HONG LIU: OK. So last time, we talked about this IR-UV connection between the bulk and the boundary. So the key thing is-- let me write down the metric here, the Ads metric in this Poincare coordinate. So this is the boundaries are equal to zero. And when you increase z, you go to interior.

And then for each constant, z, you have a Minkowski space time. You have a Minkowski d. So this is Ads d plus 1. So each slice is a d dimension Minkowski space. And due to this redshift factor, the more you go to the interior of the space time, then corresponding to the lower energy process when viewed form the field theory. So here, the same process is happening here compared to happening here, and here corresponding to the IR process, and the [INAUDIBLE] boundary corresponding to the UV process.

So this is a key relation between the bulk and the boundary theories. And also, this gives you an intuitive understanding where does this actual dimension come from from the field theory point of view. Then from the field theory perspective, this actual dimension can be considered as a geometrization of the energy scale.

And we know that the physics change with the energy scale from the field theory point of view. It's called the normalization group flow, how physics evolves when you change the energy or length scale. In the field theory, it's called normalization group flow. So you can also view that the evolution in the gravity side, say from the boundary to the interior, and that this flow in the z direction can be considered, again, to geometrize the normalization group flow of the field theory. So any questions regarding this?

Good. So now let's talk about some further aspects of the duality. So the duality is that once you realize there's such relation, since the two sides are completely different objects, so the game is that you really have to do lots of guess work.

Essentially, you have two sides if you want to relate things on two sides. And you have to do guess work. How does this quantity translate into that quantity, et cetera? And then check the consistency. Just like you don't know two languages, and then you have to guess between the two languages and then build up the dictionary. And we will be doing the same. We will be doing the same. Good.

So this is more like a review of what we already sad. So let me be more explicit. We have N equals 4 super Yang-Mills theory. We have N equals 4 super Yang-Mills theory. And then here, you have type IIB string in Ads5 times ds5. So let me just write down Ads5 just for simplicity.

So here on this side, there is a conformal symmetry which we explained before because this is a four dimensional theory. So the conformal symmetry of a d dimensional series SO d, 2. And on this side, there's precisely the same group, which is isometry of Ads5, which is precisely also SO 4, 2, which we reviewed before.

And you can write down the transformation on both sides. In your pset, you have checked some of them, that they actually one to one correspond to each other. For example, this special conformal transformation. And the others, like translation or rotation, et cetera, it's clear.

AUDIENCE: But being the right hand side, we still have some SO 5, S5.

HONG LIU: I will talk about that. So this is more like a space time symmetry. So this is a space time symmetry. From field theory point of view, this is a space time symmetry.

So in N equals 4 super Yang-Mills theory, there's also global symmetry we discussed before. This is also global symmetry. But this is global symmetry associated with space time. And there's also global internal symmetry, SO6 internal symmetry. In N equals 4 super Yang-Mills theory, we discussed last time there are six scalar fields. You can rotate them each other. And this can be considered as coming from the D3 brane. You can rotate six transverse directions the symmetry in rotating the six transverse directions.

And on this side, there's exactly the same symmetry. So now this is isometry of S5. So the isometry of S5 precisely gives you SO6. So you have exactly the same SO6.

And we will not be explicit. You can also actually map the supersymmetry between them. You can also map the supersymmetry between them. So N equals 4 supersymmetry. So there's a 4 supersymmetry, which just comes from N equal to 4.

And because we have conformal symmetry, then the conformal symmetry does not actually commute with this 4 supersymmetry, then generate another 4 supersymmetry. And anyway, so essentially, you have eight supercharge of the [INAUDIBLE]. So this all together is 32 real superchargers.

So when we say N equal to 4, so this is N equal to 4 in terms of four dimensional [INAUDIBLE]. So your supersymmetry has four [INAUDIBLE] as the supercharge. But the conformal symmetry generates another 4. So all together, you have eight [INAUDIBLE], four dimensional [INAUDIBLE], as the supercharge.

For each [INAUDIBLE] in four dimensions, there are how many components? How many components are there for [INAUDIBLE] in four dimensions?

AUDIENCE: Four.

HONG LIU: Four. Because the two components, but each component is complex. And so there are four real components. So all together, you have 32 real charges. And similarly, you find-- I will not do this side. You find the same amount of supersymmetry.

But in this case, for example, the low energy limit. Let's just talk about the low energy limit of this theory. So the low energy limit, as we said before, just has to be super gravity. So it has to be super gravity. Then you find you actually have exactly the same amount of supersymmetry in this geometry.

But the interesting thing is that by definition, the supersymmetry on the gravity side is actually local. So you actually have the same amount of local supersymmetries.

So if you look at this correspondence between each other, then you actually see a pattern. So now let me make some remarks. So if you look at the structure, this math in here, you see a pattern.

