vacuum.

So now let's forget about this thing, OK? Now let's forget about this thing. Start coming back from here, OK? So now let's coming back from here. Now let's coming back from here just consider-- again, I take limit t goes to plus infinity, and t prime goes to minus infinity. Let's look at this object, xt x x prime t prime, OK? OK?

So now, again, we are going to insert identity, OK? Now we are again going to insert the identity. And now we insert the identity in a different way. Again, we insert the identity here and insert identity here. But now we insert identity expressed in the complete set of energy eigenstates, OK? So now we insert the complete set of energy eigenstates. So I just formally labeled by m, OK? Now, let's just insert in the two places, OK? Yeah.

OK, so now we have two identities. Then we have n and m. We have n and m. So now I will-- so you keep this in mind. I will not copy this over and over, OK? So keep this limit in mind, always t prime goes to minus infinity. So now we have-- and then we have xt with m and mx with n and n with x prime t prime, OK?

So now if we look at this sum, so now if we look at this sum, if n and m equal to 0, this is the object we want, OK? This is the object we want. But this also contains many, many other stuff. So now we will use a trick to isolate n equal to 0 and m equal to 0 piece, OK, to isolate that piece, OK? That's what we are going to do now.

So this is the commonly used technique in quantum field theory, not only in quantum field theory, actually in many areas of physics. So now let's first-- to explain the trick, let's look at this object. So let's look at this object, OK? So this object is limit t prime goes to minus infinity, some energy eigenstate n, and x prime t prime. So again, this is written in the Heisenberg picture. So we need to so translate to your more familiar language of the Schrodinger picture. So we need to write it as n, and then you have exponential ht prime and then x prime, OK? So now you see the standard Schrodinger picture state.

So now since n is an energy eigenstate, we can just act this one, OK? So this is just equal to limit t prime goes to minus infinity exponential i En t prime and x prime, OK? So now we will try to select the ground state, and we do that by doing the following. So now imagine giving En a small imaginary part, OK? OK, imagine-- yeah, take En goes to En 1 minus epsilon. So epsilon is a small positive number, OK, which is equivalent to taking-- so all you just take H, your Hamiltonian, take into 1 minus i epsilon, OK, give you a Hamiltonian, a slightly imaginary part, OK? It's equivalent, OK?

So now with this, and now we have-- so limit t prime goes to minus infinity. So now we have exponential i En 1 minus i epsilon t prime and nx prime, OK? So this is just equal to that. So now you see, suppose we normalize our state so that-- yeah, so by definition, En are greater than E0, greater than E0 for n greater than 0, OK? So the effect of this putting an epsilon here, when you multiply it, it will give you a factor like this, epsilon En times t prime, OK? So give you a real factor like this, OK?

So now t prime goes to minus infinity. And the En is greater than E0. So for any state which is now ground state this factor will be exponentially small, goes to 0. So the ratio between this factor and the ground state goes to 0 for any excited states, OK, because of the En minus E0 greater than 0. And this goes to minus infinity. And epsilon is positive, OK?

So this implies-- so this implies, so we continue-- sorry, it's just-- so let's continue over here. So this means we get essentially 0 for n not equal to 0. And for the exponential i E0 1 minus epsilon t prime for n equal to 0, OK? Yes.

AUDIENCE: If you want to formally show is that all energy levels that are not [INAUDIBLE] state get exponentially suppressed divide by the ground state energy to the [INAUDIBLE] energy? Or how do you do that?

PROFESSOR: Oh, you just take out the overall factor corresponding to the ground state. Yeah, then all the other coefficients wil go to 0. Yes.

AUDIENCE: What is the justification or why are we allowed to add a small imaginary part to the energy?

PROFESSOR: It's a mathematical trick. And at the end of the day, we set epsilon equal to 0. And then you go back to your original Hamiltonian. So this is just a trick. If you're doing this order properly, it just naturally select the ground state for you. Yeah, yeah, just a pure mathematical trick. Yes.

AUDIENCE: The epsilon doesn't have a certain sign?

PROFESSOR: Yeah, epsilon-- when we write this epsilon we always assume epsilon is greater than 0. So yeah, yeah, you always assume epsilon-- yeah.

AUDIENCE: [INAUDIBLE]?

PROFESSOR: Hmm?

AUDIENCE: I don't see why you can't do that [INAUDIBLE].

