MARKUS KLUTE: Welcome back to 8.701. So we continue our discussion now of electron and proton scattering. And we dive deep into the structure using deep inelastic scattering. Inelastic here means that we are destroying the structure of the proton in the scattering process. But we have a way to look at the remnant of the proton and also of the scattered electron, and then compare our theoretic expectation for the cross-sections with the finding in experiments.

Let me just talk about this in more general terms. The energy of the probing electron or the photon in the scattering process allows us to look at the proton with varying resolution. So at very low energies, we basically see a point-like particle. And then the scattering process looks very much like the scattering of an electron with a muon.

If we increase the energy of the electron, we can see that there's an extended charge distribution in the proton. Further increase allows us to resolve the fact that the proton is made out of three quarks. And if you increase the energy further, we see a lot of new particles appearing, quarks and antiquarks and gluons, which make up the structure of the proton.

The picture here is-- I like this very much, I drew this myself some years ago-- is what I would like to have you remember. So in deep inelastic scattering experiments, we basically use the photon scattered-- radiated off the electron as a magnifying glass for the proton. So we can look into the structure of the proton here, and we see the distribution of the charged particles in the proton-- only the electrically-charged particles. We don't have scattering between-- direct scattering between photons and gluons.

So in measurements, what we can do is we can test scattered electron, and we can look for the remnant of the proton in our measurement, and then we do differential cross-section measurements, compare with our theory, and can infer information about the structure of the proton. To do this, we have special kinematic variables which turn out to be very useful. The most important one is probably this x year, which is called the Bjorken scaling x or Bjorken x, which you can think about-- so this is q squared, the momentum transfer of the photon, p is the momentum of the proton, q is the momentum transfer, this q here, factor of 2.

And what this is basically the fraction of the momentum carried by the parton here in the scattering process. There's a few other useful variables, but I don't want to go into any of the details yet.

So there's a number of very important scattering experiments. The first one I mentioned before is SLAC-MIT experiment, which led to the discovery that the proton is made out of quarks, and the Nobel Prize in Physics 1990 to Jerry Friedman, Henry Kendall, and Taylor. And what they did is they basically had a beam at SLAC of electrons of 5 to 20 GeV. And they scattered this beam off of hydrogen target protons.

And they used this spectrometer here in order to then make a differential measurement of the scattered electrons. So that's very cool. Even higher energies were available at HERA, the electron-positron collider, where the energies of the electrons were in the order of 30 GeV of the protons up to 830 GeV.

And so what we find then in those collisions, the differential cross-section measurements, is shown here. And it's very-- not an easy plot to read. So you see our structure functions here in the logarithmic plot. Remember, this is the a log 10 plot here. And you see here q squared, the momentum transfer of the photon, so the energy used in the scattering process.

When we try to read this here, we can look at a fixed q squared, for example. And at a fixed q squared, you see that if you probe-- if you are testing for a fixed fraction of the partons taking away partons, a fraction of the parton's momentum of the proton, you see that the lower the fractions, the more particles you see. So at a fixed energy, you see many, many more particles the lower you go in the fraction. So there seem to be like an increase of particles the lower the fraction is.

If you then check for a fixed fraction, let's say 0.4-- it means that the parton carries 40% of the proton's energy-- you see that it's almost flat as a function of q squared. So it seems like there's 40%-- the number of particles you see at 40% of momentum fraction is constant, this q squared.

However, if you look at smaller energies-- sorry, smaller momentum fractions, you see that higher energies seem to show even more particles at this momentum fraction. And the ways to understand this is two diagrams. So the first one is this one here, where you see a quark radiating a gluon.

And so what you see is that the deeper you look, you are able to then resolve this part here. And so you see more quarks and gluons which carry even smaller momentum fractions than the initial quark here. You also see diagrams like this, which is called gluon splitting, as a gluon splits into a quark/antiquark here. And you start resolving those. And those also carry lower momentum.

The evolution of our-- of this parton distribution function, or the structure functions, can be calculated and described in the so-called DGLAP equations. And all you do here is calculate the contributions from the so-called quark and gluon splitting. So you calculate the splitting functions, these higher-order corrections to a very simple quark model. And you find that you can actually very nicely describe those curves here. So you see this yellowing-- yellow here is the QCD fit, which basically uses those splitting functions as input.

All right. So we learned quite a bit about this proton already. If we want to now calculate a cross-section of a proton scattering with a proton, we are actually interested in the energy distributions or the momentum distributions of the partons in the protons. And so we'll come back to how we use this later.

But I want to introduce parton distribution functions which do exactly that. They're defined as the probability to find a parton in the proton that carries energy between x and x plus dx. You can write them using the structure functions before. But they literally describe this probability.

And so what you find inside the protons are now the valence quark, the down quark and the up quarks, c quarks and antiquarks, and gluons. So you want to describe those momentum distributions or energy distributions of those particles.

There's a number of sum rules. If you integrate momentum fractions from 0 to 1, 1 being the momentum of the proton, if you integrate them all together, you have to find 1, because that is the momentum of the proton you start with. If you integrate the down quarks and the up quarks, you find 1 or 2, meaning that those are the number of valence quarks we have available.

If you integrate the distributions of strange and antistrange and charm and anticharm, you get 0, because there needs to be the same number of strange and antistrange and charm and antistrange. And because of energy conservation, the sum needs to-- this sum, those sums need to be 0.

And then we can look at those. So what's shown here is x times the Parton Distribution Functions, the PDFs, as a function of f. And what you see here for our valence quarks, you see a distribution which is kind of what you expect-- it almost peaks at 0.3, a surge, and has the distributions because there's kinematics involved, and also interactions-- because of the interactions with the gluons.

And then you see the c quarks and antiquarks. And you see them increasing in numbers quite significantly here as you go to small fraction of the momentum carried. It's exactly what we just discussed in the previous plot already. An interesting way to look at this very same distribution function is if you plot them proportional to the momentum fractions, or the area proportional to the momentum fractions.

And what you see there is that a very significant part of the momentum of the proton is carried by the gluon. So you see here again, our valence quarks, our c quarks, and the gluons itself.

So that's all I wanted to say on the structure of the proton. We'll later in a lecture see how we can use those PDFs, those Parton Distribution Functions, in order to calculate a cross-section in proton scattering.