

8.701

Introduction to Nuclear
and Particle Physics

Markus Klute - MIT

4. QED

4.10 Noether's Theorem



Lagrangian for the free Dirac field

$$\mathcal{L} = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi.$$

Unchanged by the global U(1) phase transformation

$$\psi \rightarrow \psi' = e^{i\theta}\psi$$

Invariance can be expressed by infinitesimal phase transformation

$$\psi \rightarrow \psi' = (1 + i\varepsilon)\psi \quad \text{and} \quad \bar{\psi} \rightarrow \bar{\psi}' = (1 - i\varepsilon)\bar{\psi}.$$

Lagrangian for the free Dirac field

Changes in the fields and their derivatives are

$$\delta\psi = i\varepsilon\psi, \quad \delta(\partial_\mu\psi) = i\varepsilon(\partial_\mu\psi), \quad \delta\bar{\psi} = -i\varepsilon\bar{\psi} \quad \text{and} \quad \delta(\partial_\mu\bar{\psi}) = -i\varepsilon(\partial_\mu\bar{\psi})$$

Global symmetry implies

$$\delta\mathcal{L} = \frac{\partial\mathcal{L}}{\partial\psi}\delta\psi + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\delta(\partial_\mu\psi) + \frac{\partial\mathcal{L}}{\partial\bar{\psi}}\delta\bar{\psi} + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\bar{\psi})}\delta(\partial_\mu\bar{\psi}) = 0.$$

From which we get

$$i\varepsilon\frac{\partial\mathcal{L}}{\partial\psi}\psi + i\varepsilon\frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}(\partial_\mu\psi) - i\varepsilon\frac{\partial\mathcal{L}}{\partial\bar{\psi}}\bar{\psi} - i\varepsilon\frac{\partial\mathcal{L}}{\partial(\partial_\mu\bar{\psi})}(\partial_\mu\bar{\psi}) = 0.$$

Lagrangian for the free Dirac field

The terms involving the derivative with respect to $(\partial_\mu\psi)$ can be expressed as

$$\frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}(\partial_\mu\psi) = \partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\psi\right) - \left[\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\right)\right]\psi.$$

So we find

$$i\varepsilon\left[\frac{\partial\mathcal{L}}{\partial\psi} - \partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\right)\right]\psi + i\varepsilon\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\psi\right) - \{\bar{\psi}\text{ terms}\} = 0.$$

which leaves us

$$i\varepsilon\partial_\mu\left[\frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\psi - \bar{\psi}\frac{\partial\mathcal{L}}{\partial(\partial_\mu\bar{\psi})}\right] = 0.$$

Conserved current

$$j^\mu = (\rho, \mathbf{J}) = -i \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)} \psi - \bar{\psi} \frac{\partial \mathcal{L}}{\partial(\partial_\mu \bar{\psi})} \right)$$

The partial derivatives of $\mathcal{L} = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi$.

are $\frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)} = i\bar{\psi}\gamma^\mu$ and $\frac{\partial \mathcal{L}}{\partial(\partial_\mu \bar{\psi})} = 0$,

From which we find the conserved current $j^\mu = (\rho, \mathbf{J}) = \bar{\psi}\gamma^\mu\psi$.

MIT OpenCourseWare
<https://ocw.mit.edu>

8.701 Introduction to Nuclear and Particle Physics
Fall 2020

For information about citing these materials or our Terms of Use, visit: <https://ocw.mit.edu/terms>.