PROFESSOR: Oh, I can-- I have a path integral. And essentially I can design the rule I want as far as at the end of the day I get the desired quantity. Yeah, but the order of limit is very important. You have to be very careful. Yeah. Yes.

AUDIENCE: Can you like get around this with some sort of a stationary phase or a saddle point expansion because in the end you're going to be integrating exponential factors. And so the term that dominates--

PROFESSOR: No, no, here you cannot do stationary phase because if you just-- yeah, yeah. Yeah, you really need to suppress the other contribution, not only suppress them, they have to go to 0, yeah. [INAUDIBLE] go to 0. OK, so similarly, this is the same trick, just the same scene in one shot, OK? Select n equal to 0, but you see, actually also select m equal to 0 because you see? If it's the same sign, if it's the same sign, you can easily check yourself when t goes to plus infinity. So these now become limit t equal to plus infinity. So exponential minus i Em 1 minus epsilon t times xm. And then again this become 0 for m not equal to 0. And then the ground state.

Well, n equal to 1 for m to 0, OK? So this simple trick beautifully select the m equal to 0 and n equal to 0 piece, OK? So now we can write-- so this is just the ground state wave function. And this is just the ground state wave function, OK? So now we have limit t prime goes to minus infinity, and t goes to plus infinity, x tx x prime t prime. So that equal to-- so you have the ground state wave function star x prime x-- no, 0 x 0. And then you have this phase factor. And everything else goes to 0, OK?

So equal to that, OK? So now after those factors-- after those factors we get what we want, OK? We get what we want. But still you have lots of loading factors, but they can be easily got off. They can be easily get rid of us using the following. So now, you also, similarly, the same exact same reasoning-- so in this discussion, whether you have x there or what's the form of x doesn't matter, OK? x can be anything. So x can be anything.

So you can just take x equal to 1, OK? And then you also have limit t goes to plus infinity. t prime goes to minus infinity, the xt x prime t prime. So this is also equal to psi 0 x psi 0 star x prime. And now it's the 0 with 0, OK, because you've set x equal to 1, OK? And then you just ground state. And so this is just equal to 1. And then you have the same factor, OK?

So now, we can just find the Gn, OK? So this object is Gn. So Gn can be defined as the ratio between the two. Take the limit, t plus infinity, t prime goes to minus infinity of xt x prime t prime, and xt and x xt, OK? You can just find that as a ratio between the two. So this gives you the expression for calculating the scoring function, OK, this correlation function. So on the axis, any time order the product, OK? And so now, for each of them, we know the path integral. And then you can just write down the path integral for each there.

So now you see this Gn is in principle a physical observable that actually involves in the ratio of two path integrals, one without any x and one with some x, a visual desired correlation functions. So that I mentioned before that when you evaluate the path integrals you will get that kind of C determinant of those since they just cancel between upstairs and downstairs. Later we will see explicitly, OK?

So yeah, so this is a beautiful formula, OK? And yeah, and then you don't have to-- yeah, yeah, so all those factors canceled. You don't need to know the ground state wave function. You can just perform that path integral, OK, perform that path integral. And here, in principle, you can also take arbitrary x and x prime, OK? You can take up to x and x prime. Doesn't matter. So conventionally, we just take x equal to x prime equal to 0 just for simplicity, OK? We can just take them to 0. OK, any questions on this? Yes.

AUDIENCE: Why can you specify both points in your [INAUDIBLE]?

PROFESSOR: Yeah, the only thing matters is the-- so they only come into this prefactor and then cancel between the two. It doesn't matter. Yeah, so all the factors there, and these factors, they just cancel. And we're just left with this x minus 0 left. So it doesn't matter. They cancel anyway. So it doesn't matter where you put the x and x prime. Yes.

AUDIENCE: So we didn't actually take when the limit as epsilon goes to 0 or anything. So it doesn't actually matter--

PROFESSOR: So normally, when you calculate that at the end, and then you take epsilon goes to 0. So we just normally leave that implicit. Yeah, when you do the path integral you keep a small epsilon. And so after you finish doing the path-- after you finish in the end you take the epsilon goes to 0. Yes.

AUDIENCE: Sorry, also on why would we can take x and x prime equals 0. Doesn't our path integral, the bounds depend on x and x prime [INAUDIBLE]?

