8.701

Introduction to Nuclear and Particle Physics

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1. Fermions, bosons, and fields

1.4 Decays

Measuring properties of forces

Three basic properties that can be experimentally determined

1) Masses (or energies) of bound states

2) Decay rates or widths of unstable particles

3) Reaction rates expressed as cross sections

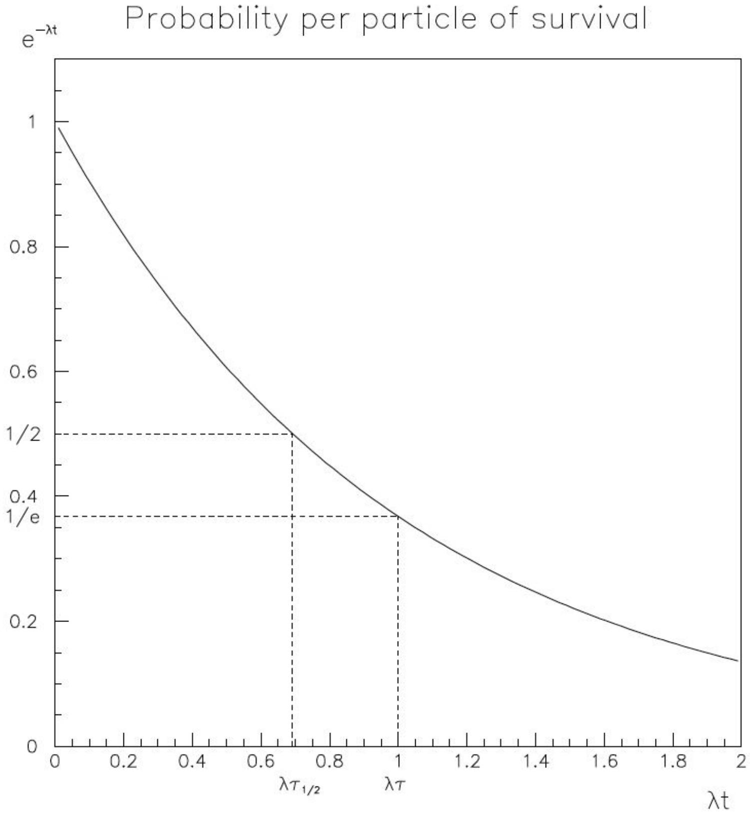
Decays

Decay rate λ

S(t) probability that particle will survive at least until time t

$$S(t)(1 - \lambda dt) = S(t + dt) = S(t) + \frac{dS}{dt}dt$$

$$\frac{dS}{dt} = -S\lambda \qquad \frac{dS}{S} = -\lambda dt \qquad \ln S = k - \lambda t$$



with $S(0) = 1$ $k = 0$ we get $S = e^{-\lambda t}$ the exponential decay law

Decays

The average time the particle lives is

$$\tau = \langle t \rangle = \int_0^\infty t\lambda e^{-\lambda t}\, dt$$

and we find

$$\tau = \left[-te^{-\lambda t}\right]_0^\infty + \int_0^\infty e^{-\lambda t}\, dt = \left[-\frac{1}{\lambda}e^{-\lambda t}\right]_0^\infty = \frac{1}{\lambda}$$

The average is called the lifetime. In terms of lifetime

$$S(t) = e^{-t/\tau}$$

For numbers instead of probabilities

$$N(t) = N_0 S(t) = N_0 e^{-\lambda t}$$

Decays

Half-life, $\tau_{1/2}$ often used in nuclear physics

$$N(\tau_{1/2}) = \frac{N_0}{2} = N_0 e^{-\lambda \tau_{1/2}}$$

$$e^{\lambda \tau_{1/2}} = 2$$

$$\tau_{1/2} = \frac{\ln 2}{\lambda} = \tau \ln 2 = 0.693\tau$$

Decays

Unstable states do not have exact energy state

$$\Delta t \Delta E \geq \frac{\hbar}{2}$$

Quantised by the particle width

$$\Delta m c^2 = \Gamma = \frac{\hbar}{\tau} = \hbar\lambda$$

Complication if decay in several modes is possible.

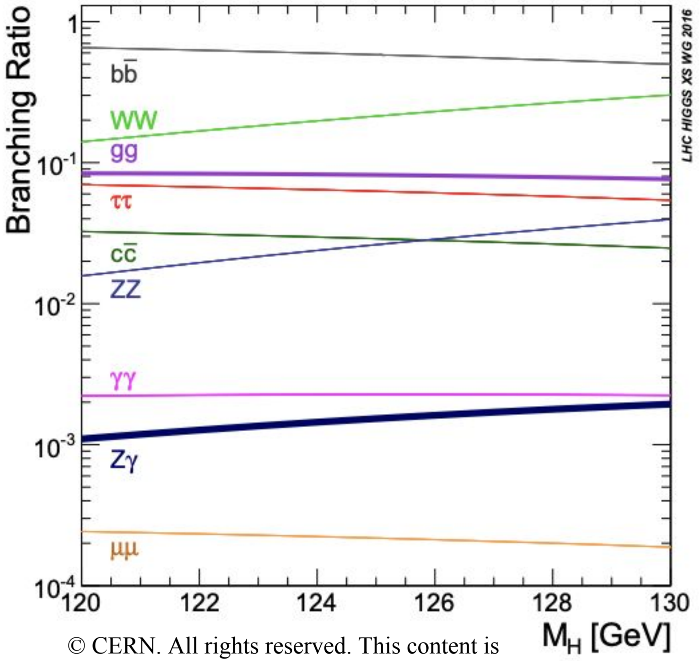
Define particle width

$$\Gamma_i = \hbar\lambda_i \qquad \Gamma = \hbar\lambda = \sum_i \hbar\lambda_i = \sum_i \Gamma_i$$

Branching fraction

$$\mathcal{B}_i = \frac{\Gamma_i}{\Gamma} = \frac{\lambda_i}{\lambda} \qquad \sum_i \mathcal{B}_i = 1$$

Example: Higgs Boson Decay



Decay channel	Branching ratio	Rel. uncertainty
$H \to \gamma\gamma$	2.27×10^{-3}	$+5.0\%$ -4.9%
$H \to ZZ$	2.62×10^{-2}	$+4.3\%$ -4.1%
$H \to W^+W^-$	2.14×10^{-1}	$+4.3\%$ -4.2%
$H \to \tau^+\tau^-$	6.27×10^{-2}	$+5.7\%$ -5.7%
$H \to b\bar{b}$	5.84×10^{-1}	$+3.2\%$ -3.3%
$H \to Z\gamma$	1.53×10^{-3}	$+9.0\%$ -8.9%
$H \to \mu^+\mu^-$	2.18×10^{-4}	$+6.0\%$ -5.9%

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