[SQUEAKING]

[RUSTLING]

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PROFESSOR: Welcome back to 8.701. So in this lecture, we talk about the Higgs mechanism. As you might know, the Higgs boson was discovered in 2012 by the LHC Experiment, but the theoretical discovery of the Higgs boson happened much, much earlier than that did. In the mid 1960s, Peter Higgs and a few others proposed a mechanism which gives rise to masses of the gauge boson, the w and the z boson. And the Higgs boson, or the Higgs field then can also be used to give masses to the fermions.

So let's have a look at this and start with a simple observation. When we have written down our Lagrangian for a simple spin one field, gauge field, like a photon, we find that we want to have local gauge invariant for this equation, which means that we can do a local gauge transformation of our fields, and the physics should be unchanged of this. So the physics, meaning that the description by the Lagrangian, should be invariant under this transformation. All right?

The problem, however, is that, if you want to have a spin one gauge field which is massive, you have to have terms in your Lagrangian, like this one here, where you have a mass term for your fields. So in general, this is not possible without breaking gauge invariants, and this is a guiding principle of our Lagrangian theory. So this is a real bummer.

So if you want a stopping point, so you have a beautiful theory which describes all the interactions, but one important characteristic, the masses of the particle is missing. But you are able to actually do this, not by adding specific mass term but by breaking the symmetry, by breaking the local gauge symmetry. There's various ways to do this, and one of the ways is to use spontaneous symmetry breaking.

So what is spontaneous symmetry breaking? Imagine you have a symmetry of rotation, a symmetry like this pen here, and by applying some force on top, this pen would bend. And by bending this pen, it bends in one specific direction, that breaks the rotational symmetry. Another way to look at this is to just let this pen drop. Let it go to its ground state, lowest possible energy state, and it will land somewhere on the table and by doing this breaking the symmetry, and it does this spontaneously.

Let's look at spontaneous symmetry breaking in a toy model first. So what we're going to do here just add a complex scalar field and the corresponding potential for this field. Potential is shown here, and this general potential can have multiple forms.

The first form would just be this parabola here. This is a solution where mu square, this term, mu square, is greater than 0. In this term, there's this unique minimum. The minimum is here at 0, and because of that, the mass of this field would be equal to 0, and the mas of our gauge field would also be equal to 0.

But what happens now if we have through this potential a breaking of the symmetry? So in this case here, the vacuum itself, the lowest energy state, breaks the symmetry. You go away from the 0 point, and you're breaking the symmetry. So this minimum is at v over square root 2. v is the vacuum expectation value of this field, and you can simply rewrite then this field itself by evolving it around its minimum. And so you find two fields here, this chi and this h. The h is already kind of pointing towards the Higgs boson, and the vacuum expectation value.

Now, if you add this back into your Lagrangian-- and I'll do this again. This is shown here, but also on the next slide-- you can start to identify terms which look like mass terms for your particle. And the first one is here which can be identified as a mass term for our gauge field. The mass is e time v.

e times v is the strength of the coupling of this gauge field e times the value of the vacuum expectation value. All right? So this is interesting. So we used this new scalar field to break spontaneously the symmetry, and then the mass term appears which is proportional to the strength of the coupling and the vacuum expectation value. So the mass is generated through the spontaneous symmetry breaking and the coupling to the field.

You also find a mass term for the six field here, for this h field. This is not the Higgs boson. It's just a field which looks like it, and so this mass term is here. But remember that mu squared is less than 0, and then this chi, or the so-called Goldstone boson, its mass is 0.

But then we have those terms left over here, which we cannot really interpret it very well. And it's possible to remove them by choosing a specific field. So we do a gauge transformation by just relabelling things, and then the new Lagrangian is independent of this field. Just as a reminder, Goldstone, the Goldstone boson, you find those Goldstone bosons in many places in physics. And Jeffrey Goldstone is a retired faculty at MIT, so you might, in the spring or next summer, you'll him walking across the corridors.