PROFESSOR: No, the bound does not depend on. The path integral is only limited by t and t prime t. Yeah, initial value, they're the same. Of course, specific value of this path integral will depend on x and x prime. But they cancel between upstairs and downstairs. So they cancel, yeah. And so in the end this right-hand side does not depend on x and x prime.

Yeah, also you see here. If this formula make any sense, the left-hand side does not depend on x and x prime. So the right-hand side must not depend on x and x prime. Otherwise, this formula immediately wrong. Yes.

AUDIENCE: [INAUDIBLE] the same. They have to keep a small epsilon [INAUDIBLE].

PROFESSOR: Yeah.

AUDIENCE: So again, [INAUDIBLE] goes to 0.

PROFESSOR: Yeah, that's right. That's right. Yeah, just when you do the calculation you keep a small epsilon. When you do the path integral you keep a small epsilon. But when you finally calculate the final answer, you can set the epsilon to 0.

AUDIENCE: Do we do that before we take a t [INAUDIBLE] t also [INAUDIBLE]?

PROFESSOR: No, you have to keep an epsilon. This you have to take first. So this is part of your calculation. You have to always take this first because the final answer also does not depend on t and t prime. And because this is the vacuum, this correlation function does not depend on your t and t prime. So you always set the-- yeah, you keep a small epsilon. Take this limit first, and do the calculation. Do the calculation. And then when you calculate everything at the end of the day, then you set epsilon to 0. And then that guarantee you choose the vacuum, OK?

Good. There's one more trick we have to do, OK? Sorry, there are too many tricks.

[LAUGHTER]

There's one more trick we have to do. So now we have told you how to calculate this guy for arbitrary x. But often, it's actually not-- it's not the smartest way to directly calculate such an endpoint function, OK? It's often not the smartest way. So there's something a little bit more clever. It's called generating functionals. So some of you may have seen this, say, in your high school math competition.

[LAUGHTER]

So suppose we want to do an integral like this, OK? So let's do-- again, let's go back to our ordinary integral. So let's just suppose we want to do an integral like this so. So do you have a parallel. Let's just say, so let's sat we have some phase, like what you do in the path integral. And we want to calculate x to the power n, OK? n is some integer. Here, you have an endpoint function. Suppose we want to calculate this.

So you can just calculate this guy. It's fine, OK? But there's a better way. Often, there's a better way. Instead of calculating this for individual n, you can actually consider the following quantity, za, which is defined to be dx expansion i lambda fx then plus i ax. So you can see that this object-- so this object, when you expand this exponential, yeah, so [INAUDIBLE], I think I don't need this anymore. Yeah, let me just erase this.

So now, why we are interested in this object, because imagine you expand this exponential factor. And then essentially you get the a is exponential n equal to 0 to infinity n factorial i an zn. So if you expand this thing in power series of a, a power series-- if you expand this, and then just n's term will give you an x power n. And then it's the same in that we just expand this za in power series. And then the zn would be the n's coefficient, OK?

So if you compute za in one shot, and then [INAUDIBLE] you can just do a Taylor series, OK? So sometimes we also write zn [INAUDIBLE] i to the power n. Then you take n's derivative of the a over a. And then you set a equal to 0, OK? So this is the same thing. So when you take n derivatives of a, you get rid of the lower powers. And then the higher powers you get rid of by setting a equal to 0, OK? And then let's just pick this term, OK, pick the n term.

So the a is called the generating function, OK, generating function. Oh, OK, so now when we want to-- so one final remark, so now we want to compute this guy. So this Gn is equal to t x1 to xn, so x t1 x tn, again, instead of computing each one of them, we can consider such a so-called generating function a generalization of this idea. We can consider z jt. You can consider this object d xt exponential is xt. And now, add i dt from minus infinity to plus infinity jt xt, OK?

So remember this is just the path integral. So let's just look at the upstairs, OK? Exponential x exponential is xt. So now this starts the same thing. So now if you take this, imagine you expand this factor, OK? And then to nth power, then you will generate this term as a coefficient, OK? We will discuss that in more detail next time, OK? So often, instead of computing this object directly, we compute a generating functional, so-called generating functional, which often makes things much easier, OK, often makes things much easier.

Yeah, so with this preparation, and then we are ready to tackle how to treat this thing, how to actually find in such object in quantum mechanics. And once we have that, doing field theory will be automatic. And then you will then know how to actually calculate the scattering amplitude, which is a good achievement. Yeah. OK, so let's stop here.