Recitation 10

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April 28, 2023

1 The Electroweak Theory and the Higgs Mechanism

1.1 A Toy Model

We now have enough background to discuss one of the most interesting aspects of the Standard Model. In particular, we'll be focusing on the electroweak sector.

The electroweak sector of the Standard Model is a chiral gauge theory, described by the gauge group $\mathrm{SU}(2)_L \times \mathrm{U}(1)_L$. Don't worry about the word 'gauge', the takeaway is that the electroweak Lagrangian is invariant under local $\mathrm{SU}(2)_L \times \mathrm{U}(1)$ transformations. The subscript L here is important, because it means that the SU(2) transformation only acts on left-handed particles. This is why the Standard Model is a chiral theory–it treats left-hand particles differently from right-handed ones. The U(1) is called hypercharge, which is related to but different from $\mathrm{U}(1)_{\mathrm{EM}}$.

Toy Model: Symmetry Structure
Let's make this concrete with a toy model with 2 global symmetries $\mathrm{U}(1)_L \times \mathrm{U}(1)_Y$. Instead of $\mathrm{SU}(2)_L$ we'll consider $\mathrm{U}(1)_L$, which rotates left and right-handed Weyl fields as

$$\psi_L \to e^{i\alpha Q}\psi_L, \qquad \psi_R \to \psi_R$$

We also have a hypercharge symmetry, which rotates left and right-handed fermions equally:

$$\psi_L \to e^{i\beta Y}\psi_L, \qquad \psi_R \to e^{i\beta Y}\psi_R$$

This one we've seen, but the chiral $\mathrm{U}(1)_L$ is new. Note that here Q and Y are charge operators: let's assign different charges to different particles. The particles we will consider are electrons and neutrinos (there is no evidence of right-handed neutrinos, but more on this later)

What does this have to do with the Standard Model? After all, the spinors around us, like electrons and positrons, are Dirac spinors. This is not so in the Standard Model: the fermionic matter is described by Weyl spinors. To see why, consider a Dirac spinor charged under $\mathrm{U}(1)_L$ with $Q = 1$, $\Psi = (\psi_L, \psi_R)^\dagger$. The problem is that the mass term is not invariant under the $\mathrm{U}(1)_L$ symmetry:

$$\bar{\Psi}\Psi = \psi_R^\dagger\psi_L + \psi_L^\dagger\psi_R \to e^{i\alpha}\psi_R^\dagger\psi_L + e^{-i\alpha}\psi_L^\dagger\psi_R$$

This shouldn't be surprising, since the mass term couples the left and right Weyl fermions, which transform differently. Therefore, the Lagrangian has 2 independent Weyl fermions $\psi_{L,R}$.

Toy Model: Matter Structure
So what are our fields? We need to work with Weyl fermions. Scalars, vectors, and so on can be added, but there is no point in adding Dirac spinors, since massless ones are equivalent to Weyl spinors. Since ψ_L and ψ_R are decoupled, they should be thought of as distinct particles, and for instance we can have

more of one-handed particles than we do the other. Inspired by the real world, the fermion fields we'll work with are a LH electron, RH electron, and LH neutrino:

$$e_L \qquad\qquad e_R \qquad\qquad \nu_L$$

I stress that e_L and e_R are decoupled fields: so far, there is no reason to treat them as chiral components of the same field. Now that we have our fields, we need to specify their charges under each symmetry. We do this in a table:

Field	U(1)$_L$	U(1)$_Y$	Lorentz
e_L	$-1/2$	$-1/2$	$(1/2, 0)$
e_R	0	-1	$(0, 1/2)$
ν_L	$1/2$	$-1/2$	$(1/2, 0)$

The U(1)$_L$ and U(1)$_Y$ charges are taken from the Standard Model, and may seem arbitrary to you. However, in our world they are almost uniquely constrained by anomaly cancellation conditions.

We can now write down mass terms. Any mass term in our Lagrangian must be Lorentz-invariant, which constrain us to Majorana and Dirac terms. To add them to the Lagrangian, we must also check they transform trivially under both $U(1)_L$ and $U(1)_Y$.

- Majorana masses: $e_L^T\sigma^2 e_L$, $\quad e_R^T\sigma^2 e_R$, $\quad \nu_L^T\sigma^2\nu_L$, $\quad \nu_L^T\sigma^2 e_L$, $\quad e_L^T\sigma^2\nu_L$
  These are not invariant under either the U(1)$_L$ or U(1)$_Y$ symmetries, so are not valid terms in $\mathcal{L}$.
- Dirac masses: $e_L^\dagger e_R + \text{h.c.}$, $\quad \nu_L^\dagger e_R + \text{h.c.}$
  These are not invariant under $U(1)_Y$.

