MARKUS KLUTE: Now back to 8.701. So in this section, we'll talk about relativistic kinematics. Let me start by saying that one of my favorite classes here at MIT is a class called 8.20, special relativity, where we teach students about special relativity, of course, but Einstein and paradoxes. And it's one of my favorite classes.

And in their class, there's a component on particle physics, which has to do with just using relativistic kinematics in order to understand how to create antimatter, how to collide beams, how we can analyze decays. And in this introductory section, we're going to do a very similar thing. I trust that you all had some sort of class introduction of special relativity, some of you maybe general relativity. What we want to do here is review this content very briefly, but then more use it in a number of examples.

So in particle physics, nuclear physics, we often deal particles who will travel close to the speed of light. The photon travels at the speed of light. We typically define the velocity as v/c in natural units. Beta is the velocity, gamma is defined by 1 over square root 1 minus the velocity squared. Beta is always smaller or equal to 1, smaller for massive particle. And gamma is always equal or greater than 1.

The total energy of a particle with 0 mass-- sorry, with non-zero mass, is then given by gamma times m c squared. And the momentum is given by gamma times mv or gamma times m beta. The total energy squared of a particle, considering one massive particle, or one particle, is given by energy squared equal momentum squared plus mass squared.

And if you now consider a particle with 0 mass, you see that the energy and the momentum are equal. If you consider a particle at rest, meaning the momentum is 0, you see that the energy is equal to the mass. You get Einstein's famous formula, the energy is equal-- E equal m c squared. Energy is equal to the mass or the equivalence between those two.

But we want to fully understand and control our Lorentz transformations. Here shown for example, we have a boost or a transformation in x-direction. So you see that energy and momentum transform like time and space. And you just-- I really encourage you to just review this in more general cases, but you can always, when you have a boost in one direction, just do rotation and get to this more simplified case.

So here is the first example I would like you to actually go through. The Lorentz transformation here, I decided to use the z-direction, just to change things up a little bit. And the velocity of the boosted frame is vb. So we want to calculate the quantity m squared s squared in the transformed frame.

And what you will find, if you actually do the calculation-- and the solutions are in the backup slides-- it's that z quantity doesn't change in the Lorentz transformation. It is invariant. And we'll talk about an invariant mass in this context.

So now in particle physics, we often have the case that we are not considering just one particle and want to describe just one particular and measure it, but often the case of particles, or multiple particles, which are involved in the reaction. So we can look at the total energy, just the sum of the energy of all particles, and total momentum, the sum of the momentum of all particles.

And those two quantities are always conserved. They are not invariant. So be aware of the distinction between conserved properties and invariant properties. Invariant here means perform a transformation like the Lorentz transformation, and the property doesn't change. Conserved here means we have a reaction, and in that reaction the property is not changing. Those are two different, distinct things.

So now you can look at the invariant property, or the one which is conserved in this collision, which is this mass term or mass-squared term. The total mass, we'll define this total mass as equal to the energy squared minus momentum squared.

And then you can consider the two cases of a laboratory frame and the so-called center-of-mass frame. So in the laboratory frame, you have a particle. It's moving when we observe this particle, and then it decays into, in this example, three daughter particles.

In the center-of-mass frame in this example, we put ourselves into the rest frame of the particle we are interested in. And then that frame then, three particles emerge. And we can describe the three particles. So the momenta between the three daughter particles are not going to be [INAUDIBLE].

But because this total mass is an invariant property, it's the same in both frames. And it's equal to the mass of the parent particle which we [INAUDIBLE] So when you measure the energy and momentum of the daughter particles, you can infer in any frame the mass of the parent particle by calculating the total mass.

And so you can infer from those measurements the identity of the mother particle. And that's, for example, how we discover the Higgs boson. We measure the Higgs boson decay into a pair of photons, and then we calculate the mass of those two photons in our laboratory frame. And that mass, then, is equal to the Higgs mass.

So now here we want to compare or look into those two cases a little bit more. The first case is a case where we have a particle 1 colliding with a particle 2, where the particle 2 is at rest. Particle 1 has a certain energy E1. And the second example-- this is called a fixed-target experiment. So the second particle is fixed, the first one is colliding.

The second example is the one where you have two particles, and both have energies, and we bring them to collision. Often, the two particles are in nature, like two protons, an electron and positron, and the energies of the beams are the same. But this doesn't have to be the case.

Later in the class, we'll look at heavy iron collisions. It's the collision of heavy ions like lead. It's a proton. And here the masses are different, and the energy of the particles can be different.

All right. And here's another exercise now. So we want to actually create a Z boson, which has a mass of about 91 GeV. Note I dropped the c squared, 1 over c squared here. And you want to produce this particle colliding a positron with an election. This has happened at LEP at CERN in the late '80s and '90s.

The center-of-mass energy, often what's called square root of s, is equal to 91 GeV. So that's the energy we need in order to produce this new particle. The mass of the electron and the positron are 511 KeV or 0.511 MeV. So the energy needed is 45 GeV, 45.5 GeV.