On this side, all these symmetries are global symmetries. But on this side, all these symmetries are local symmetries. So this, I just said that in super gravity, the supersymmetry is local. And the space time isometry is just subset of a space time coordinate transformation. And space time coordinate transformation are local symmetries.

So the isometry is a subgroup of diffeomorphism. So diffeomorphism just means the coordinate transformations. And these are local symmetries.

Now we find the mapping is on the field theory side, the global symmetries is mapped on the gravity side into the local symmetries. So for each global symmetry in the field theory side, there's a corresponding local symmetry on the gravity side.

But you may immediately ask the question, on the gravity side, if you talk about diffeomorphism, then this is a huge group-- certainly much, much larger than what we are talking about the isometry here. So why we are only talking about isometry? Why we don't talk about other parts of the diffeomorphism, only talk about isometry?

So what's special about the isometry? So the second remark, why isometry? Why only isometry? Why only look at the isometry? But let me just save time.

So the isometry is important for the following reason. Because even though this is a subgroup-- so as I mentioned last time, when we talk about quantum gravity, when we talk about string theory and Anti-de Sitter times S5 in terms of AdS5 times S5, you should keep in mind that this AdS5 times S5 refers to the asymptotic geometry. Because as a quantum theory, the theory can fluctuate in the middle.

And the isometry can be considered as a subgroup. But the level is even though the space time fluctuates, but the AdS5 times S5 specifies the asymptotic geometry of the space time. And the isometry is precisely the subgroup which leaves the asymptotic form of the metric invariant.

So these are not ordinary diffeomorphisms. So the ordinary diffeomorphisms-- so ordinary gauge transformations, let me just say more generally. By gauge transformations, also means the diffeomorphism or just general gauge transformations.

So when we talk about gauge transformations, say for example, in QED or in QCD, et cetera, we always assume-- so the classical gauge transformation, we can see that we always take them to be fall off sufficiently fast at infinity. When we talk about gauge transformations, when we talk about the typical diffeomorphisms, these are the kind of transformations we assume goes to 1, say, at infinity.

And this isometry, which leaves the asymptotic over the metric invariant, so these are essentially the large gauge transformations. So-called large transformations is that they don't go to the Identity at infinity. And of course, this is precisely the large gauge transformations which leaves the asymptotic invariant.

So in a sense, these large gauge transformations can be considered as the global part of the diffeomorphism. So any questions on this? Yes?

AUDIENCE: What does it mean that those isometries are local? What do you do with isometry [INAUDIBLE]? What do you mean when you say that isometries--

HONG LIU: Yeah. These are just coordinate transformation. A coordinate transformation is always defined point by point, right? Just these are specific coordinate transformations.

AUDIENCE: And what happens to gauge symmetries [INAUDIBLE]?

HONG LIU: Sorry?

AUDIENCE: What happens to gauge transformations in the Yang-Mills part?

HONG LIU: No. The gauge transformation in the Yang-Mills part, you don't see it. Gauge freedom is just redundant freedom. You never see it on the other side.

AUDIENCE: Does this list exhaust all symmetries of the Yang-Mills particle?

HONG LIU: Yeah. Exhausts all global symmetries in the Yang-Mills theory part.

AUDIENCE: Not local?

HONG LIU: All the global theories on the Yang-Mills side, and not local symmetries.

AUDIENCE: [INAUDIBLE] discuss them, [INAUDIBLE].

HONG LIU: There's a u(n) gauge group there, but they don't have correspondence in the gravity side. The reason we don't consider the gauge symmetry, because they correspond to redundancies and they don't have to be present on the other side. You only need to do the physical [INAUDIBLE] to be identical.

And this also is the reason I make this remark here. Even though all these global symmetries, they find the corresponding local symmetries on the gravity side. But precisely due to this remark, those things which really map to the global symmetries in the field theory can be considered as a global part of the local symmetries on the gravity side. So even on the gravity side, in some sense, you should think of them as global symmetries. And they are large gauge transformations. Because the ordinary gauge transformation is just corresponding to redundancy of degrees of freedom. And that should not be reflected on the other side.

AUDIENCE: What do you mean by large gauge transformation?

HONG LIU: Large gauge transformation means the gauge transformations which don't vanish at infinity. So the ordinary gauge transformation, just like in QED, in quantum electrodynamics, when we talk about gauge transformation. The gauge transformations we consider all the gauge transformations which go to zero at infinity. And if you consider those gauge transformations which don't go to zero at infinity, and that's what we call large gauge transformations. And those large gauge transformations essentially is like the global part of the u(1) gauge symmetry, in some sense can be considered as.

AUDIENCE: But that is the gauge symmetry being gravity side.

HONG LIU: That's right. So this is the part of the gauge symmetry on the gravity side. But the part corresponding to the boundary is the part associated with the infinity.

