MARKUS KLUTE: Welcome back to 8.701. So we'll continue the discussion on QED. In the last video, we looked at wave equations and we discussed Dirac equations. Now we want to look at solutions to the Dirac equations.

All right, so remember that the overall goal now is to find a description of spin-half particles, which we can then use in our matrix element calculation in order to get to cross sections or decay rates of particles.

If you just ad hoc or natural choice for a solution would be a wave equation, which is a product of a spinor, which depends on energy and momentum, and an exponent. So we have a free plane wave as a solution to our free particle base waveform.

We have to show or we have to make sure that this wave equation satisfies the Dirac equation as shown here. Since the spinor depends only on energy and momentum here, it's rather simple to write down the derivatives, because they only depend on the exponent.

So we can do this here. And we find those solutions here for the four components. We can rewrite this by putting those derivatives here back into the Dirac equation. What we find then here's this simplified form for the spinor.

Note that this does not depend on derivatives anymore. So this is a rather simple form. And then we can study-- what happens now if they have a particle at rest. So it further simplifies the Dirac equation. It further simplifies here to items E times gamma 0, u-- that's our spinor-- equal m times u.

Since gamma 0 is a diagonal, or is diagonal, we can immediately find the eigenstates to this equation. So we find four different eigenstates, and they are orthogonal. And you find that they look very similar.

So n here is just the normalization factor, which is the same for all four. And we find those four different values here. You can find two with a negative sign in the exponent and two with a positive sign in the exponent.

Now this is for particles at rest, fine. We can interpret those solutions as positive and negative energy states of a spin-half particles or a particle with two spin states. But now we want to see what happens if you have a particle which is not addressed.

So the way to approach this is, so once, we can just apply Lorentz transformation and see how the solutions transform. But it is even easier to just look directly at the Dirac equations for the spinor.

So we start again from our equation here. We just write this down. And then we rewrite the equation using the gamma matrices until we find those factors p times gamma 1-- px times gamma 1, py gamma 2, p3 gamma 3.

Can we write this using the Pauli matrices in this form? OK, so what this gives us is this coupled form of equations. So here we revised our spinor as a two vector, uA and uB. And you find the coupled form between those two if we look at those set of equations.

Great. But this is rather cumbersome and complicated. However, now we can try to find the solutions or try to find eigenstates to the equation. We know that the solutions are of this form here. That's how we started.

If you then try to find a specific [? state, ?] you can start from the simplest alternate solution, which is uA equal 1, 0. And then just put this in here. And you find for u1 those solutions here. And then you turn this around. For uB, you find 1 and 0 and you find the other solution.

So similarly as for the solutions at rest, we find here for our spinors that there is four different spinors, which are independent. You can interpret them now as, again, the positive and negative energy states.

So if you then, for example, say, OK, let's make sure that this is all consistent, we want to see that when the momentum is 0, you come back to the previous solution. If you look at the momentum, if they're 0, those components are all here-- become 0, you find the very same solution as we had on the previous page.

You can also ask yourself what happens now if you don't use this idea of positive and negative energy solutions. If you want to do that and you define that as all are either positive for energy solutions, and not two positive and two negative, you find that you can divide them as linear combinations of the others, so they're not independent solutions. So in order to have four independent solution of the Dirac equations, two have to be positive and two have to be negative in energy.

All right, so I recommend just trying the exercise of playing around with the Pauli matrices and the gamma matrices. If you have not seen this before, it's not easy to follow the algebra. But once you get a hang of it, it's actually not that complicated.

In the next lecture, we look at the solutions-- specifically the solutions for antiparticles a little bit more and discuss interpretations of those solutions.