8.701

Introduction to Nuclear and Particle Physics

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7. Higgs Physics

7.1 Higgs Mechanism

Gauge Boson Masses

QED: U(1) local gauge theory with single spin-1 gauge field

Lagrangian:

$$\mathcal{L} = \bar{\psi}\left[i\gamma^\mu(\partial_\mu - ieA_\mu) - m\right]\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ ψ: fermion field; e: electric charge m: fermion mass

Invariant under local gauge transformations

$$\psi \rightarrow \psi' = e^{i\alpha(x)}\psi; \qquad A_\mu \rightarrow A'_\mu = A_\mu + \frac{1}{e}\partial_\mu\alpha$$

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Symmetry Breaking

A mass term for A_μ would destroy the gauge invariance

$$\mathcal{L}_{mass} = \frac{1}{2} M_A^2 A^\mu A_\mu$$

In general, gauge bosons must be massless and gauge invariance is a guiding principle.

To generate a mass for gauge bosons, the local gauge symmetry must be broken.

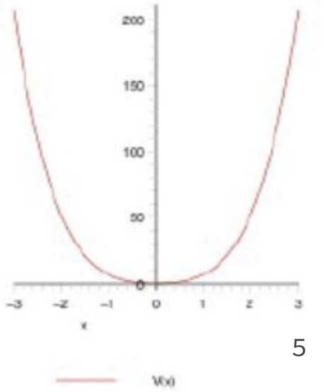
This leads to spontaneous symmetry breaking.

Spontaneous Symmetry Breaking

Toy model: a single complex scalar ϕ field coupled to U(1) gauge field

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |D_\mu\phi|^2 - V(\phi)$$

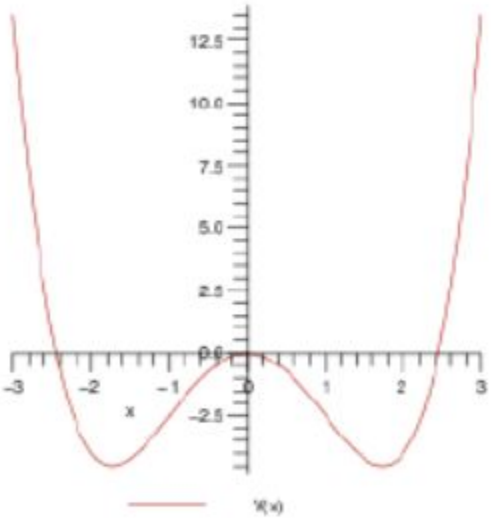
where $D_\mu = \partial_\mu - ieA_\mu$, $V(\phi) = \mu^2|\phi|^2 + \lambda|\phi|^4$



If $\mu^2 > 0$: V(ϕ) has unique minimum at =0

$m_\phi = \mu$ and $m_A = 0$.

Spontaneous Symmetry Breaking



If $\mu^2 < 0$: $V(\phi)$ has minimum at

$$\langle \phi \rangle = \frac{v}{\sqrt{2}} = \sqrt{-\frac{\mu^2}{2\lambda}}$$

Vacuum breaks U(1) symmetry

Rewrite $$\phi = \frac{1}{\sqrt{2}} \exp(i\frac{\chi}{v})(v+h)$$

Lagrangian becomes

$$\begin{aligned}\mathcal{L} \quad = \quad & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}e^2v^2A^\mu A_\mu + \frac{1}{2}(\partial_\mu h\partial^\mu h + \partial_\mu\chi\partial^\mu\chi) + \mu^2h^2 \\ & -evA_\mu\partial^\mu\chi + \ h,\ \chi \text{ interaction terms}\end{aligned}$$

Spontaneous Symmetry Breaking

$$\begin{aligned}\mathcal{L} &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}e^2v^2A^\mu A_\mu + \frac{1}{2}(\partial_\mu h\partial^\mu h + \partial_\mu\chi\partial^\mu\chi) + \mu^2h^2 \\ &\quad -evA_\mu\partial^\mu\chi + h,\ \chi \text{ interaction terms}\end{aligned}$$

$M_A = ev$

$m_h = \sqrt{-2\mu^2}$ (recall: $\mu^2 < 0$)

$m_\chi = 0$ (χ is a so-called Goldstone Boson)

how to interpret the $-evA_\mu\partial^\mu\chi$ term?