AUDIENCE: Here in isometry of AdS5, we were showing, like in p-set, it's point to point local symmetry. It's like just a space time point. But how about this conform in this Yang-Mills theory? What is the symmetry of the object? What kind of object does this symmetry correspond to because it seems--

HONG LIU: It's conformal symmetry.

AUDIENCE: Conformal symmetry of the component of the field?

HONG LIU: No, no, no. It's the conformal symmetry of the Minkowski space. So on this side, you have isometry of AdS. On the other side, you have conformal symmetry of the Minkowski space. So you're asking what are you corresponding to on the field theory side, right? So that's mapped to the conformal symmetry of the Minkowski space.

AUDIENCE: So the isometry is actually large gauge transformations.

HONG LIU: Right. So the story actually is more general. So the story actually works more general. This is the statement for the N equal to 4 super Yang-Mills theory. So the story is more general.

So in the case which you have, say, more general, say duality between a CFT in d dimensions in Minkowski d with the AdS d plus 1 dimensional gravity. In any such duality, then you always have the conformal symmetry, which is SO(d, 2) on this here mapped to AdS isometry. And any internal gauge symmetry, say if you have some u(s) global symmetry here, then this will be mapped into a u(1) gauge symmetry. Again, in the sense that this u(s) global symmetry can be considered as the global part of this u(1) gauge symmetry, the part which does not vanish at infinity.

And it's always the case that global supersymmetry here would be corresponding to the local supersymmetry here. And also in exactly the sense that even for the local supersymmetric transformation in the gravity side, there's also a part which does not vanish at infinity. And that's the part corresponding to the global supersymmetry on the field theory side. Yes?

AUDIENCE: What are the fermions supersymmetry on the AdS side?

HONG LIU: Sorry?

AUDIENCE: In the low energy limit on the AdS part, what are the fermion supersymmetry?

HONG LIU: It's the same thing. What do you mean, what about fermions? So there will be some fermions.

AUDIENCE: [INAUDIBLE] fermions? So there is more than gravitons on the AdS side involved in low energy limits?

HONG LIU: Yeah. There are gravitons, and there are also some fermions. There's something called a [INAUDIBLE]. There are some other fermions, et cetera.

AUDIENCE: Is it they who carry the u(1) gauge charge?

HONG LIU: No. Here is more general. In this case, indeed-- so this isometry of S5, once you deduce to AdS5, that just again becomes a gauge symmetry, SO(6) gauge symmetry. And some fermions indeed are charged on this SO(6) gauge symmetry. And here, it's just more general. So whatever things, anytime if you have some global symmetry here should be mapped to some gauge symmetry here. And we will say this a little bit more a little bit later. Do you have any other questions?

So now let's move to the matching of parameters. Again, this is more like a review of what we discussed before. And again, first N equals to 4 super Yang-Mills theory. And then this is type IIB on AdS5 times S5.

So previously we said, from the relation of the d-brane, so the G Yang-Mills square here is related to the 4 pi GS here, string coupling. And also, from the d3-brane solution, we find is that the G Yang-Mills square-- we find, for example, here the curvature radius R has the following form, 4 pi GS and alpha prime squared.

So the N is the same N on this side. So the N is the gauge group N. I should say the flux N. Anyway, let me add something. So the N is the number of d-branes on this side which translate into the flux. So the flux N. And then here is corresponding to you have SU(N).

So this R, this curvature radius is related to the alpha prime squared and the gs in this way. And so on this side, the dimension parameter is given by R squared divided by alpha prime. So if you look at this relation, if you look at alpha prime 4 to alpha prime squared, then you find this is just relating the alpha.

From this relation, 4 pi GS is equal just to G Yang-Mills square. And the N is N. This is the relation between the parameters. Any questions regarding this?

So on the gravity side, we said these are the two parameters. And of course, you also have this N. But these are the two basic parameters. And we can also, instead of using GS, as we said before, you can also use the Newton constant. So the 10 dimensional Newton constant is length dimension eight. Then the dimensionless parameter would be GN divided by R to the power 8. And the GN is related to the GS and alpha prime by this formula.

So now you can just use the relation between the GS to exactly translate this into Yang-Mills coupling. You just can use this relation and then use that to translate this into the parameter in the Yang-Mills theory side. Then you find, once you plug all those relations in, how the GS and alpha prime, et cetera related to N, then you find here actually, G Yang-Mills have disappeared in this relation.

What you find is that the Newton constant essentially just related to 1 over N squared. Up to some parameter, just related to 1 over N squared. So if you're expanding G Newton, just like expanding 1 over N squared.

So as we said before, we often do dimensional reduction on S5. Let me get a five dimensional Newton constant. So five dimensional Newton constant is equal to the 10 dimensional Newton constant. And the difference is the volume of S5. We wrote this down before.

