

# Massachusetts Institute of Technology

## Department of Physics

Course: 8.701 — Introduction to Nuclear and Particle Physics

Term: Fall 2020

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### Problem Set 6

handed out November 20th, 2020

#### Problem 1: Nuclear Fission Reactor [20 points]

Describe briefly the type of reaction on which a nuclear fission reactor operates.

- In nuclear fission a heavy nucleus disassociates into two medium nuclei. In a reactor the fission is induced. It takes place after a heavy nucleus captures a neutron. For example



Why is energy released, and roughly how much per reaction?

- The specific binding energy of a heavy nucleus is about 7.6 MeV per nucleon, while that of a medium nucleus is about 8.5 MeV per nucleon. Hence when a fission occurs, some binding energies will be released. The energy released per fission is about 210 MeV.

Why are the reaction products radioactive?

- Fission releases a large quantity of energy, some of which is in the form of excitation energies of the fragments. Hence fission fragments are in general highly excited and decay through  $\gamma$  emission. In addition, the neutron-to-proton ratios of the fragments,

which are similar to that of the original heavy nucleus, are much larger than those of stable nuclei of the same mass. So the fragments are mostly unstable neutron-rich isotopes having strong  $\beta^-$  radioactivity.

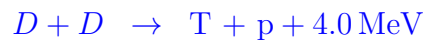
What is the role of a moderator? Are light or heavy elements preferred for moderators, and why?

- For reactors using  $^{235}\text{U}$ , fission is caused mainly by thermal neutrons. However, fission reaction emits fast neutrons; so some moderator is needed to reduce the speed of the neutrons. Lighter nuclei are more suitable as moderator because the energy lost by a neutron per neutron-nucleus collision is larger if the nucleus is lighter.

### Problem 2: Nuclear Fusion [20 points]

Give the three nuclear reactions currently considered for controlled thermonuclear fusion.

- Reactions often considered for controlled thermonuclear fusion are



Which has the largest cross section?

- The cross section of the last reaction is the largest.  
Give the approximate energy released in the reactions.
- See above.

### Problem 3: Deuteron [20 points]

A neutron and a proton can undergo radioactive capture at rest:  $p + n \rightarrow d + \gamma$ . Find the energy of the photon emitted in this capture process. Is the recoil of the deuteron important?

- The energy released in the radioactive capture is

$$Q = [m_p + m_n - m_d]c^2 = 1.00783 + 1.008672 - 2.01410 \text{ amu} = 2.234 \text{ MeV}.$$

This energy appears as the kinetic energies of the photon and recoil deuteron. Let their respective momenta be  $\vec{p}$  and  $-\vec{p}$ . Then

$$Q = pc + \frac{p^2}{2m_d},$$

or

$$(pc)^2 + 2m_d c^2(pc) - 2m_d c^2 Q = 0.$$

Solving for  $pc$  we have

$$pc = m_d c^2 \left( -1 + \sqrt{1 + \frac{2Q}{m_d c^2}} \right).$$

As  $Q/m_d c^2 \ll 1$ , we can take the approximation

$$p = m_d c \left( -1 + 1 + \frac{Q}{m_d c^2} \right).$$

Thus the kinetic energy of the recoiling deuteron is

$$E_{\text{recoil}} = \frac{p^2}{2m_d} = \frac{Q^2}{2m_d c^2} = \frac{2.234^2}{2 \times 2.0141 \times 931} = 1.33 \times 10^{-3} \text{ MeV}.$$

Since

$$\frac{\Delta E_{\text{recoil}}}{E_\gamma} = \frac{1.34 \times 10^{-3}}{2.234} = 6.0 \times 10^{-4},$$

the recoiling of the deuteron does not significantly affect the energy of the emitted photon, its effect being of the order  $10^{-4}$ .

#### **Problem 4: Interaction of photons with matter [20 points]**

Discuss the interaction of photons with matter for energies less than 10 MeV. List the types of interaction that are important in this energy range. Describe the physics of each interaction and sketch the relative contribution of each type of interaction to the total cross section as a function of energy. Note: we will discuss this topic in the week after Thanksgiving. You can prepare by reading the particle data group review on interaction of particles with matter.

- Photons of energies less than 10 MeV interact with matter mainly through photoelectric effect, Compton scattering, and pair production.

(1) *Photoelectric effect*: A single photon gives all its energy to a bound electron in an atom, detaching it completely and giving it a kinetic energy  $E_e = E_\gamma - E_b$ , where  $E_\gamma$  is the energy of the photon and  $E_b$  is the binding energy of the electron. However, conservation of momentum and of energy prevent a free electron from becoming a photoelectron by absorbing all the energy of the photon. In photoelectric effect, conservation of momentum must be satisfied by the recoiling of the nucleus to which the electron was attached. The process generally takes place with the inner electrons of an atom (mostly K- and L-shell electrons). The cross section  $\sigma_{p-e} \propto Z^5$ , where  $Z$  is the nuclear charge of the medium. If  $\epsilon_K < E_\gamma < 0.5 \text{ MeV}$ ,  $\sigma_{p-e} \propto E_\gamma^{-7/2}$ , where  $\epsilon_K$  is the binding energy of K-electron. If  $E_\gamma > 0.5 \text{ MeV}$ ,  $\sigma_{p-e} \propto E_\gamma^{-1}$ . Thus photoelectric effect is dominant in the lowenergy region and in high- $Z$  materials.

