Massachusetts Institute of Technology

Department of Physics

Course:	8.701 – Introduction to Nuclear and Particle Physics
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Discussion Problems

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Problem 1: Cloud Chamber

Carl David Anderson discovered positrons in cosmic rays. The picture below shows a cloud chamber image produced by Anderson in 1931. The cloud chamber is positioned in a magnetic field of 1.5 T with field lines pointing into the plane of the paper. A cosmic ray particle enters the chamber from below and leaves a circular track. There is a 6 mm thick lead plate in the cloud chamber visible as a horizontal line. The radius of curvature is 15.5 cm before and 5.3 cm after the passage through the lead plate.

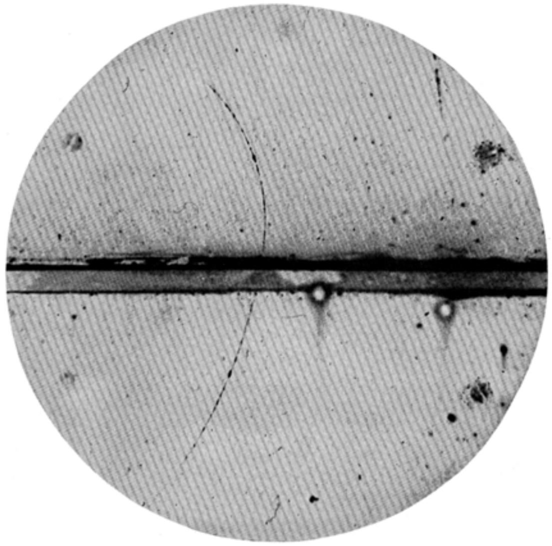

Figure 1: Cloud-chamber image of a positron.

a)

Estimate the momentum of the particle before and after the passage through the lead plate. What is the charge of the particle?

b)

Compare the energy loss during the passage through the lead plate for a proton, a pions, and an electron. For the energy loss calculation, you can use the approximate Bethe formula below and assume constant energy loss.

$$\left\langle \frac{\mathrm{d}E}{\mathrm{d}X} \right\rangle_{\text{Ion}} = -4\,\pi\, N_{\mathrm{A}}\, r_{\mathrm{e}}^2 m_{\mathrm{e}}\, c^2\, z^2 \frac{Z}{A} \cdot \frac{1}{\beta^2} \cdot \ln\left(\frac{m_{\mathrm{e}}\,\gamma^2\,\beta^2\,c^2}{I}\right) \tag{1}$$

Explain why this is sufficient to exclude the proton and pion hypothesis. The constants in the equation are $N_{\mathrm{A}} = 6,022 \times 10^{23}\,\mathrm{mol}^{-1}$, $\epsilon_0 = 8,85 \times 10^{-12}\,\frac{\mathrm{A\,s}}{\mathrm{V\,m}}$, $m_{\mathrm{e}} = 511\,\mathrm{keV}$, $r_{\mathrm{e}} = \frac{\mathrm{e}^2}{(4\,\pi\epsilon_0)m_{\mathrm{e}}\,c^2}$, $Z = 82$, $A = 207$, and $I = 820\,\mathrm{eV}$, the ionisation energy in lead.

c)

To further aide the hypothesis, calculate the energy loss of the particle through bremsstrahlung.

$$\left\langle \frac{\mathrm{d}E}{\mathrm{d}X} \right\rangle_{\text{brem}} = -4\,\alpha\, r_{\mathrm{e}}^2 N_{\mathrm{A}}\, \frac{Z^2}{A} \cdot \ln\left(\frac{187}{Z^{\frac{1}{3}}}\right) \cdot E = -\frac{E}{X_0}\,,$$

Use $X_0 = 0.56\,\mathrm{cm}$ for the radiation length in lead.

- **a)**

For the momenta before and after the lead plate we find:

$$\vec{p} = e \cdot \vec{r} \times \vec{B} \qquad p\,[\mathrm{GeV}] = 0.3 \cdot r\,[\mathrm{m}] \cdot B\,[\mathrm{T}]$$
$$p_i = 0.3 \cdot 0.155\,\mathrm{cm} \cdot 1,5\,\mathrm{T} = 70\,\mathrm{MeV}$$
$$p_f = 0.3 \cdot 0.053\,\mathrm{cm} \cdot 1,5\,\mathrm{T} = 24\,\mathrm{MeV}$$

The $\vec{B}$-field point into the plane of the paper, therefore, this must be a positively charged particle.

b)

For the energies and velocities needed for the Bethe equation we find from **a)**:

	m [MeV]	E_i [MeV]	E_f [MeV]	β_i	γ_i
Proton	938	940.6	938.3	0.074	1.003
Pion	140	156.5	142.0	0.447	1.118
Elektron	0.511	70.0	24.0	> 0.999	137

Table 1: Energy, E_i, velocity, β_i, and γ-factor, γ_i, for the particle before and energy E_f, for the particle after the passage through the lead plate for the proton, pion, and electron hypothesis.

For the Bethe equation we find:

$$\left\langle \frac{\mathrm{d}E}{\mathrm{d}X} \right\rangle = -4\,\pi\,N_A\,r_e^2 m_e\,c^2\,z^2 \tfrac{Z}{A} \cdot \tfrac{1}{\beta^2} \cdot \ln\left(\tfrac{m_e\,\gamma^2\,\beta^2\,c^2}{I}\right)$$

$$K \equiv -4\,\pi\,N_A\,r_e^2 m_e\,c^2 \tfrac{Z}{A} = -0.121\,\tfrac{\mathrm{MeV}}{\mathrm{g/cm^2}}$$

$$L \equiv \tfrac{z^2}{\beta^2}$$

$$M \equiv \ln\left(\tfrac{m_e\,\gamma^2\,\beta^2\,c^2}{I}\right)$$

$$\rho_{\mathrm{Pb}} = 11,34\,\mathrm{g/cm^2}$$

c)

	L	M	$\mathrm{d}E/\mathrm{d}X$ [MeV/g cm^{-2}]	$\mathrm{d}E/\mathrm{d}x$ [MeV/cm]	ΔE [MeV]
Proton	183	1.24	−27.30	−310	−185
Pion	5	5.05	− 3.07	− 35	− 21
Elektron	1	16.3	− 1.97	− 22	− 13

Table 2: Expected energy loss from passage through a 6 mm thick lead plate for the proton, pion, and electron hypothesis.

The critical energy for lead can be estimated from:

$$E_c \approx \left. \frac{610\,\mathrm{MeV}}{Z + 1.24} \right|_{Z=82} = 7.3\,\mathrm{MeV} \tag{2}$$

The exact value is 7.16 MeV (for a positron). Therefore, the dominate energy loss mechanism for the electron / positron is through bremsstrahlung. For the mean energy loss through bremsstrahlung we can use:

$$\left\langle \frac{\mathrm{d}E}{\mathrm{d}X} \right\rangle_{\mathrm{Brem}} = -4\,\alpha\, r_e^2 N_A\, \frac{Z^2}{A} \cdot \ln\left(\frac{187}{Z^{\frac{1}{3}}}\right) \cdot E = -\frac{E}{X_0}$$

We need to add the energy loss from ionisation and bremstrahlung to E_i and find:

$$E_i + \Delta E_{\mathrm{brem}} + \Delta E_{ion} = 70\,\mathrm{MeV} - 46\,\mathrm{MeV} - 13\,\mathrm{MeV} = 11\,\mathrm{MeV}$$

The value of $E_f = 24\,\mathrm{MeV}$ is compatible with the hypothesis for a electron-like particle.

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