[SQUEAKING] [RUSTLING] [CLICKING]

MARKUS KLUTE: Welcome back to 8.701. So we switch gears now and talk about quantum electrodynamics, QED. And we start the discussion by going back to free wave equations.

Now could argue that we are interested in collisions and we're interested in decays of particles. So why do we discuss free wave equations?

But the theory we discussed last week, which we used in order to get a hold on Feynman diagrams and calculations, was very simplified. And one of the aspects not considered in this theory was the fact that particles carry spin. So we had a theory which not only was applicable to scalars.

Now by walking through wave equations, we can see how we can incorporate or make use of the fact that particles actually do carry spin. So let's do this one by one.

So we start off with our relativistic energy-momentum relation-- e squared is equal to p squared plus m squared. We express energy and momentum via quantum mechanical operators.

And so immediately by putting this in, we find this equation here, which is the so-called Klein-Gordon wave equation. So if you look at this equation, we see that the second derivative here in time, there's no derivative in space. So there's an asymmetry between space and time. And that is a not really useful feature of our wave equation as we want them to be, Lorentz invariant, for example.

So what we want is a first-order equation in both derivatives. So we'll just start writing this down in general terms, and then make sure that this equation holds to the relativistic information we just saw on the previous slide.

We'll just write this down here. We have a first derivative time and a first derivative space. And we'll just say there's a constant between those two, relating those two.

So the sigmas are just unknown constants. So if you now try to find by squaring this, trying to find the Klein-Gordon equation and relates the coefficients, you find this relationship here. So the sigma squared are all the same and equal to 1.

But you also see that the sigmas, they're anti-commutate-- sorry-- which is not possible for numbers. So sigmas need to be matrices. You also see that this is only holding true here for m equal to 0. So this equation here is true for a massless particle.

All right, so if we then try to find solutions for those relations, we find that they can be fulfilled by the 2 by 2 Pauli matrices. We might have seen this already hopefully in the discussion of atomic physics. And there, those Pauli matrices associate spin to electrons. So this is exactly what we have in mind here also.

Now using this definition, we can rewrite the Weyl equations to energy times the field is equal to minus sigma times the momentum times the field. And to find a second equation, we'll just design flips. The chi here and the phi spinors, they're two-dimensional vectors and the sigma are our Pauli matrices.

Good. So we have the relation of [INAUDIBLE]. So we can go a step further. Now we want to introduce mass term as well. Those hold for massless particles, so we're going introduce mass.

So we can rewrite this equation and introduce its mass term here, again, with the coefficient. And we find now this alpha here being-- sorry. So this phi here is a core component spinor.

And it stands for the particle, its antiparticle, and the two spin states. So that's combining that two equations we had here. So you'll see one is for particles and one is for antiparticles, for the two spin states. So we combine this in one equation and we added this mass term.

So if you try to find the solutions here, you find that alpha is a matrix-- 4 by 4 matrix which has the sigma, supporting matrices on the off-diagonal elements. And beta is a diagonal matrix-- a 4 by 4 matrix with identity on the upper two components, and minus 1 in the lower two components.

So now with this, this is already the Dirac equation. We can rewrite the Dirac equation in the covariant form where you have just defined a new matrix here, so-called gamma matrix which you build out of this matrix beta and alpha. Which are defined on the previous slide.

Good, so good. So we have this new matrix, this new equation here, which is the Dirac equation. And it holds for particles with two spin states, examples with spin half states. And it holds for particles which have masses.

So that's great. So this is now on starting point for the discussion. The next lecture we'll look at solutions of this equation.