Massachusetts Institute of Technology

Department of Physics

Course: 8.701 — Introduction to Nuclear and Particle Physics
Term: Fall 2020
Instructor: Markus Klute

Problem Set 5

handed out November 4th, 2020

Problem 1: The T2K Experiment[50 points]

The T2K experiment uses an off-axis ν_μ beam from $\pi^+ \to \mu^+\nu_\mu$ decays. Consider the case where the pion has velocity β along the z-direction in the laboratory frame and a neutrino with energy E^* is produced at an angle θ^* with respect to the z'-axis in the π^+ rest frame.

(a) Show that the neutrino energy in the pion rest frame is $p^* = (m_\pi^2 - m_\mu^2)/2m_\pi$.

(b) Show that the energy E and angle of the production θ of the neutrino in the laboratory frame are $E = \gamma E^*(1+\beta\cos\theta^*)$ and $E\cos\theta = \gamma E^*(\cos\theta^* + \beta)$ where $\gamma = E_\pi/m_\pi$

(c) Using the expressions for E^* and θ^* in terms of E and θ, show that $\gamma^2(1-\beta\cos\theta)(1+\beta\cos\theta^*) = 1$.

(d) Show that the maximum value of θ in the labortory frame is $\theta_{max} = 1/\gamma$.

(e) In the limit $\theta \ll 1$ show that $E \approx 0.43 E_\pi \frac{1}{1+\beta\gamma^2\theta^2}$ and therefore on-axis ($\theta = 0$) the neutrino energy spectrum follows that of the pions.

(f) Assuming that the pions have a flat spectrum in the range 1-5 GeV, sketch the form of the resulting neutrino energy spectrum at the T2K far detector (Super-Kamiokande), which is off-axis at $\theta = 2.5°$. Given that the Super-Kamiokande detector is 295 km from the beam, explain why this angle was chosen.

Problem 2: Nuclear Stability [30 points]

The Weizäcker formula or semi-empirical mass formula is a parametrization of nuclear mass as a function of A and Z. Following this formula, the mass of an atom with Z protons and N neutrons is given by the following:

$M(A,Z) = NM_n + ZM_p + Zm_e - a_V A + a_s A^{2/3} + a_c \frac{Z^2}{A^{1/3}} + a_a \frac{(N-Z)^2}{4A} + \frac{\delta}{A^{1/2}}$
with $N = A - Z$.

The exact values of the parameters a_V, a_s, a_c, a_a, and δ depend on the range of masses for which they are optimized. One possible set of parameters is given by the following:
$a_V = 15.67$ MeV/c^2, $a_s = 17.23$ MeV/c^2, $a_c = 0.714$ MeV/c^2, $a_a = 93.15$ MeV/c^2 and $\delta = -11.2, 0, +11.2$ MeV/c^2 for even Z and Z, odd A, or odd Z and N, respectively.

For fixed A find the proton number Z for the most stable nucleus, and plot Z as a function of A. Each term captures an aspect of the atom. Explain briefly how the individual terms can be interpreted.

Problem 3: Decay time dating [20 points]

Naturally occurring uranium is a mixture of the ^{238}U (99.28%) and ^{235}U (0.72%) isotopes.

How old must the material of the solar system be if one assumes that at its creation both isotopes were present in equal quantities? The lifetimes are $\tau(^{235}\text{U}) = 1 \times 10^9$ years and $\tau(^{238}\text{U}) = 6.6 \times 10^9$ years.

How much of the ^{238}U has decayed since the formation of the earth's crust 2.5×10^9 years ago?

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