8.701

Introduction to Nuclear and Particle Physics

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3. Feynman Calculus

3.2 Fermi’s Golden Rule

Fermi’s Golden Rule

- Calculations of decay rates and cross sections have two ingredients
  - **the amplitude M** – dynamical information calculated by evaluating Feynman diagrams using Feyman rules
  - **the available phase space** – kinematic factor depending on masses, energies, and momenta of particles involved

- Fermi’s golden rule says that the transition rate is given by the product of phase space and the square of the amplitude

Golden Rule for Decays

- Suppose: 1 -> 2 + 3 + 4 + … + n
- Decay rate:

$$\Gamma = \frac{S}{2\hbar m_1} \int |\mathscr{M}|^2 (2\pi)^4 \delta^4 \left(p_1 - p_2 - p_3 \cdots - p_n\right)$$

$$\times \prod_{j=2}^{n} 2\pi \delta \left(p_j^2 - m_j^2 c^2\right) \theta \left(p_j^0\right) \frac{d^4 p_j}{(2\pi)^4}$$

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- To assess the phase space, we integrate over all outgoing particle four-momenta, with three kinematical constraints
  - Outgoing particle lies on mass shell

$$\delta \left(p_j^2 - m_j^2 c^2\right)$$

  - Outgoing energy is positive

$$\theta \left(p_j^0\right)$$

  - Energy and momentum must be conserved

$$\delta^4 (p_1 - p_2 - p_3 \cdots - p_n)$$

Golden Rule for Decays

- For two-particle decays

$$\Gamma = \frac{S|\mathbf{p}|}{8\pi\hbar m_1^2 c}|\mathcal{M}|^2$$

Golden Rule for Scattering

- 1+2 -> 3+4+…+n

$$\sigma = \frac{S\hbar^2}{4\sqrt{(p_1 \cdot p_2)^2 - (m_1 m_2 c^2)^2}} \int |\mathscr{M}|^2 (2\pi)^4 \delta^4(p_1 + p_2 - p_3 \cdots - p_n)$$

$$\times \prod_{j=3}^{n} 2\pi \delta\left(p_j^2 - m_j^2 c^2\right) \theta\left(p_j^0\right) \frac{d^4 p_j}{(2\pi)^4}$$

Golden Rule for Scattering

- Two body scattering in CM frame: 1+2 -> 3+4

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar c}{8\pi}\right)^2 \frac{S|\mathcal{M}|^2}{(E_1 + E_2)^2} \frac{|\mathbf{p}_f|}{|\mathbf{p}_i|}$$