And the S5 is equal to pi cubed R to the power fifth. And then from here, you can just work out. So G5 has dimension 3. Then G5 divided by R cubed, again only related to N given by pi divided by 2N squared. So these relations are often useful in the future.

So now let's look at [INAUDIBLE] limits of this relation. So let's first look at the classical gravity limit on the gravity side. As we discussed at the beginning of this class, by classic, we always use h bar equal to 1. So our h bar is always equal to 1.

But quantum gravity is captured by this parameter, h bar times GN. So even though h bar equal to 1, in the limit when GN goes to zero, then you are in the regime in which you can ignore the quantum gravity effect. So that's what we mean by the classical gravity limit is that this should go to zero, and then alpha prime should go to zero. So this means the string effect is not important. And this is all in the unit of R. And here, when I write these relations, I have all set h bar equal to 1.

So when I say the classical gravity limit, which is this limit in which h bar equal to 1, but the effect of the Newton constant goes to zero. And the reason I emphasize it is that if you have some matter field in this geometry, then those matter fields should be treated as full quantum. Just don't treat the gravity as quantum, but those matter fields should be treated as quantum.

So the classical gravity limit is the same as QFT, Quantum Field Theory in curved space time. So gravity does not fluctuate. So you have rigid curved space time. But your matter field can fluctuate, h bar equal to 1. So essentially, we are dealing with quantum field theory in curved space time. So in the type IIB super gravity, there are many, many such kind of matter fields, and they all should be treated quantum mechanically. It's just that you should consider this small.

So let's consider what this means. So GN small as a dimensionless parameter, this translates into field theory side if we use this relation. So that means N goes to infinity. So this is the large N limit of the Yang-Mills theory. This is the large N limit of the Yang-Mills theory.

And then alpha prime goes to zero. From the relation between the alpha prime and here, when alpha prime goes to zero, so this is in the downstairs. Then that means this should go to infinity. So this is the t Hooft coupling we defined before. So this means the t Hooft coupling goes to infinity.

So now we see a remarkable relation. So this is what we expect. If you still remember what we did before in the large N gauge theory, in the large N gauge theory in the large N limit, the fluctuations become very small, et cetera. So this is consistent that on the gravity side, the fluctuation in the geometry is very small.

And now, we see that the decoupling of the string effect requires on the field theory side the strong coupling. This is also something roughly we said before. Remember, when we talked about large N gauge theory, large N gauge theory, you have planar diagram, non-planar diagram, et cetera. And at each level, say at the planar diagram, you need to sum over infinite number of [INAUDIBLE] diagrams which are all planar topology.

And if you look at just those [INAUDIBLE] diagrams, of course you don't see a space time interpretation because they are just [INAUDIBLE] diagrams. You don't see a continuous surface. And I already alluded before that the continuous surface can emerge if that diagram becomes sufficiently complicated. And the way to make that diagram sufficiently complicated is to make it to be a strong coupling. The strong coupling, then the diagram with many, many vertices will dominate. And then the most dominated diagrams are those diagrams with not a lot of vertices. And they essentially are going to continue to limits.

And so this relation [INAUDIBLE] realizes that kind of intuition. If you don't remember, go back to your loads regarding the large N gauge theory.

So this relation is also remarkable for the following reason. Because this limit on the gravity side is simple, we can just deal with quantum field theory in the curved space time, which we know how to do. But on this side, it's highly non-trivial because this is an infinite coupling limit. So this will tell you that the strong coupling limit is described by classical gravity. So that means that we can actually use classical gravity to, in principle, solve problems which are strongly coupled. So also, of course, there are corrections beyond this.

So quantum gravity corrections on this side, so this is a classical gravity limit if you take those parameters to go to zero. But suppose those parameters are not zero, just small. Then you can just expand in those parameters. For example, can you expand in GN divided by 8. And that expansion essentially gives you quantum gravity corrections [INAUDIBLE].

And now, from this relation, the expansion of GN then is translated. So this is expansion in G Newton. So from that relation, we see this has become the expansion 1 over N squared in the field theory side. Just expansion 1 over N squared.

So on this side, you can also take into account that the alpha prime is non-zero. Then the alpha prime corrections in the expansion in alpha prime. So from this relation, translates in the expansion 1 over square root lambda.

So in principle, the corrections beyond this limit can again be studied on the gravity side. And then the 1 over square root lambda corrections then corresponding to the string G corrections. And the 1 over N squared corrections corresponding to the quantum gravity corrections.

So this is the classical limit. You also can see the classical string limit we considered before, we discussed before. In the classical string limit, still you can see the N go to infinity, which corresponds to GN. R8 goes to 0. But here, the alpha prime can be arbitrary. And here the [INAUDIBLE] coupling can be arbitrary. So it can be finite. So let me just say alpha prime finite no longer zero. And then this is just corresponding to lambda finite, which is no longer infinite.

