MARKUS KLUTE: Welcome back to 8.701. So now after we introduced the weak interaction and the Feynman rules for weak interaction, we can now look at decays of muons, and in this case, the decay of a pion. Decay of the pion is specifically interesting. And we discussed the decay of the pion before when it came to the discussion of helicity states. Now, let's look at this again with the information we have and what we learned.

Now, if you look at the pion decay, the two or three leading decay modes are given here. The one is where the pion, in this case a negatively charged pion, decays into an anti-electron neutrino and an electron, or V or the W in a muon and an antimuon neutrino.

If you look at this in the rest frame of the pion, we can see that the neutrino and the lepton, charged lepton, are produced back-to-back. Now, the spin of the pion is 0, which means that the opposite-direction outgoing leptons have to have the same helicity states. Since the neutrino is massless, the antineutrino is massless, the antineutrino is produced right-handed. It is always right-handed. The chiral state of the neutrino and the helicity state of the neutrino are essentially the same, because they're massless. Means the projection of the spinor is basically the same as a projection of the spin on the momentum [? direction. ?]

All right. But the charged lepton is massive. If the charged lepton would be massless, the decay would not be allowed. There would not be a right-handed helicity state for a charged lepton.

Now, this causes quite some confusion. And I've seen, even in this course, some students being confused by this. I can write the, let's say, right-handed charged lepton and decompose its right-handed helicity state. So this is, let's say, the right-handed helicity state. And I can decompose this through the chiral states, the right-handed and the left-handed.

And you have seen in the previous lectures that only the left-handed component participates. Now, you can also see from this equation here that if the momentum and energy would be the same, as it is the case for massless particle, this would be 0, this would be 1, this would be 1. And therefore, this right-handed helicity state would be the same as the chiral state, and it wouldn't be coupling to the weak interaction.

Now let's erase this really quickly, because you want to actually look at this decay. And so now we have all the tools together-- almost all the tools together to calculate this, the decay rates, or the ratio of decay rates. And you want to do this in the pion rest frame, so the momenta are given here. See that the pion momentum is 0. And for momentum, the energy is equal to the mass for the charged lepton. And for the neutrino, just produce in an opposite direction. So neutrino in this case goes into negative d direction.

Then we can write the leptonic current, as we have just seen in the previous lecture. You see this 1 minus gamma 5 term here. Good. And I could have just called this left-handed here and put this into the definition of the spinor. Fine. When we put a real spinor, this comes out immediately [INAUDIBLE]. Immediately. You have to keep this in mind.

The matrix element, then, is a little bit more complicated. And here is an additional-- so you see the current here again. You see the propagator, and I went into the low-energy approximation here. You see that instead of having a [? q ?] square minus m square, I'm just keeping the m square component of this.

And then I have this part here for the current, for the pion current here. And I simply parameterize my missing understanding or missing ability to calculate [? non-prohibitive ?] QCD with a form factor. So I introduce this form factor for a pion. This is not an important part of the discussion, we just keep track of this here.

All right. Then we can calculate this matrix element fine. We then have to be explicit about the spinors we are using, and we use the momentum as defined above. So this step here I'm not doing explicitly. If you want, you can go to Thomson and read in chapter 11. He gives quite some detail on this.

All right. So moving on, there is one extra thing. When we try to calculate the spin-averaged matrix element, we find that we don't have to do any work because there's a spin [? 0 set, ?] there's only one state contributing, so we don't have to do any work. We just have to square the matrix element.

We find this as a solution here, and there's an additional factor we haven't introduced yet. This is the Fermi coupling. Again, this comes out in the low-energy approximation. And G Fermi is simply defined over the coupling to the W over the W mass squared, as shown here.

All right. Again, this is just a factor which is not relevant to the discussion at this point. But we can then, using Fermi's golden rule, calculate the partial decay width of the pion decay. OK? So we just put in the matrix element here, and we replace the momentum with the energy, being equal to the mass of the pion. And voila-- we get this as an answer for the partial decay width. OK?

If you now want to know some experimental information, like the partial decay width of the pion, charged pion, to electrons over the muon, you want to know what this factor is, we immediately can do this. We don't need to know any of the details F, G Fermi as a structure function of the pion. All of those factors cancel out. And what is left here are the parameters of the electron mass, the muon mass, and the pion mass. And if you just use values like this, the mass of the muon with 205 MeV and the mass of the pion 240 MeV, you find 10 to the minus 4 as a value for this ratio of part [? indicators. ?]

And you see where this comes from. This basically comes from the fact that the electron mass is much, much smaller than the mass of the muon. And factually, you can expand this by the fact that a right-handed helicity state for a muon can have a much larger contribution of the left-handed chiral state of a muon, while this is not possible-- or, that is, the component is much smaller for the lighter electron. And again, only the left-handed component of the charged lepton contributes to the weak interaction.