## MITOCW | L3.3 Feynman Calculus: Toy Theory

## KLUTE:

All right, so welcome back to 8.801 . We'll continue our discussion on Feynman calculus. And here we dive into toy theory.

So this theory is a toy. And it's just an example to illustrate Feynman rules. What we do, the simplification we employ here is leaving out the spin of the particle involved.

We consider the spin, we add another algebraic complication which is quite now confusing. So we leave this out for now. We come back later to this.

So we're supposed to have three kinds of particles involved here-- particle A, B, and C. And so we can have a primary vertex where these all three particle are interacting like shown here. So particle A, might decay into particle $B$ and $C$. You can assume that particle $A$ is heavier than the sum of $B$ and $C$, so this is schematically allowed.

We might also have corrections involved here as shown here. And what we'd be interested in is now, for example, calculating the lifetime of this particle A. We might do this just for this primitive vertex. Or we might do this for this complicated set of corrections.

We might also be interested in calculating scattering processes where particle A is scattered with particle A and produces particle B and particle B. Or we scatter particle A with particle B and so on.

So in this theory, at the end of the lecture, we have all tools in hand to calculate this. No worries-- I will not leave you alone with this. This lecture, we go through the recipe. And then later on we'll see how we actually apply this.

So let's look at this recipe. So the recipe has a number of steps. And the key is to just follow those steps in order to get to the desired result.

The first step is to label incoming and outgoing four-momenta of particles. We label them with p1, p2, and up to pn. We also want to label all internal momenta. So we have an internal line, then we want to label this internal momenta with q1, q2, and so on.

We want to add arrows to each line to keep track of what is a positive direction, as we discussed before, particles might travel backwards in time. Those are typically antiparticles. And for those we, have to make sure that we correctly account for the momenta.

For each vertex, we have a factor. We write this factor minus ig, where $g$ is the coupling constant. So this is a measure of the strength of the interaction involved.

Then we have a propagator. So for each internal line, the internal lines are also called propagator. We write down a factor, i over qj squared minus mj squared t squared.

Note that qj squared doesn't have to be mj squared c squared, meaning that the parity can be off shell, off mass shell. You also see that there is a complication in the integral when you actually have those vectors being the same.

You want to make sure that energy and momentum is conserved. So for each vertex, you write down a delta function with the conditions. This is for this three vertex where momentum of the first one plus the second plus the third is equal to 0 . Only then the value of the delta function is 1 . Remember, there's a minus sign somewhere, most likely here for this kl value.

Then you want to integrate overall internal momenta. So for each internal line, we write a factor-- 1 over 2 pi to the fourth power. d 4 is on your momenta. And then, this all will result in a delta function, which you just eliminate. And you do that by multiplying it. You erase this delta function and you replace it by a factor i.

So this seems like very confusing. Why do you add delta functions first, and then you erase them later? Note in Fermi's golden rule, we use the squared of the amplitude. And you also saw that the [? phase-based ?] factors already have this kind of delta functions included.

So we get out of the complications that we don't really know what to square of the delta function is by erasing, by adding the $i$ and then keeping track of the momentum conservation, this conservation here when we apply the [? phase-based ?] factor. And then, voila, you just calculated a matrix element.

All right, so those are the rules. Now the key is to practice how to apply them. So what we do next is to practice using this toy experiment in how to calculate the matrix element, the [? phase ?] base, and the $z$ decay rates and cross sections.

And then as a following step, we will see how this all unfolds. Then we have a real series, like QED, like the weak interaction and the strong interaction.

