Welcome back to 8.701. Our second part of the chapter on QCD is on elastic electron-proton scattering.

So elastic electron-proton scattering in general has a long history, going back into the 1960s. Very famous examples are the MIT-SLAC or SLAC-MIT experiment which led to the Nobel Prize in physics for Jerry Friedman, Henry Kendall, and Richard Taylor in 1990. And the most recent and highest energetic electron-positron experiments were conducted at DESY in Hamburg, at the so-called HERA ring. And we'll go back into those experiments. They are famous for not just elastic electron-positron scattering, but inelastic and deep inelastic electron-positron scattering, which is going to be subject of future lectures.

What we are trying to do here-- you might still wonder why this is topic of a QCD lecture. What we're trying to do is study the structure of protons. And the way to look at this is in these Feynman diagrams, we're using this photon here to probe into the structure of the proton. We can start from this scattering process, which we already calculated, and we derived the Mott scattering cross-section formula. But we really want to reconsider this proton now as a prop. In elastic scattering, we are not destroying the proton, so we leave the proton intact in the scattering process.

We do know that the proton is not a point-like particle. So if you want to now study the proton, we have to take into account that it's built out of constituents and that it has an extension. It's not point-like anymore. And one way to do this is by analyzing the Fourier transform of the charge density functions. Remember, the photon couples to charged particles. And so when we use the photon to probe a proton, it probes the charge distribution inside the proton.

So we build a Fourier transform of the charge distribution, and then can extend the cross-section from a point-like cross-section via this Fourier transform of this charge distribution. Great.

In electron-positron scattering, there is another point. It's not just the extension of the proton, but also the fact that the proton carries recoil. And so we need to use two form factors in order to describe the cross-section fully.

So let's have a look at the amplitude. Again, we can start from where we left off with the electron-positron scattering, or electron-muon scattering, and write down our matrix elements, and just now for the proton. What we do here is we modify the vertex of the proton. And the modification is parameterized in those two factors, describing the cross-section of the matrix elements' amplitude with two form factors.

The one, which just looks like a modification of our spin-1/2 particle-- you remember there's this gamma factor, and then this is going to be a number. And the other one is a little bit more complicated. There are the Pauli matrix here and another number, and it's normalized by the mass of the photon.

All right. So this is just a parameterization. We haven't done much physics here. We have just parameterized the distribution.

If you then use this, we can, as I just described, calculate the cross-section again, using this very same parameterization, and get to this formula here in the laboratory frame. This looks rather complicated, but if you go back to our form factor definitions and set this one to 0 and this one to 1, you get back-- and the same here-- we should get back to our Mott scattering result which we had before, so really extended the discussion to extended objects, and considering the charge distribution and also the recoil in the proton.

So this is great. Historically-- so this is not a new idea. This has been done for generations. Historically, the parameterization was done slightly different. And so we introduced the linear combination of those form factors, and those are typically referred to as the electric and the magnetic form factors, so GE and GM.

This formula here-- and this is just algebra, going from previous formula to this-- it's called the Rosenbluth cross-section formula. All right? So this is just, if you find this in particle physics booklets or in nuclear physics booklets, this is what is meant by this. The only thing we did here is extended the Mott scattering formula using extended objects, extend the charge distribution to the [INAUDIBLE] [? Rosenbluth ?] formula.

All right. But we have done that by using a Fourier transform of the charge distribution. So if you now measure the cross-section, we can infer the charge distribution of the proton, and with that the radius, the charge radius, of the proton. This has been done, and we find that the RMS-- the root mean square-- of the proton charge is 0.81 femtometers. So that's the charge.

And this measured charge is still today a hot topic in particle physics, because its measured distributions do not quite agree with the theory predictions. But you see this here parameterized for this value of the proton charge. The theory and the global fits to the data don't quite agree. You would have to go to slightly higher values of the proton radius in order to have theory agree with the experiments.

So I'll leave it here. The next lecture now-- we split this lecture in two parts, the elastic and the inelastic scattering. The next lecture will be on inelastic scattering, where we will break the proton apart and learn about the structure of the proton.