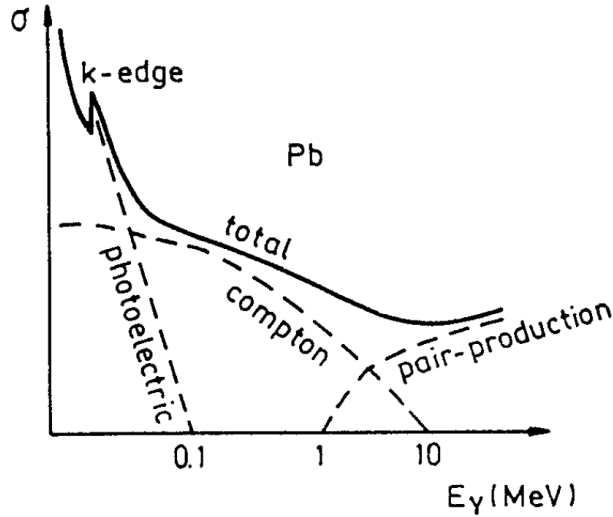


Figure 1: Cross section of the three interactions versus  $E_\gamma$ .

(2) *Compton scattering*: A photon is scattered by an electron at rest, the energies of the electron and the scattered photon being determined by conservation of momentum and energy to be respectively

$$E_e = E_\gamma \left[ 1 + \frac{mc^2}{E_\gamma(1 - \cos \theta)} \right]^{-1},$$

$$E_{\gamma'} = E_\gamma \left[ 1 + \frac{E_\gamma}{mc^2}(1 - \cos \theta) \right]^{-1},$$

where  $m$  is the electron mass,  $E_\gamma$  is the energy of the incident photon, and  $\theta$  is the angle the scattered photon makes with the incident direction. The cross section is  $\sigma_c \propto ZE_\gamma^{-1} \ln E_\gamma$  (if  $E_\gamma > 0.5$  MeV).

(3) *Pair production*: If  $E_\gamma > 2m_e c^2$ , a photon can produce a positron-electron pair in the field of a nucleus. The kinetic energy of the positron-electron pair is given by  $E_{e^+} + E_{e^-} = E_\gamma - 2m_e c^2$ . In low-energy region  $\sigma_{e^+e^-}$  increases with increasing  $E_\gamma$ . While in high-energy region, it is approximately constant. Figure 1 shows the relative cross section of lead for absorption of  $\gamma$ -rays as a function of  $E_\gamma$ . It is seen that  $E_\gamma \gtrsim 4$  MeV, pair production dominates, while for low energies, photoelectric and Compton effects are important. Compton effect predominates in the energy region from several hundred keV to several MeV.

### Problem 5: Proton decay [20 points]

The possible radioactive decay of the proton can be tested in a very large reservoir of water instrumented with devices to detect Cerenkov radiation produced by the

products of the proton decay.

Suppose that you have built a reservoir with 10000 metric tons of water. If the proton mean life time  $\tau_p$  is  $10^{32}$  years, how many decays would you expect to observe in one year? Assume that your detector is 100% efficient and that protons bound in nuclei and free protons decay at the same rate.

- Each  $\text{H}_2\text{O}$  molecule has 10 protons and 8 neutrons and a molecular weight of 18. The number of protons in  $10^4$  tons of water is then

$$N = \frac{10}{18} \times 10^{10} \times 6.02 \times 10^{23} = 3.34 \times 10^{33},$$

using Avagadro's number  $N_0 = 6.02 \times 10^{23} \text{ mole}^{-1}$ . The number of expected decays per year is therefore

$$\Delta N = \frac{3.34}{\tau_p} \times 10^{33} = 33.4/\text{year}.$$

A possible proton decay is  $p \rightarrow \pi^0 + e^+$ . The neutral pion immediately (in  $10^{-16}\text{s}$ ) decays to two photons,  $\pi^0 \rightarrow \gamma\gamma$ . Calculate the maximum and minimum photon energies to be expected from a proton decaying at rest.

- In the proton rest frame (lab frame), the momenta of particles are:

$$\begin{aligned} p_p &= (m_p, \vec{0}), \\ p_\pi &= (E_\pi, \vec{p}), \\ p_e &= (\sqrt{m_e^2 + p^2}, -\vec{p}). \end{aligned}$$

So

$$\begin{aligned} m_p &= E_\pi + \sqrt{m_e^2 + p^2} \\ m_p^2 &= E_\pi^2 + 2E_\pi\sqrt{m_e^2 + p^2} + m_e^2 + p^2 \\ m_p^2 &= E_\pi^2 + 2E_\pi(m_p - E_\pi) + m_e^2 + E_\pi^2 - m_\pi^2 \\ m_p^2 &= 2E_\pi m_p + m_e^2 - m_\pi^2 \\ E_\pi &= \frac{m_p^2 + m_\pi^2 - m_e^2}{2m_p} \\ &= 479 \text{ MeV} \end{aligned}$$

Suppose, in the pion rest frame, the two photons with energy  $E'$  are emitted back to back in a line at an angle of  $\theta'$  w.r.t the boost direction from the lab frame. Then citing the results from Problem 1(b) of PSet5, we have the photon energy in the lab frame

$$E_\gamma = \gamma_\pi E' (1 + \beta_\pi \cos \theta') = \gamma_\pi \frac{m_\pi}{2} (1 + \beta_\pi \cos \theta') = \frac{E_\pi}{2} (1 + \beta_\pi \cos \theta'),$$

where  $\gamma_\pi = \frac{479}{135} = 3.548$  and thus  $\beta_\pi = 0.9595$ .

So, when  $\theta' = 0^\circ$ ,  $(E_\gamma)_{max} = \frac{E_\pi}{2}(1 + \beta_\pi) = 469.3 \text{ MeV}$ ; when  $\theta' = 180^\circ$ ,  $(E_\gamma)_{max} = \frac{E_\pi}{2}(1 - \beta_\pi) = 9.7 \text{ MeV}$ .

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