So this is just a standard t Hooft limit, which we take N go to infinity but keep lambda to be finite. Standard t Hooft limit, which we talked about before in the large N gauge theory. And then again, the corrections in GN will be corrections in 1 over N squared. So any questions? Yes?

AUDIENCE: Doesn't small alpha over R squared mean large GS?

HONG LIU: No. That has nothing to do with GS. These are two independent parameters.

AUDIENCE: But expression parentheses, so there's also N.

HONG LIU: This is--

AUDIENCE: So in the classical [INAUDIBLE], it means that since lambda is finite, so this means G Yang-Mills is 0.

HONG LIU: Sorry.

AUDIENCE: I mean lambda is finite, so G Yang-Mills is 0 and it means weak coupling.

HONG LIU: No. That's what we discussed before in the large N gauge theory. The effective coupling is lambda. Lambda is your effective coupling in the large N limit.

AUDIENCE: When you draw the diagram, [INAUDIBLE].

HONG LIU: Yeah. So the lambda is your coupling. The G Yang-Mills indeed, in the t Hooft limit, the G Yang-Mills will go to 0 when N goes to infinity. But G Yang-Mills is not the right parameter to look at things. Any other questions? OK, good. So now we can move on further.

We can talk about the matching of the spectrum on two sides. So from now on, I will just restrict to essentially the semi classical. Here I should call semi classical gravity limit because we still treat the matter fields essentially as quantum. So we call it semi classical gravity limit. So from now on, so we will mostly just consider the semi classical gravity regime in the gravity side. Because essentially, we know very little about the string theory in this geometry.

And also, I will often use the phrase which applies to the general correspondence, and not necessarily just N equal to 4 super Yang-Mills theory and the type IIB string theory. I just use the language assuming there's a general correspondence between some conformal field theory and some AdS gravity theory.

So if the two theories have to be the same, two sides have to be the same, then you should be able to map their spectrum. So you should be able to map their [INAUDIBLE] space, for example. So again, [INAUDIBLE] now use the general language of the boundary and the [INAUDIBLE].

So the boundary theory is a conformal theory with this symmetry. So for the moment, I don't have [INAUDIBLE] symmetry. So that means that [INAUDIBLE] space should be organized as the representations of the conformal group, say of this SO(d, 2). And similarly here, because here, the SO(d, 2) is the isometry group which lives in infinite invariants.

And again, you should be able to organize your [INAUDIBLE] space using the representations of the SO(d, 2). And those representations, of course, should be the same. So if there's one representation here, then there must be a [INAUDIBLE] representation here. And if there's two [INAUDIBLE] representations here, there would be two identical representations here. The representations must match.

And on the boundary side, the local operators should also transform on the representations of the conformal symmetry. And again, they conform local operators can also be mapped to, say, the field on the gravity side.

So on the gravity side, the fields should also transform under this SO(d, 2) isometry. That means that it should be a one to one correspondence between the local operators here and the bulk fields on the gravity side. There should be a one to one correspondence on this side.

For example, in the boundary, if there is some scalar operator, there must be a corresponding scalar field on the gravity side. Similarly, if there is some vector operator in the boundary theory, there must be a corresponding bulk vector field in the gravity side. And similarly, if you have some symmetric tensor, then this must also be related to some symmetric tensor.

So now I will often use the notation that I've used. Mu mu refers to the boundary indices. And the capital M, capital N refers to the bulk indices. Because they are on different dimensions. So then they are not quite the same.

AUDIENCE: I have a question.

HONG LIU: One second. Let me just finish. So here, I'm just talking about the conformal symmetries. And if the theory has some other symmetries, say some global symmetries or supersymmetries, then again, all those fields and the state, they should transform on the representations of those symmetries. And again, they should all match together. They should all match together. Yes?

AUDIENCE: So we proved that the super Yang-Mills theory really lives on the boundary of the AdS?

HONG LIU: No. We did not prove that. This is just a postulate. First, let me just repeat again. Yang-Mills theory lives on Minkowski space. And it's just an observation that the Minkowski space is the boundary of AdS. And then you say you can imagine that this is the boundary, this relation is related to the bulk and the boundary. And this is a postulate based on that fact. Yes?

AUDIENCE: I thought one of the motivations for thinking about the holographic duality was to try to escape [INAUDIBLE] theorem. And all of a sudden, it strikes me, so we're trying to get on the boundary spin to massless particles. Then they will also exist in the bulk.

HONG LIU: Sorry. In the field theory side, there's no massless spin to particles. They map to some field in the gravity side in the five dimensions. So this is some four dimensional operator. Then this maps to some five dimensional fields.

AUDIENCE: I need to think about my question better.

HONG LIU: Yes?