remove by gauge transformation

$$A'_\mu = A_\mu - \frac{1}{ev}\partial_\mu\chi$$

new Lagrangian:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}e^2v^2A'^\mu A'_\mu + \frac{1}{2}\partial_\mu h\partial^\mu h - V(h)$$

In this (so-called unitary) gauge, $\mathcal{L}$ contains only physical particles

χ is gone! It has been "eaten" by the gauge boson field

Pocket Guide

Spontaneous breaking of a U(1) gauge symmetry by a non-zero vacuum expectation value of a complex scalar field results in massive gauge boson and one real, massive scalar field. The second scalar field (the Goldstone boson) is eaten by the gauge boson field and is transformed into its longitudinal component

Higgs Mechanism

- now we generalize from a U(1) to a SU(n) gauge group
- scalar field is in n-dimensional fundamental representation of SU(n)
- gauge fields A_μ^a are in $n^2 - 1$ dimensional adjoint representation
- Lagrangian:

$$\begin{aligned} \mathcal{L} &= (D_\mu \Phi^\dagger)(D^\mu \Phi) - V(\Phi) \\ V(\Phi) &= \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2 \end{aligned}$$

- $\mathcal{L}$ is invariant under ($i, j = 1, \ldots, n$; $a = 1, \ldots, n^2 - 1$; ϵ^a are small parameters, g is the coupling constant, τ^a are the group generators [$\tau^a = \sigma^a/2$ for SU(2)])

$$\begin{aligned} \Phi_i &\rightarrow (1 - i\epsilon^a \tau^a)_{ij} \Phi_j \\ D_\mu \Phi &= (\partial_\mu - ig\tau^a A_\mu^a) \Phi \end{aligned}$$

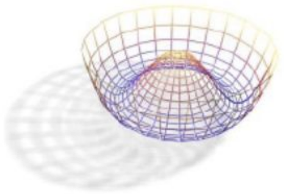
Higgs Mechanism

- Now consider the SM SU(2)×U(1) gauge group
- introduce one complex Higgs SU(2) doublet

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

- if $\mu^2 < 0$ in $V(\Phi)$, then spontaneous symmetry breaking occurs
- minimum of $V(\Phi)$ at

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$



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- choice of minimum breaks SU(2)×U(1)
- why is $\mu^2 < 0$?

W and Z Boson Masses

- now couple the Higgs field to the SU(2) and U(1) gauge bosons:

$$D_\mu = \partial_\mu - i\frac{g}{2}\sigma^a W_\mu^a - i\frac{g'}{2}B_\mu$$

W^a, $a = 1, 2, 3$: SU(2) gauge bosons;

B: U(1) gauge boson

g: SU(2) coupling constant,

g': U(1) coupling constant

- gauge boson mass terms (... after some algebra)

$$(D_\mu\Phi^\dagger)(D^\mu\Phi) = \cdots + \frac{v^2}{8}\left(g^2(W_\mu^1)^2 + g^2(W_\mu^2)^2 + (-gW_\mu^3 + g'B_\mu)^2\right) + \ldots$$

W and Z Boson Masses

- mass eigenstates (A is the photon):

$$W^{\pm}_{\mu} = \frac{1}{\sqrt{2}}(W^1_{\mu} + W^2_{\mu}) \qquad M_W = \frac{gv}{2}$$

$$Z^0_{\mu} = \frac{gW^3_{\mu} - g'B_{\mu}}{\sqrt{g^2 + g'^2}} \qquad M_Z = \sqrt{g^2 + g'^2}\,\frac{v}{2}$$

$$A_{\mu} = \frac{g'W^3_{\mu} + gB_{\mu}}{\sqrt{g^2 + g'^2}} \qquad M_A = 0$$

- Define weak mixing angle:

$$\sin\theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}$$

- W and Z masses are related then by

$$M_W = M_Z \cos\theta_W$$

Summary

start with one complex scalar SU(2) doublet (4 degrees of freedom): this is the minimal case

Higgs vacuum expectation value breaks $SU(2)\times U(1)\rightarrow U(1)_{em}$

The $W^{\pm}$ and Z^0 bosons acquire mass

Three Goldstone bosons are absorbed into the W and Z

One massive scalar (Higgs) boson remains

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