MARKUS KLUTE: Welcome back to 8.701. So in this section, we look at electroweak unification. So the aim is to combine the weak and the electromagnetic interactions. The issues we can see here are first, the strength of the interactions are very different. This can be mitigated by the fact that we have heavy gauge bosons involved, and we have seen our heavy particles as being used as mediators to kind of change the strength of the interaction. So this might not be a big issue.

The second problem is that the structure of the coupling is very different. We have seen for QED that they are the vector coupling and for weak interaction that they are the vector-axial coupling. This 1 minus gamma 5 is vector minus axial coupling. So one way to mitigate this problem is to simply absorb this 1 minus gamma 5 term in the definition of the particle spinors.

And I have to warn you, this is a little misleading. And I think that led also to some of the confusion we had in the class before. So what we are doing here simply is, we take our spinor, and we project out with the 1 minus gamma 5 term what the left-handed component of this spinor is. This is just a projection and the definition of this. And we can do this for antiparticles as well as for the right-handed components as well. Good.

So now we can look at the current again. And we look at this weak current that you have seen you can write as our antineutrino here, gamma mu times 1 minus gamma 5 1/2 times e. Now, if we now define our particles which gamma matrices, we find that this simplifies quite a bit, because we now find the current, which can be simply written as a vector current. So we mitigated this quite nicely.

So what now happens to our electromagnetic interaction here? So we have an electron coupling to a photon. We can project out a right-handed and left-handed component, and then we have to add them together again.

When we do this, you find that this component of the left-handed component, or the current corresponding to the left-handed particle and the current corresponding to the right-handed particle. There's no mixed term here because of the way gamma matrices or gamma 5 matrices multiply.

So that's nice. This also explains why the helicity is not changed in QED. You basically see this from the algebra involved in those equations. Good. So far, we haven't done anything. We have just changed the notation.

So we can go one step further. And we use the concepts we introduced when we talked about QCD or the strong isospin. And since we can nicely describe those currents here of those particles, we can maybe see if we can write a neutrino and an electron as part of a duplet.

And when we do this, we rewrite the currents, the positively charged and the negatively charged current as simply the left-handed components of those duplets. We introduced a new matrix here, tau plus and tau minus. And they're simply combinations of tau 1 and tau 2, which are, in fact, the Pauli matrix. This is just a relabelling as well.

So there's a lot of relabelling going on, not to confuse you. But we have simply written this current as positively charged weak current and negatively charged current, where we rotate a neutrino into an electron or an electron into a neutrino using the weak interaction.

Great. So now we can write this current here as the third component of this current. And we see when we write down the third component of this current using tau 3 here, we find that something looks-- something which looks like a neutral current. So this is something like a neutral current, where we have a neutrino coupling to a neutrino and a left-handed electron coupling to a left-handed electron, on a vertex vector [?] where there's an interaction that was going on.

This is not quite the full story yet. Let me remind you about the definition we used also in isospin, which is the Gell-Mann-Nishijima equation, which connected the electric charge to the isospin and the strangeness of the particle. And we do the very same thing. We have an isospin component and a so-called hypercharge component. It's similar to the strangeness we had before. And Q is the electric charge of the particle involved.

And so with this, we can now define an iso-- a hyperspin current, which is given as 2 times the electromagnetic current minus 2 times the third component of the-- third component of the weak current here. And so now we find interesting effects here. There's a new component which also couples to right-handed particles.

So the missing part here-- some of you might have seen this already-- is this neutral current in the upper equation didn't connect or didn't have a contribution from right-handed particle. And since there's right-handed electron which coupled to Z boson, there needed to be this kind of additional term. So now we have a current which includes the right-handed particles as well.

So that's great. We can generalize this by writing those duplets for all particles we know. There's an additional kind of caveat here. We haven't talked about this too much. We have to consider the fact that mass eigenstates are not really the same eigenstates which participate in the weak interaction. Right now we can ignore this. We'll later come back to this question.

And then we can write the three components of our isospin current and our hypercharge current as well. Note that this EM current here is our electromagnetic current.

Good. So now we rewrote this, and we find somehow very close a consistent picture. We find that there is a charged current, and then there is a neutral current in the weak interaction. But what we actually wanted to do is combine the weak interaction and electromagnetic interaction.

Now, let's look at this again and start over again. So we have an isospin current here, which copies to the three components-- the triplet, the isospin triplet. So this is a W1, W2, W3 triplet. And then we have the singlet here which couples to the hypercharge. Very good.

So now, if I try to identify now components which we already know the first thing we can do, we have to make sure that we find our W plus and W minus bosons again. And they're simply linear combinations of the W1's and the W2's.

And then the next thing I have to do is I have to find my electromagnetic interaction. And we can do that by binding this A-- this is the photon-- as a linear combination of the third component of our triplet, isospin triplet, and our singlet. And what you see here is that there's actually mixing going on. So we basically rotate those with this mixing angle, which we already introduced, sine omega weak mixing angle theta omega-- sorry, theta W.

So we find that the photon can be made out of a mixing of the third component of the isospin triplet and the singlet component B mu. And similarly, we can find the Z boson as the other component in this mixing, the other state we find in this mixing.

The way we find those mixings here is through the couplings we already know, that we find this omega g times sine theta W is equal to g prime cosine omega W. And that's equal to the electromagnetic coupling. And then similarly, we find a solution for gz.

So what we have seen now is that apparently we are able to combine the weak and the electromagnetic interaction by mixing-- by introducing the weak isospin and by mixing isospin triplet components with a singlet component. And so we find a picture which is consistent with the W plus, a W minus photon, and the Z boson. So that's very nice.