AUDIENCE: So if it's a postulate that-- I mean, it's not required that the theory live on the boundary. It seems that that's just sort of a convenient way of thinking about it. But what implications would it have if it actually lived on the boundary? Would it change anything?

HONG LIU: That's what I said before. So this discussion does not depend on that. This discussion does not depend on that. You can say I don't need to worry about that whether this is on the boundary or bulk, et cetera. These are just some different theories. I want them to be the same.

AUDIENCE: Right. OK.

HONG LIU: But, as I said last time, if you believe this bulk and boundary relation, then this is powerful because then, you can immediately deduce that the Yang-Mills theory on the S3 times time then is related to the gravity theory in the global AdS. And that is a nontrivial prediction from thinking the boundary and the bulk relation. Now you can generalize it. Because if I just look at this relation, I have no reason to suspect why. You can argue from the symmetry point of view also, but that language is more direct.

In principle, after establishing the relation between the Yang-Mills theory in Minkowski space time and this gravity in the Poincare patch, I may also, just based on the group theory aspect, to say that should generalize to the Yang-Mills theory on the S sphere and the global AdS. You may also be able to do it that way. But thinking from the holographic point of view will give you a direct way to argue that, give you an alternative way to argue that.

Also, in your p-set, you have checked this holographic bound. And so that's a confirmation of this. Yes?

AUDIENCE: So what does the massless [INAUDIBLE] field on the right map to if it's the same representation of SO(d, 2)?

HONG LIU: Sorry. Say it again.

AUDIENCE: So in the representation line, so if you have the massless [INAUDIBLE] field on the right, what does it map to on the left?

HONG LIU: Yeah. We are going to talk about it. We are going to talk about it. So if there are other symmetries, then everything should match. So now, as an immediate check, you can now just open your old papers. And if you load them-- because in the '80s, people have worked out the supergravity spectrum precisely on this space.

So even though this relation was discovered in '97, actually in the '80s, people already worked a lot to consider this type to be supergravity on this space. Because this is a maximally supersymmetric space, et cetera. Anyway, so people have already spent lots of trouble to actually work out that spectrum.

And now you can just open their paper. Then there's a big table about the different fields and their representations, et cetera. And then you can immediately see they actually map to certain representation of operators on the N equal to 4 super Yang-Mills theory. Then you can immediately check them. Just based on the group theory, check them.

So I won't go through those details. But let me just mention the most important such kind of mapping for these two theories. And actually does have consequences for the general story.

So the most important mapping. So as we said, on the string theory side, you always have this dilaton. So these is some scalar field. And this dilaton will appear in the scalar field, in AdS5, say, for example.

And it turns out this dilaton field is mapped to the Lagrangian of the N equal to 4 super Yang-Mills theory. So the Lagrangian is a Lagrangian of the N equals to 4 super Yang-Mills theory, say, trace f squared plus phi squared, et cetera. So that's a local operator. And it turns out that operator is mapped to this dilaton.

And on the N equal to 4 super Yang-Mills theory side, we have this SO6 gauge symmetry. Did I just erase them? We have this SO6 gauge symmetry. Then we have the SO6 conserved current. And it turns out this, on the gravity side, just maps to sO6 gauge field.

AUDIENCE: You said it was an SO6 gauge symmetry?

HONG LIU: No. In the N equal to 4, this is a global symmetry. But on this side, so this appears as isometry on the S5. But when you dimensionally reduce on S5, then in AdS5, then there will be a pure gauge field corresponding to each symmetry generated [INAUDIBLE] S5. And that naturally maps to this conserved current on the field theory side.

And then another universal operator on the field theory side is the stress tensor. So no matter what theory you consider, you have stress tensor. And this is mapped to-- turns out, to the metric perturbations. It's a deviation from the AdS metric.

Again, so at the representation level, this is very natural. So this is a symmetric tensor. This is a symmetric tensor. But physically, this is also natural. Physically, also natural. I will elaborate this a little bit further from a different perspective in a few minutes. So do you have any questions regarding this?

So now, given this mapping, any operator is due to a bulk field. Then you can ask some immediate questions. For example, the quantum numbers of these operators will map to the quantum numbers of the bulk fields. And that's something I said you can check their symmetries. For example, not only the representations under this conformal symmetry should map under that representation. And also, the representation under that SO6 we also match, et cetera.

So for local operator on the field theory side, so once we have this mapping, we can immediately ask questions related to operators on this side, and try to ask what's the counterpart on the other side. And ask the story about the field on the gravity side, and ask what's happened on this side. We can start developing the relations.

So now let's start discussing them. So the first thing you can do, so immediate question you can ask is that given the operator, say operator O in the field theory. So there are some natural questions one can raise about this operator. For example, what is correlation functions, et cetera? And that we will discuss a bit later. But now let me discuss another natural question.

