MARKUS KLUTE: Welcome back to 8.701. So in this lecture, we look at the interaction of W bosons with quarks, or the charged weak interaction of quarks.

Let's just make a number of observations. Now, we observe that the weak interaction respects the lepton generation, meaning that a W couples to an electron and an electron neutrino, but not an electron muon neutrino. But in the case of the quarks, there is violation of this. So there is a disrespect of the quark generation when it comes to the interaction with Ws.

So when you investigate these two diagrams here, you find that the W couples to the V quark and the U quark, but it also couples to the S quark. S quark and the U quark, all right? In order to encapsulate this, we have to make a correction. And the corrections are typically called cosine theta C and sine theta C. Theta C is the Cabibbo angle, so theta Cabibbo. Turns out this is rather small, so it's a decrease. So it is a correction, a small correction.

We studied the partial decay width of the kaon in leptonic decays over the partial decays to a pion of the pion in leptonic decays. We found that there is a set of forms that are the form factor for the pion decay and a form factor for the kaon decay. It turns out that the form factors are due to this additional correction, so you find the tangent-square of the Cabibbo angle as part of this correction.

Good. So far, so good. Now we have made an observation. We haven't explained anything yet. We can make one more observation, or discuss one, and that's the decay of neutral kaons to a pair of muons. It turns out that those are not very likely. Even so, you would expect that the amplitude has a factor here of sine and cosine of Cabibbo, so the amplitude should be on the order of sine theta Cabibbo times cosine theta Cabibbo.

So when this was studied, the charm quark hadn't been discovered. And the explanation to why this decay is suppressed comes from the fact that there is a second diagram here, where we just replace the U quark in this rule with a C quark. This diagram contributes there's a minus sign to the amplitude. And therefore, those two diagrams, they cancel. Right? They are the same magnitude, about the same magnitude, and they have an opposite sign. So this was the first indication that there must be a force quark contributing to this kind of process. That's the charm quark.

Let me now try to understand what's going on here. Why is the W coupling modified? Or why is not the full down quark or charm and strange quark participating in the weak interaction? We can do the following here. We can rewrite-- we note that the weak interaction eigenstate, the eigenstate which participate in the [INAUDIBLE], is not the eigenstate of the particle itself, the so-called mass eigenstate. So we have to write the weak eigenstate as the linear combination of the mass eigenstate or [INAUDIBLE].

This can be done in this matrix form here, where we simply multiply the weak eigenstates with a matrix, and just basically rotate it into the mass eigenstate. So this was proposed by Cabibbo, and rather successful. But it didn't incorporate the third-generation particle. And this was done by Kobayashi and Maskawa, who generalized the scheme and proposed the so-called CKM matrix, the easier form Cabibbo-- Cabibbo, Kobayashi, and Maskawa.

Because of constraints we'll discuss in one of the recitations, this matrix can be parameterized as only three independent angles and one complex phase as independent parameters. So you can choose different parameterization to capture that there's only four parameters in this matrix, which has nine components. One is by thinking about this matrix as three independent rotations and this complex phase here.

In terms of numerical values, you see that the diagonal elements of this matrix are very close to 1, meaning that this mixing is on the block sector of the [INAUDIBLE] effect. You find that those next-nearest off-diagonal elements are on the order of 20%, and the next-to-next off-elements are even smaller. OK?

This leads us, then, to the discussion that we can use different parameterization in order to capture [INAUDIBLE]. We already discussed the standard parameterization, which you can really think about three different rotation. And the values of those angles are give here, together with the value of this additional phase.

Another way to look at this is the so-called Wolfenstein parameterization. And this captures the fact that it seems like that there's a correction being applied to the actual particle. So you find elements of the order of lambda. Lambda is about 22%. And you find elements which are in the order of 1 minus the lambda-square correction. And then there is elements which are of lambda-square and lambda 3rd power. So this captures the matrix, and then there's higher-order corrections to that which are of order lambda to the 4th power. OK?

Because there's constraints on this matrix-- and specifically, unitarity constraints, meaning that we have three generations that will make a mixing of those three mass eigenstates to weak eigenstate-- then unitarity, the total number of particles in this discussion, is conserved. This will change if there would be, for example, a force generation particle. So the study of weak-charged interaction with quarks helps us to understand whether or not there might be a force generation.

We'll not go into too much detail here, but also, the complex phase explains part of our understanding of CP violation. And we might discuss this in a little bit of a later lecture. But nevertheless, what we can achieve from here is those unitarity constraints, just simply summing over the matrix elements, the scalar product of matrix elements. And those where the contribution vanishes, so those where j and k are not equal, those can be represented as a triangle.

That's kind of interesting. You can just rewrite this. You just say that those three elements of the sum are equal to 0. Then you normalize by one element. In this case here, normalize by Vcd, Vcb. And so then this makes this point being 0, and so we have this nice triangle here, which has three angles, alpha, beta, and gamma, and this point here, rho and eta. And so this is a nice way to illustrate actual measurements of the elements of the CKM matrix [INAUDIBLE].

And without actually explaining how we do this experiment, you can assume that all measurements have-- well, you can understand that all measurements have to do with the weak interaction with quarks. That's how we have access to the CKM matrix elements. Sometimes this results in the modification of masses or splitting of mass states, and sometimes the direct measurement cause a recoupling.

When you put all of those measurements back together, you can look at this. So we see our triangle here. We see this point, eta and rho, which is given here in this right-angular plane. And you see various number of measurements which correspond to elements of this CKM matrix.