So we find our new Lagrangian which has our mass terms here, which has a term for the Higgs field, and has our potential for the Higgs field. So this specific gauge we just decided to use, this so-called unitarity gauge, and it's important to note that the Lagrangian itself contains all physical particles. But this chi, the Goldstone boson, is gone, and the lingo we sometimes use here is that the Goldstone boson has been eaten by the physical bosons. And the way it has been eaten is through the longitudinal polarization of this boson. It's the equivalent of saying that it has acquired mass.

So the pocket guys here for spontaneous symmetry breaking is such that spontaneous symmetry breaking of a u1 gauge symmetry by a non-zero vacuum expectation value of a complex scalar results in a massive gauge boson and one real massive scalar field. So we created mass, but as a side product, we also have an additional field. And that field itself has a mass term, so it's massive. The second scalar we had just disappeared. The Goldstone boson has been eaten by the longitudinal component of the gauge field itself.

All right. That was a simplified toy model. Let's look at the standard model. So now, here, we have to generalize from u1 to su2 or su-n gauge groups. The scalar field is now an n-dimensional fundamental representation of that group for the standard model. That will be su2.

The gauge fields are n squared minus one-dimensional joint representations, like our photon, for example, or our unmixed w boson field. And the Lagrangian looks very similar to the one we just before with our potential. Again, we have this mu squared term. We also have a lambda term here, and then we require local gauge invariants again.

OK. So now, for the standard model, again, su2 cross u1 gauge groups. We introduce a complex field, complex six field in su2. It's a duplex, meaning that it has four components. It's complex and has two components, which there's a total of four components to it.

So we already know that we want to use m square less than 0 for our potential to allow for spontaneous symmetry breaking to occur. The minimum is then at 1 over square root 2. 0 for the upper component, and v, the vacuum expectation, for the lower component. That's a choice already.

Again, why mu square less than 0? Because if we would have chosen to use a positive value for mu square, we wouldn't have spontaneously broken the symmetry. OK? So we need to have potential which looks like this Mexican hat here.

All right. So now, what happens now to our w and z bosons. We have discussed electroweak mixing already. Good. So that was the first step. Now, we will understand where the mass terms actually come from.

So now, we just did the spontaneous symmetry breaking, and now we are looking at what happens now if we also couple the Higgs field to the bosons. So again, we write this this way, and then we just try to find terms. It's really like a mechanical writing of the individual terms, and you find again terms which have a vacuum expectation value here and the coupling here. And you find the coupling to the u1 term and the coupling to the su2 and the coupling to the u1 term representative to the coupling to the original photon field and our gauge field for su2.

All right. The rest is rewriting and identifying terms. If you do this-- and this is like a couple of pages of writing, fine-- but if you do this, you find, again like before, that you find the first and second component of our su2 gauge field. It gives us the charge at the boson, the w plus and the w minus. And then the z boson and the photon mixtures of the third component and the field b. All right?

So these are all physical fields, and then we try to identify the mass terms you find for the w. That the w mass is proportional or equal to the coupling strands of the su2 group times the vacuum expectation value over 2. And the mass of the z boson is given by both couplings times the square root of the sums of the square times v over 2.

OK. If you're trying to look for a mass term for the photon, you find none, meaning that the photon is massless. And then we can look again at weak mixing angles, and they are now defined directly through the couplings in those two gauge groups. The masses of the w and the z bosons are related via this weak mixing angle, cosine theta w. All right. Those elements we already saw before. Now, we find that the masses of the gauge bosons are given by spontaneous symmetry breaking via the vacuum expectation value and the strength of the coupling of the gauge field to the Higgs field.

All right. So in summary, we started with a complex scalar field. A representation of su2 is four degrees of freedom. The Higgs vacuum expectation value breaks the symmetry spontaneously. The w plus and w minus and the z boson require mass, and the three Goldstone bosons are each absorbed into the w's and the z bosons.

We also find an additional scalar Higgs boson that remains. And so that was the understanding in the '60s and '70s, and the standard model was further developed. And then it took us all the way to 2012 to actually find this new scalar particle, the Higgs boson itself.