And the natural thing to do is to deform your original theory by adding this operator to the Lagrangian. So I have an operator. I can put the source. I can put the coefficients. And these coefficients may depend or not depend on the space time coordinates. And I can always add such a term to a Lagrangian. And then this deforms my theory away from the original theory. So this is a natural thing to do.

And this phi 0 is often called the source. So this phi 0 is called the source. So immediate question you can ask-- and when phi 0 is equal to constant, then this corresponding to a change in the coupling for this operator.

So if your Lagrangian previously already included this operator, and then if you add such a term, so phi 0 equal to constant, then you are just changing the coupling for that operator. And if this operator is not there, then you just add the new coupling. But in general, you can make it space time dependent.

AUDIENCE: Why is it coupling since there's no other operators coupling without if we say phi 0 is a constant?

HONG LIU: Sorry?

AUDIENCE: If phi 0 is a constant?

HONG LIU: No. O is the operator. It's the local operator.

AUDIENCE: Yes, but there is no other operator this operator O coupling with.

HONG LIU: No, no, no. What is the meaning of operator? The meaning of operator is that O is a sum of the product of fields. And so there's already-- so this O may be phi cubed, or may be phi 4, et cetera. So example is O in the scalar field theory. If a scalar field theory has a gauge field, then I can write O as trace F squared. Then this corresponding to changing the coupling for trace F squared.

So immediate question is, what does this operation-- so in the field theory point of view, you can always do this operation. So the immediate question, what does this corresponding to in the bulk? What does this operation correspond to in the bulk?

So now I'll try to answer this question. As I said before, in establishing such dictionary, you often have to do lots of guesswork, and then you check. And the guesswork is based on some very small clues. So people who are able to make good guesses is that they are able to grasp important clues sometimes from very simple facts. And so that is what's good physicists do is that they can see non-ordinary things from ordinary things.

So here, I will try to deduce the answer to this question by starting from this relation. So let me start with that relation. Related to the GS. Let's forget about p for pi.

And now, we've talked about before GS string coupling can be considered as the expectation value of the dilaton field. Let me actually call it capital Phi. So we mentioned before when we talked about string theory that this string coupling can be considered as expectation value of the dilaton field.

Of course in general, again, the dilaton can fluctuate, et cetera. So normally, in flat space, when we say the dilaton has expectation value, what we normally say is that the value of the dilaton at infinity because the boundary by infinity, that's how you fix.

And similarly, for space time like AdS with a boundary, and the expectation value of this dilaton can be identified with the value of the dilaton at the boundary of AdS. So here, I write partial AdS means the boundary of AdS. So this is the value of AdS, value of phi at the boundary. So expectation value essentially can be associated with the boundary value of the field.

AUDIENCE: You mean if it is a constant? You mean if expectation value of the phi is constant? Or if it's, function-wise, equal to the boundary limit?

HONG LIU: No, no, no. Think about the following thing. As I said, phi in principle can fluctuate. And its expectation may also be able to fluctuate in the space time. But the only sensible way to talk about expectation value is to talk about its value at the boundary because we assume the boundary conditions don't fluctuate. It's the same thing for flat space. And we always specify the boundary condition at the spatial infinity, for example. And in the AdS, which is also space time with a boundary, than we can associate the constant parts of the expectation value as the value at the boundary.

Now you have to use a little bit stretch of imagination. The following fact, you don't need to. So now, with this relation, we have established a connection between the Yang-Mills theory coupling with the value of phi at the boundary of AdS. And now remember, this Yang-Mills coupling, so I will not be careful about 1 over Yang-Mills squared. This is related. This is the coupling or source for the Lagrangian of N equals to 4 super Yang-Mills theory.

So now we can deduce here two things. In particular, if we deform the Lagrangian, say, change this coupling, deform the boundary by changing the coupling, which corresponds to you add some delta G, say the Lagrangian of N equal to 4 super Yang-Mills, deform the boundary set by changing the coupling. And this corresponding to you essentially change the boundary value of dilaton because these are related.

So now, this example gives us the answer to this question. This example gives us this question. From here, we can deduce first that the operator corresponding to the dilaton must be the N equal to 4 super Yang-Mills Lagrangian, as you can already deduce from some other methods, say using group theory, et cetera.

And the second is that phi 0 O, in the boundary theory, adding a phi 0 O in the boundary theory must be related-- now I'm generalizing this story. The bulk field phi-- so now I'm no longer not necessarily using the dilaton. The bulk field phi due to O has a boundary value phi 0. Any questions about this? Yes?

AUDIENCE: Your choice of operator O should be consistent with all symmetries, right?

HONG LIU: Hm?

AUDIENCE: You're not pre-choosing O?

HONG LIU: No. O can be anything.

