Quantum Field Theory II (8.324) Fall 2010
Assignment 4

Readings

- Peskin & Schroeder chapters 6 and 7
- Weinberg vol 1 chapters 10 and 11.

Note

- The purpose of Prob. 1 is to remind you of techniques used in tree-level calculations involving vector and spinor fields.
- In Prob. 2 you will have an opportunity to practice the procedure we discussed of removing UV divergences. If you have problem working it out yourself, you can consult sec. 16 (part I) of Srednicki's book, from where the problem was devised.

Problem Set 4

1. **Physics of massive boson (50 points)**

   Peskin & Schroeder prob. 5.5

2. **One-loop correction to the vertex in ϕ^3 theory (50 points)**

   Consider the ϕ^3 theory in d spacetime dimension

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 + \frac{g}{6}\phi^3 + \mathcal{L}_{ct} \tag{1}$$

   with the counter terms given by

$$\mathcal{L}_{ct} = -\frac{1}{2}A(\partial\phi)^2 - \frac{1}{2}Bm^2\phi^2 + \frac{Cg}{6}\phi^3 \ . \tag{2}$$

Note that $A, B, C \sim O(g^2)$ and we have determined A and B to this order in lectures. C will be determined below.

We can define an exact three-point vertex function $V_3(k_1, k_2, k_3)$ as the sum of *one-particle irreducible diagrams* with three external lines carrying momenta k_1^μ, k_2^μ, and k_3^μ, all incoming, with $k_1 + k_2 + k_3 = 0$ by momentum conservation.[1] The physical coupling g is defined by requiring

$$V_3(0,0,0) = g \ . \tag{3}$$

(a) Write down all the 1PI diagrams which contribute to V_3 at order $O(g^3)$.

(b) Use Feynman's formula and Wick rotation to evaluate the diagrams in (a). In your final expression you should have done all the momentum integrals, but leave the integrals over the Feynman parameters.

(c) What are the spacetime dimensions for which the vertex corrections at this order are UV finite?

(d) Now take $d = 6$ and determine C (use the dimensional regularization) using (3).

(e) In d=6, evaluate your $V_3(k_1, k_2, k_3)$, which is now finite and fully determined, in the limit $|k_3^2| \gg m^2, |k_1^2|, |k_2^2|$. Comment on your result.

[1] In adopting this convention, we allow k_i^0 to have either sign; if k_i is the momentum of an external particle, then the sign of k_i^0 is positive if the particle is incoming, and negative if it is outgoing.

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