The takeaway from this is that any mass term we can construct is not invariant under our toy electroweak symmetry U(1)$_L$ × U(1)$_Y$. But real world electrons and neutrinos have masses, and the 2 components $e_{L,R}$ can be thought of as a Dirac fermion with a Dirac mass. We must account for this somehow if our toy theory (and the Standard Model) is to hold any water.

1.2 The Higgs Mechanism

Enter the Higgs boson. The Higgs is a scalar H whose existence allows e_L and e_R to couple together. In the electroweak sector, this is done via a Yukawa coupling,

$$\mathcal{L}_{\text{EW}} \supset \lambda(H^\dagger e_R^\dagger e_L + \text{h.c.})$$

Since this is a term in our Lagrangian, it must be invariant under U(1)$_L$ and U(1)$_Y$. We are therefore able to deduce the U(1) charges of the Higgs.

Field	U(1)$_L$	U(1)$_Y$	Lorentz
H	$-1/2$	$1/2$	$(0, 0)$

The details of the Higgs mechanism are quite intricate, but here I'll provide a basic description. Through a process called spontaneous symmetry breaking (explain if have time: Mexican hat potential), below the energy of $v \sim 246$GeV, the Higgs boson takes on a 'vacuum expectation value' (vev), and can be parameterized as:

$$H(x) = \frac{1}{\sqrt{2}}(v + h(x))$$

v is this constant energy scale, and $h(x)$ is the physical Higgs boson, which is a real scalar field. It parameterizes fluctuations about its classical value v. Note that this parameterization is only valid for energies $E \ll v$: for larger energies, $h(x)$ is no longer a perturbation and this breaks down. However, 246GeV is an absurdly high energy: the LHC can only probe up to 14TeV. It is of the same order as the Higgs mass $m_h = v/2 \sim 125$GeV, and this is why it took so long to find the Higgs.

Let's take this parameterization, and substitute it into our coupling terms in our Lagrangian:

$$\begin{aligned}
\mathcal{L}_{\text{EW}}|_{E\ll v} &\supset \frac{\lambda}{\sqrt{2}}[(v+h)^\dagger e_R^\dagger e_L + e_L^\dagger e_R(v+h)] \\
&= \frac{\lambda v}{\sqrt{2}}(e_R^\dagger e_L + e_L^\dagger e_R) + \frac{\lambda}{2}(h^\dagger e_R^\dagger e_L + e_L^\dagger e_R h) \\
&= \frac{\lambda v}{\sqrt{2}}\bar{e}e + \frac{\lambda}{\sqrt{2}}h\bar{e}e
\end{aligned}$$

In the second line, we have generated a Dirac mass out of $e_{L,R}$. In the last line, we have combined e_L and e_R into a Dirac spinor field, $e = (e_L, e_R)^T$. We have also generatde 3-point interactions between the electron and the Higgs. (Draw Feynman rules before and after SSB.)

The takeaway from this is that physics splits into 2 regimes:

- $E \gtrsim v$ (very high energy): SSB has not occered, and left/right handed e_L, e_R are massless, independent particles which couple to the Higgs via a 3-point interaction.
- $E \ll v$ ('low' energy): the Higgs has given the electrons a Dirac mass by taking on a vev., in a way that respects chiral gauge symmetry.

The Higgs mechanism in the Standard Model is slightly more complicated, owing to that the chiral symmetry is $\text{SU}(2)_L$ instead of our toy $\text{U}(1)_L$. It also allows for a host of other processes, including the generation of a mass for gauge bosons, and the generation of a photon as a linear combination of the $\text{U}(1)_L$ and $U(1)_Y$ symmetries.

Neutrino Masses

Note that the Higgs cannot generate a neutrino mass not happen for neutrinos, because of the absence of a right-handed ν_R. This coincides with our world: all neutrinos we have observed are left-handed, and we do not know why. Similarly, the Standard Model does not predict neutrino masses. On the other hand, we know neutrino masses exist via observing neutrino oscillations. This discrepency can be accounted for by adding neutrino masses *to* the Standard Model. There are two easy ways to do this, but the form of the masses depends on whether neutrinos are Dirac or Majorana, something else we do not know.

- If they are Dirac, then there must be a right-handed neutrino ν_R which we have not observed. If it exists we expect it to be very heavy. Neutrino masses can thus be generated by the Higgs, as

  $$\mathcal{L}_{\text{SM}} \supset \lambda(H^\dagger \nu_R^\dagger \nu_L + \text{h.c.})$$

- If they are Majorana, there does not need to be a right-handed ν_R. We can write the mass term

  $$\mathcal{L}_{\text{SM}} \supset \lambda(\nu_L \sigma_2 H)(\nu_L \sigma_2 H)$$

  From dimensional reasons, we expect the coefficient λ to by incredibly small, which explains why neutrinos are so light. But even if this is the case, we still need to explain the absence of the right-handed ν_R.

All in all, neutrinos are one of the least understood aspects of the Standard Model, and a fertile source to look for new physics.

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Spring 2023