8.701

Introduction to Nuclear and Particle Physics

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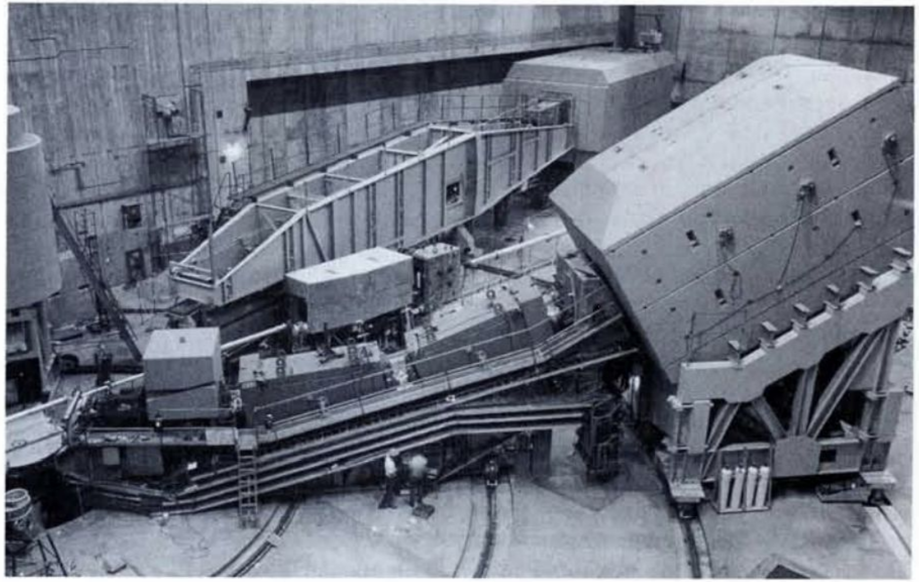
5. QCD

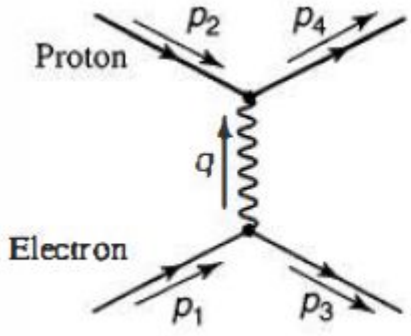
5.2 Elastic Electron-Proton Scattering

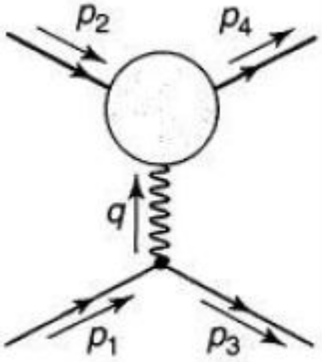
Electron-Proton Scattering

Fig. 1 — The two magnetic spectrometers used for the SLAC-MIT experiment. The 8 GeV spectrometer is in the foreground and the 20 GeV unit is to the rear. The bulk of the detectors comprise shielding (weighing 450 tons for the 8 GeV device).



Elastic electron-proton scattering





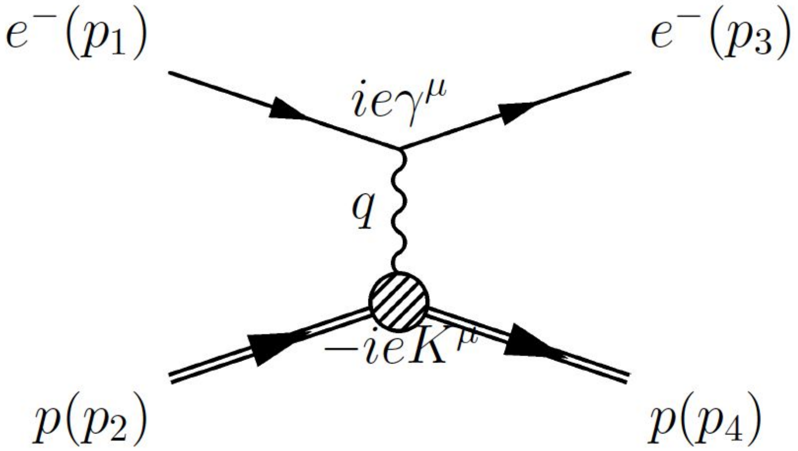
Form Factors

$$F(\vec{q}) = \int d^3\vec{r}\, \exp\{i\vec{q}\cdot\vec{r}\}\, \rho(\vec{r}) \Rightarrow F(0) = 1$$

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{extended}} \approx \left.\frac{d\sigma}{d\Omega}\right|_{\text{point-like}} |F(\vec{q})|^2$$

For e-p scattering, two form factors are required to capture the electric charge distribution and the recoil of the proton

Amplitude



$$\mathcal{M}(e^- p \to e^- p) = \frac{e^2}{(p_1 - p_3)^2} \left(\bar{u}_3 \gamma^\mu u_1\right) \left(\bar{u}_4 K_\mu u_2\right)$$

with

$$K^\mu = \gamma^\mu F_1(q^2) + \frac{i\kappa_p}{2m_p} F_2(q^2) \sigma^{\mu\nu} q_\nu$$

Cross section

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{lab}} = \frac{\alpha^2}{4E_1^2 \sin^4\frac{\theta}{2}}\frac{E_3}{E_1}\left\{\left(F_1^2 - \frac{\kappa^2 q^2}{4m_p^2}F_2^2\right)\cos^2\frac{\theta}{2} - \frac{q^2}{2m_p^2}\left(F_1 + \kappa F_2\right)^2 \sin^2\frac{\theta}{2}\right\}$$

For a point-like spin-½ particle, F_1=1, χ=0, the equation reduces to the Mott scattering result.

Cross section (common notation)

Using linear combinations of the form factors

$$G_E = F_1 + \frac{\kappa q^2}{4m_p^2} F_2 \qquad G_M = F_1 + \kappa F_2$$

Referred to as electric and magnetic form factors.

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{lab}} = \frac{\alpha^2}{4E_1^2 \sin^4\frac{\theta}{2}} \frac{E_3}{E_1} \left(\frac{G_E^2 + \tau G_M^2}{1+\tau} \cos^2\frac{\theta}{2} + 2\tau G_M^2 \sin^2\frac{\theta}{2} \right)$$

With $Q^2 \equiv -q^2 = (p_1 - p_3)^2 > 0$ and $\tau = Q^2/4m_p^2$.

Proton charge radius

$$\rho(r) = \rho_0 \exp(-r/r_0) \qquad 1/r_0^2 = 0.71\ \mathrm{GeV}^2 \qquad \langle r^2 \rangle = 0.81\ \mathrm{fm}^2$$