8.701

Introduction to Nuclear and Particle Physics

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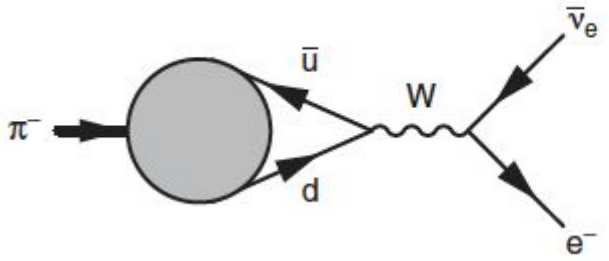
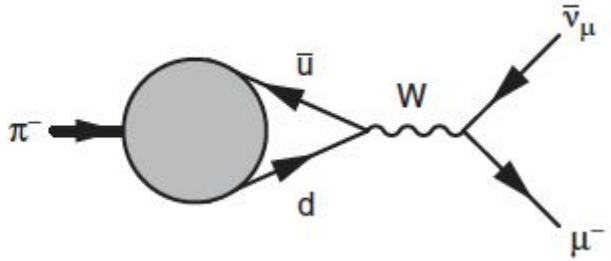
6. Weak Interaction

6.3 Pion Decay

Pion Decay



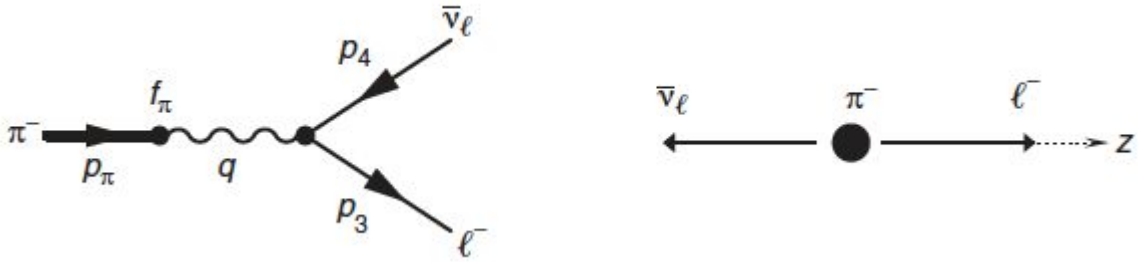


$$\frac{\Gamma(\pi^- \to e^- \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)} = 1.230(4) \times 10^{-4}.$$

$$u_\uparrow \equiv \tfrac{1}{2}\left(1 + \frac{p}{E+m}\right) u_R + \tfrac{1}{2}\left(1 - \frac{p}{E+m}\right) u_L$$

Pion Decay Rate

Pion rest frame



$$p_\pi = (m_\pi, 0, 0, 0), \quad p_\ell = p_3 = (E_\ell, 0, 0, \mathrm{p}) \quad \text{and} \quad p_{\bar{\nu}} = p_4 = (\mathrm{p}, 0, 0, -\mathrm{p}),$$

Weak leptonic current

$$j_\ell^\nu = \frac{g_\mathrm{W}}{\sqrt{2}} \bar{u}(p_3) \tfrac{1}{2} \gamma^\nu (1 - \gamma^5) v(p_4).$$

Matrix element

$$\mathcal{M}_{fi} = \left[\frac{g_\mathrm{W}}{\sqrt{2}} \tfrac{1}{2} f_\pi p_\pi^\mu \right] \times \left[\frac{g_{\mu\nu}}{m_\mathrm{W}^2} \right] \times \left[\frac{g_\mathrm{W}}{\sqrt{2}} \bar{u}(p_3) \gamma^\nu \tfrac{1}{2} (1 - \gamma^5) v(p_4) \right]$$

$$= \frac{g_\mathrm{W}^2}{4 m_\mathrm{W}^2} g_{\mu\nu} f_\pi p_\pi^\mu \bar{u}(p_3) \gamma^\nu \tfrac{1}{2} (1 - \gamma^5) v(p_4),$$

Pion Decay Rate

$$\mathcal{M}_{fi} = \frac{g_{\mathrm{W}}^2}{4m_{\mathrm{W}}^2} f_\pi m_\pi \cdot \frac{m_\pi + m_\ell}{\sqrt{2m_\pi}} \cdot \left(\frac{m_\pi^2 - m_\ell^2}{2m_\pi}\right)^{\frac{1}{2}} \cdot \frac{2m_\ell}{m_\pi + m_\ell}$$

$$= \left(\frac{g_{\mathrm{W}}}{2m_{\mathrm{W}}}\right)^2 f_\pi m_\ell (m_\pi^2 - m_\ell^2)^{\frac{1}{2}}.$$

$$\frac{G_{\mathrm{F}}}{\sqrt{2}} = \frac{g_W^2}{8m_{\mathrm{W}}^2}.$$

$$\langle |\mathcal{M}_{fi}|^2 \rangle \equiv |\mathcal{M}_{fi}|^2 = 2G_{\mathrm{F}}^2 f_\pi^2 m_\ell^2 (m_\pi^2 - m_\ell^2),$$

$$\Gamma = \frac{4\pi}{32\pi^2 m_\pi^2} \mathrm{p} \langle |\mathcal{M}_{fi}|^2 \rangle = \frac{G_{\mathrm{F}}^2}{8\pi m_\pi^3} f_\pi^2 \left[m_\ell (m_\pi^2 - m_\ell^2)\right]^2$$

$$\frac{\Gamma(\pi^- \to \mathrm{e}^- \overline{\nu}_{\mathrm{e}})}{\Gamma(\pi^- \to \mu^- \overline{\nu}_\mu)} = \left[\frac{m_{\mathrm{e}}(m_\pi^2 - m_{\mathrm{e}}^2)}{m_\mu (m_\pi^2 - m_\mu^2)}\right]^2 = 1.26 \times 10^{-4},$$

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