MARKUS KLUTE: Welcome back to 8.701. So we continue our discussion on the Feynman Calculus, and continue now talking about Fermi's Golden Rule. The heart of calculating decay rates and cross sections is Fermi's Golden Rule. And it simply tells you how we can use the calculation of amplitudes and the available phase space to make assessment of decay rates and cross sections.

So the amplitude M holds all the dynamical information. And we can see that we can calculate the amplitude by evaluating Feynman diagrams directly using Feynman rules. The available phase space holds is a kinematic factor, and it depends on masses and the energies and the momentum of the particles involved. And then again, Fermi's Golden Rule simply says that the transition rate or decay rates and cross sections are given by the product of the phase space and the square of the amplitude.

How does this now look like? If you look at the Golden Rule for Decays, here we suppose having one particle decaying into a second, third, fourth, n-th particle. This is that the decay rate is given by matrix element, the amplitude squared, and a term which is C, the phase space factor. There's also a factor here in front. This S has to count-- we have to account for the fact that we might have the same particle in the final state, or the same particle occurring multiple times. And we have to make sure that we don't have double counting. In this double counting has to be correct. If all particles are different, this extra factor is 1. We'll look at this some more later.

Now, at first glance, this looks like rather complicated. But if you try to assess now what those individual terms mean, you will see that it's very accessible. So when we try to calculate a use from this golden rule for the case, we to integrate over all outgoing particle four-momenta. But we have three kinematical constraints. The first one is that the outgoing particles have to be on mass.

So they have to be on mass shell. We talked about this issue of virtual particle [? inertia ?] before. But simply, they have to-- the energy of the particle has to follow this condition. This is a delta function, which simply means that this gives us, if the argument is 0, the delta function, which was 1. The argument is non-zero. The function returns 0. So this [INAUDIBLE] simplifies this term here. So this first part here in our Fermi's Golden Rule simply accounts for the fact that outgoing particles have to be on mass shell.

Outgoing particles also have to have positive energies. And this explains our second factor here, this factor. And so this factor is the heavy side function, and this is simply 0 for negative values and 1 for positive values. And the last one means that energy and momentum of the particle have to conserve. So the first particle minus the second, third, and so on for each of the component for energy and for those [? three ?] components of the momenta have to be 0 for this to return 1. So again, another delta function.

That's also factors of pi. And the simple rule here is for each delta function, you have to account for a vector of 2 pi in your function. So this basically explains already everything we see here on this slide. So now we can calculate. And I recommend to have a look at Griffiths chapter 6 for this.

If you look at two particle decays. So one particle into two particles. This simplifies quite tremendously because of all the delta functions here and the heavy side functions. This equation simplifies directly to a factor. You have the momentum of the particle here, and a matrix element. Again you have this statistical factor here to account for the fact that there might be duplicates and you want to keep track of the statistical factor.

For scattering, the equation looks almost the same. So you have the same phase space factor. Almost the same phase space factor. Matrix element. Again, this is the transition way it is given by, as Fermi's Golden Rule tells us, by the matrix elements squared. And the phase space factor. This overall effect, as we look at later, they are slightly different. But here, as a rule, we want to make sure that we have a way to assess cross section. Did this make sense? We'll see later in more detail.

For two body decays in the center of mass frame. You know that the initial momenta in the center of mass frames of particle 1 and particle 2 have to be the same. The outgoing momenta also have to be the same. They don't necessarily have to be the same as the [INAUDIBLE] But if any uses of the differential cross-section can be calculated quite straightforwardly. Again, there's a matrix element squared. You have the final state momenta, the initial state momenta. And divide this by the sum of the energies squared together with an extra factor here.

So what we have seen here is just we have looked at it. I didn't explain how we got to this. But we have seen from this Golden Rule, which helps us to assess. And we have seen how we can calculate the phase space factor. The next lecture, we now see how we can calculate the matrix element itself. And we'll start doing this by using a toy experiment or toy model, such that the discussion simplifies, algebra simplifies quite a bit.