AUDIENCE: Consistent with original symmetries of the--

HONG LIU: No, it doesn't have to be. You can deform the theory without-- you can break the symmetry. That deformation can break the symmetry. If you want to preserve the symmetry of the original Lagrangian, then you can choose a certain O. But in principle, you can choose any O. You can break the symmetry if you want.

AUDIENCE: Do you have to always then break some symmetry on the AdS part?

HONG LIU: Yeah. Impose this kind of boundary conditions. Then you may break AdS symmetry too. So now we have answered this question. And then of course, this just provides the answer. Then you would check it. And we will check it later using other methods. Now I'm saying because I already know it's true. But in real life, what you will do from this example, you will say, ah, this must be the case. Then you will start trying to find examples to check it. Start to find other ways to check it. And we will describe it later. So in the end, what we will describe is a self consistent story. I will not contradict myself in my later discussion.

So if we assume this, I can also use this to argue. I can also use this identification to argue this too. So let me call this star and star star. I can use this identification to make star and star star natural for any duality, not only N equals 4 super Yang-Mills theory and this type IIB gravity.

It's that any conserved curve in the boundary theory must be equal to some gauge field in the gravity side, and the stress tensor should always be due to the metric. So now I'm going to use this argument to make that a little bit more natural.

So let's first consider this. So suppose I have a conserved current, J, J. I have a J mu. For simplicity, let me just take it to be u(1). Then I can deform the boundary theory by adding a source for this J mu by adding a boundary Lagrangian term like this. And a mu is the source.

And then according to this identification, the A mu must be A mu-- we should be able to identify it as the boundary value of some bulk field, A mu evaluated at z equal to 0. So this is a bulk vector field. So if this is true, there must exist a vector field due to this vector field. And this must correspond to the boundary value of that.

But now we can argue why this should be a gauge field. So now I want to argue this is a gauge field. And to see this is very easy because since J mu is conserved, then this coupling-- so let me call this star star star. Let me call it star star star.

So since this is conserved, and this star star star is invariant under the following transformation, A mu goes to A mu plus partial mu lambda x for any lambda x. So that means the A mu, this A is the boundary value of A. So that means this capital A, that means the dynamics of this A mu should also be invariant. If I change the boundary value of A by this transformation, the dynamics of A mu should also not change. Let me call it AM so the bulk and general will be different.

But this is like a gauge transformation. So this is like a gauge transformation. So we deduce that somehow, this must be some subset of the bulk gauge transformation. Now call it star to the power 4. Then star to the power 4 also must be a subset of some bulk gauge transformation.

So this does not prove it, but makes it more natural. This argument makes it more natural. The fact that the boundary value of A has such kind of gauge symmetry. So in some sense, this must generalize to some gauge symmetry in the full gravity side.

So similarly, you can do this for the metric for the stress tensor. Similarly, you can do this for the stress tensor. So for the stress tensor, so for the star star, again, we can add h mu mu x T mu mu to the boundary Lagrangian. And this is the source to the stress tensor.

And now, from your knowledge of quantum field theory, when I add such a term to the field theory, it's the same. So this is equivalent to I deform the boundary space time metric from just eta mu mu to eta mu mu plus h mu mu. So let me call this quantity g mu mu b.

So adding such a term source for the stress tensor is like you deform your space time geometry a little bit by this source. This is when h is very small. When we consider the information, we always consider the source is small.

But now, we can argue this thing, mass corresponding to the boundary value of the metric in the gravity side for the following reason. So if you look at this expression, so if you look at even the pure AdS metric before you do any deformation, so this is just due to the original theory. So from the AdS metric, so the original AdS metric, the g mu mu component.

So let me write this AdS metric in the form, say, R squared, z squared, separate the z squared with the rest. So this is essentially the component of the bulk metric along the boundary direction.

The boundary value of this metric component evaluates z equal to 0 is equal to R squared divided by z squared eta mu mu. So that just comes from the AdS metric, just this g mu mu. And when you go the boundary, just given by this. And this is eta mu mu. We recognize it just as the boundary metric.

So these correspond to deform. Since adding this term corresponds to deform the boundary metric, so we expect, when we add this term, so this corresponding to deform the g mu mu now to the 0 become. So we expect this becomes equal to g mu mu b.

So this tells you that the stress tensor must be due to the metric perturbations in the gravity side. The stress tensor must be corresponding to the metric perturbation on the gravity side because it's corresponding to perturb the boundary conditions of the bulk metric-- corresponding to preserve the boundary condition for the bulk metric.

So this is a very general statement valid for any correspondence between the gravity and the field theory. And that also tells you that if you have a theory which due into a higher dimensional theory, and then that theory, if the field theory has a stress tensor, then this bulk theory must have gravity because this bulk theory must have a dynamical metric. And if you have a dynamical metric, then you have gravity. So you can say, if any field theory is due to a theory of one higher dimension, that theory of one higher dimension must involve gravity-- nothing about quantum gravity. Let's stop here.