PROFESSOR: Welcome back to 8.701. So in this lecture, we want to write down the Feynman rules for QCD. A lot of work happened to get to this point. If you recall, we derived Feynman rules for toy theory, then we extended this to QED on the electrodynamics, introducing and keeping track of the spin of particles.

And now in QCD, We have to do one step in addition. We have to keep track of the color of particles. So color, the charge of QCD, plays a very important role here.

If we just investigate this fundamental process here, this fundamental vertex, which is very similar to a photon being radiated from an electron. Here, we have a gluon being radiated from quarks.

And when this happens, the quark changes its color. So here you have a quark with color blue and a quark with color red. The gluon adds a color. It adds red, but it also takes away the blue.

So the quark itself basically carries two colors. It's bi-colored-- one color and at the same time an anti-color. A leading order process here has two vertices. And this is a scattering, for example, of two quarks which [INAUDIBLE] through a gluon.

All right, so we have three kinds of charges, meaning that the quarks come in red, green, or blue. So we have to keep track of this when write down our amplitudes or matrix element. So we do this by just introducing a new vector. I call this c here. Very simple-- a three-element vector for those three colors.

All right, but how about the gluon itself? So QCD is based on a symmetry group which is called SU3. So it's basically a rotation in three dimensional space. There's eight independent such rotations.

And what you want to think about is just moving from one color state into the next. So you can write them down as I show here. And we do this in linear combination of color and anti-color.

If you want to keep track of this, we do this in this form. We have a vector of A elements. And the gluon is one of those components. This is given here for this very first one.

So let me introduce the notation here. Literally, we were introducing Pauli matrices, but not for SU2, as a symmetry group with SU3. They're called the Gell-Mann matrices, and they're just written here.

Again, those are the rotations I was just talking about. Those are the rotation from one color state into the next. There's commutator relations for this. So if you have two of those Gell-Mann matrices and you tried to write down the commutator, you find this 2 times i times this structure functions, or structure constants, times 1-- well, the next Gell-Mann records.

So if you're just thinking about how many combinations are there, there's eight matrices. We have 8 times 8 times 8 combinations of those constants here. And so it means that there is 512 of those constants. Most of them are 0. And the ones which are not 0, they're listed here, or combinations of those.

All right, so now we're ready to just write down the QCD Feynman rules. Again, we start from the external lines. So for the incoming quark or outgoing quark, we write our spinor, and then we keep track of color. We do this for quarks and we do this for anti-quarks in the very same way.

For the gluon now, we have to keep our polarization, keep track of the polarization of the gluon and also of the color. And those are the vectors I just introduced on the previous slide.

For the propagators, we have quarks and anti-quarks and gluons. Gluons are massless, so the propagator here looks very much like the one for the photon. And the propagator for the quarks looks very much like the one we had for the electron in QED.

The fundamental vertices, I introduced one already, but there's two more. That's because gluon carries charge and can come to itself. We already discussed this in a recitation. So we have those self-couplings of the gluon here. And those vertices are as relevant or more relevant even than the one here.

All right, so the vertices come with a vertex factor in our Feynman rules. Again, for this very first one, we find a very similar one as we had before. You see here this Gell-Mann matrix. And let's just call this a rotation in color.

But we have this three gluon vertex and the four gluon vertex as well. And those come with structure functions here to keep track of the permutations, commutations between the color involved. And it becomes more complicated for this four gluon vertex, which has pairs of structure constants each.

All right, that's all we need in order to calculate matrix elements or amplitudes for QCD. The rest is just executing the Feynman rules as we did before for our toy theory, and also for QED.