MARKUS KLUTE: Welcome back to 8.701. So the name of this plan is called Casimir's trick. But what we're actually going to do is we're going to evaluate or learn how to deal with spin information in the calculation of our matrix elements.

So now what is it we're trying to do? So the first problem we might have is that we have polarized particles. So we have here again our example of electron muon scattering. And if you assume that the electron and the muons are polarized, you will find as we discussed in the previous lecture that our matrix element is proportional to the adjoined vector of mu 1 times some sort of gamma matrix times the spinor. And we probably have a polarization that is as well included here to give the polarization of the photon involved at the propagating.

Good. Now in order to now get a number for M, we actually have to be explicit about the base function of the external particles. And you can do this-- you can just write this down.

However, in experiments, we are often interested in the scattering of unpolarized particles. Even if you have a way to polarize a beam, your polarization is not going to be perfect. So in your calculation, you might want to average or some opposite available spins of your particle.

So how are we going to do this? So we're trying to calculate the spin average amplitude. Why averaging? We want to average over the polarization of the incoming particles, again, because we don't know what the polarization states are.

And we want to sum of the polarization of all final state particles simply because each polarization state is a possible outcome of the interaction. And we have to sum over all possible outcomes. Good.

So how do we calculate the spin average amplitude? So again, we start from our matrix element where we have the adjoined vectors and gamma matrix or spinor. If you then calculate the square of this matrix, we find this solution here.

Great. And now we're just doing a few tricks. So first of all, we can write our adjoined mu 1 equal to mu [INAUDIBLE] 1, gamma 0. And then just continue this rewriting to refine this part here of he solution, which looks a little bit simpler where we now the adjoined matrix instead of the adjoined matrix given here.

So we want to evaluate what this adjoined u1 gamma u2 is times adjoined u2 gamma adjoined u1. Good. So this was just matrix algebra and working with gamma so far.

Now we're using this completion relation where I probably haven't told you yet what this syntax here is. So gamma flash is equal to gamma mu p mu. And that's just the way to simplify it writing down the equation.

So if you use this so-called Feynman's slash, you can also rewrite that Dirac equation in the simple form phi del flesh minus M y equal to 0. That's now our direct [INAUDIBLE]. Remember there was a gamma mu in here.

All right, let's just decide now again. We have to sum over all spin states, or the spin states of all those particles. Now if you start with something over particle 2, we can rewrite this here. We use this equation, this completely-filled relation, and just really find this q as equal to this part here. So now we have this equation here, which looks a little bit simpler, because we just have our incoming and outgoing spinors for particle 1 given here.

All right, now we're looking at this part of the equation now. And again, we're just doing a little bit of matrix algebra here. And you find that this is equal to q u 1 mu 1, i-i, which is just simply something over the same indices, which is the same as building the trace of this matrix. All right, so if you just put all of us together, you find that calculating the sum of matrix elements, it's similar to building the trait of a matrix. And then our final results then give you [INAUDIBLE].

All right, so summarizing this part, we have a matrix element. And we saw that the matrix elements of this form's proportional to this form. And then when we try to calculate the spin average matrix element squared, that's equal to the traits of the particles involved using the completion image.

There's an additional factor of 1/2 here, which comes from the averaging over the initial spins. So assuming exactly one of u1 and u2 corresponds to the initial particles defined as vector 1/2, if both initial particles are the same, like in a parent relation, then vector is 1/4.

And if neither is in the initial state, then the factor is 1, which you would have for fair production.

Good. So now we simplified the calculation of our matrix elements, putting the specific spin states in there and the specific polarization vectors to what the calculation of traces of matrices. So now we can look at what does it mean. So this is what Casimir's trick really is about. So summing over spins reduces to summing over matrices.

If you have antiparticles, the completeness relation, which uses two p/minus M, and then you we go ahead. So all you need to know now is how to do this trace it. So first, some general remarks on traces. We have to matches, A and B. You want to calculate the trace of A plus B. Set equal to the trace of A plus the trace of B.

If you have a multiplicator here, a vector alpha, you can just take this out of the calculation of the trace. In the traces, two matrices commutate. The trace of A times B is equal to the trace of B times A. And then you can use this in order to show a more complicated relationships.

Good. We have already started playing around with the gamma Here are a few more identities which might be of use when you calculate traces and calculate matrix elements overall.

The first one is g mu nu times g mu nu is equal to 4. The anti-commutator relations we already discussed and have shown in one of our recitation. And if you have three matrices you can rewrite them as minus 2 times the matrix, which is in the middle here.

So I'm not going through this in much detail here, but I encourage you to just follow, it's an additional exercise. We didn't have the opportunity yet to play with the quantum.

And then you can use those tricks, for example. And commutator relation to calculate the traces. For example, the gamma mu, is 4 times g mu nu.

OK, so what you see here is basically used this anti-commutator. Put it in here, use this part here, this is basically one trade-off, 1 is 4. So we get g mu-mu times the trace the 1. But 4 times 4 matrix is 4. And so you get the trace of gamma mu gamma mu is equal to 4 times g mu, and so on.

Later on, we haven't discussed gamma 5 so much, gamma 5 is defined as 5 times gamma 0, gamma 1, gamma 2, gamma 3. We will see them and discuss the weak interaction. That there's a prominent role will come up this very special gamma matrix.

And here are just some pieces of information for traces for gamma 5. The trace of gamma 5 is at 0. The trace of gamma 5 times-- sorry-- a different gamma matrix is zero. And that's also true for the product of additional gamma matrix that you can find when you just try to calculate this relation and the relation of that.

One last piece of the traces, this gamma matrices, it's shown here. Only with four or more gamma matrices can you define or find a non-zero trace involved in gamma matrices. And here's one example where you have the product of five or four gamma matrices with gamma 5.

And that's equal in this case-- 4 times 3i times the total asymmetric tensor. The total asymmetric tensor is defined as minus 1, or even for rotation of those numbers here plus 1 for odd permutations and if there's two instances of this state.

All right, again, I'm not showing you this in too much detail. But I encourage you to play around. And you will be asked in the next homework to play around with some of the compilations with that.