However, that was the setup at LEP, where you have two beams colliding. So we have this center-of-mass energy being given by the energy directly given approximately by the energy of the two beams. So now, imagine somebody would have proposed a fixed-target experiment, where you have stationary electrons, for example electrons in atoms, just a gas of some sort, and then you have produced positrons in a beam, you accelerate them and bring them to collision.

So the question now is, how large does it-- do you need an energy of this positron beam? How large does it have to be in order to produce a Z boson? So again, this is something I would like you to actually explore and just write down. Solutions for this example are also in the backup.

So now, you know, there is a number of interesting examples just coming from E equal m c squared, and from being able to use-- and that can be answered by being able to use Lorentz transformation. And so now here I give you just a set of examples. And you should work on them on your own time. Maybe we'll touch on them in recitation.

The first one is rather straightforward. Again, we are talking about LEP at CERN. After the Z bosons were produced, when it was trying to go to high energy to find some new physics, some new particle, for example the Higgs boson. The Higgs boson might be produced by a process which is called Higgs-Strahlung process. We will look at this later.

So you have an electron and a positron colliding to virtual Z boson. So that is a Z boson which is heavier than 91 GeV. We'll see later how that's possible. But then the virtual Z boson can radiate a Higgs boson. So that's why it's called Strahlung, like the German word for radiation, Strahlung process.

And so electrons and positrons were accelerated to 100 GeV each, center-of-mass energy 200 GeV. What was the gamma factor for those electrons?

Another question which is quite exciting is, how much energy do you need in order to split a proton and a neutron, which is a bound state? It's called a deuteron. And it's a fundamental-- the important particle in the evolution of our universe, in the sense that in order to generate higher mass or higher proton number elements, a neutron is rather important in this.

And so just by knowing the mass of the proton, the mass of the neutron, and the mass of the deuteron, you can now calculate how much energy is in the-- binded-- what is the binding energy between those particles. We'll talk a lot about models to calculate binding energy when we talk about nuclear physics. But here, just from the kinematics you can-- and from E equal m c squared, you can calculate how much energy needs to be in this binded or compound state.

From atomic physics, you might remember or know that particles can-- excited particles can emit photons. And so now you have a particle. It goes-- it [INAUDIBLE] excites, radiates a photon. What happens now to the photon? I mentioned this happening in a big gas or in some solid state. Can the photon be reabsorbed by the same medium, or even by the same particle?

It's not a trivial question, but what is the conditions under which-- so for example, imagine you have a gas as an excited particle. And it emits a photon. And so now the photon sees the rest of the gas. Can that rest of the gas absorb the photon? Interesting question. It's not trivial.

Another interesting question, I think, is you're trying to produce new forms of matter. Like you just produced a Z boson, but you can also produce antiprotons. So what is the minimal energy in a proton on a fixed-target experiment-- so again, you have a target of protons in some form, you shoot a proton against this target, and you try to produce an antiproton.

So that means that you have to put produce-- in this collision, you have two protons in the initial state, you have to have a proton, a proton, another proton, and an antiproton in your final state. But what-- how much energy is needed for the [INAUDIBLE] beam in order to succeed with this collision?

My counting is incorrect here. So this should be 5, but OK, fine. Decays. So assume a pion decays at rest. So a pion is at rest. You look at a pion, that's the compound state of meson out of an up quark and a down quark. And it might decay in an electron and a positron. Whatever the dynamics is in these decays, if you just look at the kinematics of this, how fast are the decay products?

In order to calculate that, you need to look at the pion mass, electron mass we just discussed, and the positron has the same mass. So how fast are electron and positron coming out of a pion decay? Assume that the pion is at rest. And you can use momentum conservation and calculate the speed of the electrons and positrons.

Again, one of those minimal-energy proton colliding experiments, very similar setup. But here we try to produce a proton, a neutron, and a pion out of proton-proton collision. And then the last one is the so-called Compton effect, where you have a photon which scatters of an electron target. And so you have an incoming photon, and the electron is at rest.

And then you look at the scattered photon angle, scattered electron angle, and in that collision the energy of the photon is going to change. So the energy of the photon is h times mu or h over lambda, the wavelength. And so the question is, how does the wavelength of the photon change in this kind of condition.

So those are just examples in how you can use relativistic kinematics in order to calculate very important aspects of collisions in particle physics without any understanding, at this point, of the underlying dynamics, the underlying forces, the underlying conservation laws, and so on. So later in this class, we'll discuss what is the likelihood of a pion to decay into an electron positron, and why that is actually not that likely. And also, the collision rates, lifetimes of particles. But here we are just looking at the kinematic of those processes and calculate how much energy is involved and what is the momentum of resulting particles.

So I'll stop here. If you scroll down on the slides, you'll find solutions to two of the problems. And we'll discuss them in recitation.