HONG LIU: So let me first remind you what we did at the end of last lecture. Say, suppose you can see the string moving in some spacetime M. So it's the following metric, given by g mu nu and the coordinate, which I've denoted by X nu. So nu typically will go from, say, mu nu, go from 0, 1 to D minus 1. And capital D will be the [? number of ?] spacetime dimension. And then the worldsheets will be characterized by this X mu as a function of worldsheet coordinate sigma tau. Then this describe the [? mapping ?] of a two-dimensional surface embedded in the full spacetime.

And then we also use the notation of X mu equal to sigma a, and sigma a will be sigma 0, sigma 1, and equal to tau and sigma, OK? And then the simplest action, which have a clear geometric meaning, is essentially just the action that is given by the area of the surface. Given by the area of the surface.

And this delta h is the induced metric. So if you have embedding in this spacetime, then this embedding will induce a longitudinal metric on the worldsheets. So this h is the induced metric on the worldsheet. And then if you take the determinant of it, then it essentially gives you the area. [? Adamant ?] on the worldsheets.

But we also showed that classically, this is action is equivalent to this action. You can rewrite it by introducing something like a Lagrangian multiplier like gamma ab. And then by eliminate gamma ab, then you cover this. OK? And the equation of motion for the gamma ab-- yeah, if you solve the equation for gamma ab, the thing you will get something like this, lambda hab. And lambda is arbitrary constant. Arbitrary function. Yeah, say if you solve the equation of motion for gamma ab, and then that's what you get.

So for this final form, so this is the action we will use to quantize it, is because this action have a nice polynomial form. Have a nice polynomial form. Not like this action, which is a square root. It's always awkward. And so this have a nice polynomial form.

And then when you quantize the action, say if you do path integral quantization, then you need to integrate both dynamic variables. So exponential i Sp gamma on X. OK. So both are dynamic variables. When you quantize, you have to do path integral over all of them.

And so let me just say a little bit more about the geometric, which we mentioned at the end in last lecture, the geometric meaning of this action. So if you just look at this action itself, this just like a scalar field in a two-dimensional curved spacetime. And this two-dimensional curved spacetime have a longitudinal metric, which is given by gamma. And the scalar field is just this X, OK?

So this gamma can be considered as some kind of intrinsic. And we can see that as an intrinsic metric on the worldsheet. And then this X still describes this embedding of this worldsheet in spacetime. But X, from the point of view of this two-dimensional worldsheet, it just behave like a scalar field. It just behave like a scalar field.

So when we quantize this action, so when we do the path integral, if you just do the path integral over DX, then this is just like a standard two-dimensional quantum field theory, OK? Just like a standard two-dimensional quantum field theory. But what's unusual here, when you quantize a string, because the gamma is also a dynamic variable, you also have to integrate over gamma. And this make it more nontrivial, and then this is no longer just like quantizing the scalar fields.

And because you are integrating over all possible intrinsic metrics on the worldsheet. So this is like quantizing 2D gravity. When you integrate over all possible metric, then this is like quantizing gravity. But this is only the gravity on the worldsheet, coupled to D scalar fields. OK? Yes.

AUDIENCE: So in what sense is it 2D gravity? Because the metric is like a D by D object.

HONG LIU: No, no. Gamma is a two-dimensional metric. It's intrinsic metric on the worldsheet. Yeah, so it's two by two. [? ab's ?] only goes two by two. ab is only going from 0 to 1.

Yeah, so ab only runs over the worldsheet at 0 to 1. Yeah. So once you write that form, then you introduce an intrinsic metric on the worldsheet. They introduce an intrinsic curved spacetime on the worldsheet.

And when you integrate over all possible metrics, then that's corresponding to take into account the fluctuations in the metric, from this two-dimensional point of view. So that's why we say this is like quantizing 2D gravity. By gravity we just mean the spacetime. OK, is this clear?

So now we see the question of quantizing the string theory, essentially [INAUDIBLE] to quantize this two-dimensional gravity on worldsheets coupled to these scalar fields. And these scalar fields describe the embedding of your worldsheet. OK?

So we will consider a simple example. We will consider the example which M is just Minkowski spacetime. Three-dimensional Minkowski spacetime, OK? So in this case, g mu nu [? come ?] [? equal ?] to eta [INAUDIBLE] nu.

And then this Polyakov action can also be simplified. So this SP now just become 1 over [? 4 ?] alpha prime, d square sigma, gamma ab, partial a X mu, partial b X mu. So now when I write this X mu, X mu should be understood that they are contracted by the standard Minkowski metric.

So now, this is really, if you just look at the X, this is like free scalar field series, because of the dependence on x, purely quadratic. So this is like a free scalar field in curved spacetime, in curved worldsheet, 2D worldsheet.

So this still slightly unusual thing about this scalar field theory, from our standard one. Actually, I might be missing a minus sign somewhere. I might be missing overall minus sign. I think I'm missing overall minus sign. Just let me put it here. I think I'm missing overall minus signs.

