MARKUS KLUTE: Welcome back to 8.701. So again, we have now all tools in place to do a next round of cross-section calculations. We have seen how to set up a matrix element. We have seen how to build spin average or to treat the spin, and then specifically to calculate spin average amplitudes using [INAUDIBLE].

All right, I'm not saying that this is all easy now, but you have seen all necessarily elements to calculate a cross-section for QED process. So let's summarize.

So we have seen that we can set up some matrix element using Feynman rules for QED. We have seen how to set up the spin average matrix element squared using the traces.

Now we would have to evaluate the traces in order to derive this formula here. So I'll spare you a precise discussion of this step here, but you can actually follow this quite straightforwardly.

Let me just step back a little bit before we proceed. My goal for the class is not to have you calculate all kinds of cross-section processes, but to understand how you would do it, for the purpose of really understanding where dependencies come from and where this kind of calculation has its limitations.

The first part is you want to see what is the dependencies on the couplings involved. You see this g squared, for example. That's a rather important effect. You also want to see, so Fermi's this golden rule, how we get actually into the cross-section from the matrix element calculation.

So if you ever had to calculate a matrix element, am I going to ask you to do this once, maybe twice, as part of the homework set. I encourage you to open the book, follow the rules, look up tricks, how to work with traces. And then you should get to a reasonable solution in a reasonable amount of time. But here for the purpose of this discussion, we want to just have a look at a few specific cases where we make assumptions and simplifications to the discussion.

So the first one is called Mott scattering. So here, again, we are at this example of a spin-half particle scattering with a spin-half particle-- a different spin-half particle, so an exchange of a photon. So we used the example of an electron-muon scattering, but this muon here could also be a proton or any other nuclei we spin off.

The assumption for Mott scattering we are using is that the mass of this particle, the muon, is much heavier than the mass of the electron. And that's true the muon 200 times heavier than an electron. A proton is even heavier. Any heavier nuclei of this feels even heavier than this.

In Mott scattering, we also make the assumption that the momenta involved are lower than the mass of the heavy particle and that the recoil of the heavy nuclei, or the muon, can be neglected. If we do that, we can then write the differential cross-section using Fermi's golden rule as a spin average matrix element squared divided by 2 pi M squared.

OK. If you then use this kinematic information, you basically start from this matrix element here. And then you use those vectors, those four vectors, for your momentum of the first, second, third, and fourth particle. You find that many of the vectors are simplifying to ME. So p2 times p3 is ME. And so are many of the others.

And there is a few important factors. For example, p1 minus p3 squared is minus 4p squared sine squared theta half. And similarly, p1 times p3.

So you put this all in-- again, starting from this very formula we just had discussed before-- and you put all the simplifications and you get this matrix element, which already that looks much more manageable. There's an M squared, there's a p squared, there's a cosine squared theta half term, and some factor which depends on the moment times the mass.

And if you then add this to Fermi's golden rule, you find this equation for your Mott scattering. Again, this is the scattering of two different spin-half particles where one is much heavier. The outgoing momenta are small. And the recoil of the heavier particles can be neglected.

So this Mott's formula describes, for example, the Coulomb scattering, so the scattering this photon on the electric charge of a nuclei. And the scattering particle is not too heavy and not too energetic, like an electron.

We also assume that everything involved here is point-like. We haven't had any discussion on the charge distribution of the nuclei or anything. We assume that this is a point-like particle.

OK, we can further discuss now the case where the initial state particles are non-relativistic. So here our momentum formulas simplify. This is simply M squared, p amplitudes is 2ME. And alpha is q1 times q2. Those are the electric charges.

And so then our differential cross-section further simplifies to something you've already seen. The Lorentzian cross-section is equal to q1 times q2 divided by 4 times the energy sine squared theta half squared. And we have seen that as already the Rutherford scattering cross-section when we discussed cross-section measurements in a geometrical kind of thing. So this closes a loop here in our cross-section discussion how we can think about those things.

The Rutherford cross-section is nothing else but a big billiard ball being hit by a small billiard ball and looking at how the cross-section differentially kind of evolves out this setup.

All right, in this sequence we have a little bit more of a discussion. What happens now if we induce higher-order terms and how can we think about those solutions? And then have two extra lectures and where we go back and discuss spin, and also how we can actually understand this in a Lagrangian setup.