MITOCW | L9.2 Nuclear Physics: Binding Energies

MARKUS KLUTE: Welcome back to 8.701. In this video, we talk about nuclear binding energies. But before we get started on this topic, I would like you to have a look at this table or this diagram, which shows nuclear abundances in our solar system, so how many atoms of the various types are present in our solar system.

And you see this super interesting structure. Most of it is hydrogen here. And then there seems to be some sort of an excess of iron. And, you know, this table goes all the way to lead and beyond. There's a little bit of a gap here.

So how is this possible? How did this particle get there in the first place? Why is there some which are more frequent than others? A very interesting question which we will be able to answer at the end of the discussion of nuclear physics. And the first starting point is to understand why nuclei are stable in the first place, what holds them together. And that's the discussion of binding energies.

So we can very simply write the binding energy down. We just sum up all the ingredients, the number of protons and electrons, times Z, the number of neutrons, A, the mass number minus Z. And then we subtract from this the mass of the nuclei itself. What remains is the binding energy.

Just for record, the mass of the proton, the mass of the proton, the mass of the neutron, you see that the neutron is slightly heavier than the proton, and the neutron itself decays into the proton. A free proton does not decay. However, inside the nucleus, a proton can also decay.

And then we have the mass of the electron. You see there's a factor of almost 2,000 between the mass scales of an electron and the mass of a neutron. For all practical purposes, we can ignore this, but if it comes to precision measurements, then the mass of the neutron which is 1/2 MeV becomes-- of an electron, which is 1/2 MeV, becomes relevant.

This plot here shows the average binding energy per nuclei as a function of the mass number. And you see that with the exception of those light elements, you see that this is fairly stable and in the range of 7.5 to 9 MeV. You also see that there seem to be a maximum around iron, which then leads to an advantage in gaining energy when you are-- gaining energy going to lower-energy state when you go in this direction and in this direction.

This part is called fission, this part is called fusion. Both processes, because we go to a more energy-preferred state, are possible. They can be used in order to attain energy from nuclear processes.

This diagram here can be parameterized. And the rest of this video, we'll talk about a very popular parameterization of the binding energy. This is semi-empirical. It's called the Weizsager formula, because it was proposed by a German called Weizsager. Sometimes it's called the semi-empirical mass formula, and sometimes the discussion is summarized in the liquid-drop model. And you'll see why in a second.

What you see here is very similar to where before we can calculate the mass and from that the binding energy by having those first elements here. And this part then here is our binding energy. And there's 1, 2, 3, 4, 5 terms, which we're discussing now on the next slide. What is shown here is a parameterization so you can fit the data and get a best estimate for the individual parameters in this equation.

All right. So as the name said, liquid-drop model, we can think about, in some essence, about a nuclide atom as being built out of a soup, a liquid of protons and neutrons which are bound together. So the first term which contributes to the binding energy is the so-called volume term. This dominates the binding energy. And it's proportional to the number-- the mass number.

Remember, the mass number is proportional to the third power of the radius, hence proportional to the volume of the nuclei. And you know, this contributes with about 16 MeV per nucleon, per proton and neutron.

And from this, you can conclude that the nuclear force must be very short range. Why is that? Because in order for the binding energy to depend dominantly on the volume, the individual nuclei can only see its nearest neighbor. So this corresponds to a short range force which is roughly of the distance of two nuclei. If any given nuclei would be able to see everybody else, we would see a term quadratic in the number of nuclei available.

As a result of this, you can calculate the central density, which is about 0.17 nucleons per cubic femtometers or an average distance between protons and neutrons of 1.8 femtometers. OK. So they are really tightly packed. The size of a proton is about a femtometer.

All right. However, the protons and neutrons which are on the surface of this construct, they see less nuclei-- they see less nuclei, nucleons around him. So therefore, the binding energy needs to be reduced. And it needs to be reduced with the area of the surface of the nucleus. So they need to be proportional to r squared, and therefore proportional to A, the mass number, to the 2/3.

OK. Then the protons in the nucleus, they are electrically charged. So they are going to get apart. And that itself also reduces the binding energy. So this is proportional to the number of charges squared. And then you normalize this by the radius. So charge squared over-- no, normalized. Charge squared over r. So this is q squared over r, Coulomb term.

All right. There's two more terms, which are quite interesting. The first one is sensitive to the asymmetry between the number of neutrons and the number of protons. And it can be explained by the Pauli exclusion principle, which allows only two identical fermions, neutrons, or two protons to occupy the same energy state. So you basically fill up the energy states.

Now, as is shown in this picture here, you reach the lowest energy state if the number of neutrons and the number of protons is actually the same. You have higher energies if there is an asymmetry between those two numbers. But we have yet another term which is sensitive to the asymmetry between the number of neutrons and protons, which reduces the binding energy.

And last but not least, the pairing term, which has a very similar origin. So what we are looking at here is the energy is lower if you have an even number of neutrons or protons. It's higher when we have an odd number. So you can have an odd number for the protons or the neutrons, in which case A is odd. And the worst case, or the worst energy state, is achieved having both the protons and the neutrons odd.

So that's why this is a little bit more complicated way to write this. You have those three different cases-- number of protons and number of neutrons even, A odd, or both Z and N odd, in which case A is even-- just to add to the confusion a little bit.

All right. Then you can make a drawing of the binding energy as a function of the mass. And you see, again, those individual terms, the volume energy, the volume term here constant, the volume plus surface reduced, the volume plus surface Coulomb further reduced, and then all terms put together here. And you see again this binding energy parameterization, which we just discussed-- we saw the actual values. There's a maximum around here for iron.