For our current purpose, it does not matter. But I think for the later purpose we need it. Because if you can see the standard field theory using this minus [? 1 ?] metric, then I need the minus sign. OK? So this is just like a free scalar field theory in this 2D curved spacetime. But the slightly [? thing ?] unusual about this one, so there are two unusual things. One is that we also need to integrate gamma.

But there's a lot of unusual things, even from the perspective of scalar field theory. And so if you look at the 0-th component of X, because eta 0,0-- we are using the eta equal to minus 1, 1. Because eta 0,0 is minus 1. So the 0-th component of X, we have a opposite kinetic term to the other component, OK? So you might have learned in your quantum field theory class that actually you cannot change the sign of your kinetic term. Otherwise you get instability, because then your Hamiltonian will be unbounded from below, et cetera.

So this action, so our story here have two unusual features. One, you have to integrate over gamma. And the second is that [? the 1 over ?] the scalar field have the wrong kinetic term. OK. The sign with kinetic term is wrong. And we will see that these two features actually solve for each other. And this is actually a consistent theory. So this is actually consistency. Any questions at this point?

OK, so before you do any theory, what you should do? So before you work on any theory, what should you do first?

AUDIENCE: [INAUDIBLE].

HONG LIU: No, no, that's too far away. Just the first thing you should do. After you write down the action [? Lagrange, ?] what should do?

AUDIENCE: Solve the equation of motion.

HONG LIU: That's still too far away.

AUDIENCE: Check units.

HONG LIU: That's something what you should do. Yeah, but that is done in kindergarten, so it's not included here.

AUDIENCE: Constants of motion.

HONG LIU: That's close. What is responsible for constant of motion?

                 [INTERPOSING VOICES]

AUDIENCE: Symmetry.

HONG LIU: Yeah, that's right, symmetry. You first have to analyze the symmetries. OK, so that's what we will do. OK, let me call this equation three, just continue from my notation of last week. Yeah.

So first we should analyze the symmetry of this action. So first, so this is a mapping to a Minkowski spacetime. The Minkowski [? spacetime ?] itself have a Poincare symmetry. And that Poincare symmetry will be reflected here.

First we have the Poincare symmetry of X mu. In other words-- and this action is invariant-- if you take X mu to go to X mu plus a constant. OK, so a mu is a constant. So it's obvious this is invariant because everything on [? X ?] we said derivative. So if we shift by X mu with a constant, then I'll call that a symmetry. And also, this is invariant under Lorentz symmetry. OK. So lambda is the constant Lorentz transformation. OK?

So this is also self-evident, because of this contraction, if you make a constant Lorentzian transformation, you don't see the derivatives. And then this is contracted as Lorentz scalar, then this is invariant. OK?

So this is the first set of symmetry. The second set of symmetry essentially by construction, because we are writing this as a scalar field in the curved spacetime. Then this is invariant in the two-dimensional coordinate transformation. And in other words, this is invariant on the reparameterization of the worldsheet. OK.

So this is essentially a less [? desirable ?] condition, which should satisfy by any string action, because as we said before, sigma tau, you just parameterize your worldsheet. And you actually should not depend on how you parameterize it, and that parameterization should be arbitrary. You should be able to change your parameterization. Because if you think about the surface in spacetime, no matter, you can say parameterization cause [INAUDIBLE] agreed on the surface. Whether you do it this way, or you do it in some other way, if you're not changing the surface, the property of the surface itself.

So that means the action itself should not be invariant on the reparameterization of your worldsheet. And the parameterization of the worldsheet translates into just arbitrary coordinate transformations. So sigma and tau should be able to, say, if you make arbitrary coordinate transformation, and the action should be invariant. OK.

And this is by construction. And this is action by construction in [? Lorentz. ?] And this number goes to action. And Polyakov action we obtained by rewriting this number called action. It's also about construction. Because its [? manner fits ?] because this is in [INAUDIBLE] in the curved space actions for scalar fields. So this automatically invariant under the coordinate transformation.

And under this coordinate transformation, this X mu just transformed [? my ?] scalar field. So the scalar field transforms. Say as the following is going to X mu prime. And X mu prime, we've evaluated here that the new coordinate tau prime and sigma prime should be the same as your original X evaluated at the original location. So this is essentially the definition of a scalar field.

You see that when you make a coordinate transformation, if you follow the point, the value of X does not change. Because when you go to the new location, the X prime evaluated at new location should be the same as your original value. OK. Yes.

AUDIENCE: So basically we have a few more [INAUDIBLE] variants in real space, and the worldsheet. That's like--

HONG LIU: No, no, this is not if your [? morphism ?] invariant. No, this is not [? different ?] [? morphism ?] invariant.

AUDIENCE: What about in a special case, though?

HONG LIU: No, this is not [? different ?] [? morphism. ?] No, this is not different [? morphism. ?] This is global symmetry.

AUDIENCE: Oh, yeah, OK.

AUDIENCE: And also, did we ever consider curved real space?

HONG LIU: Yeah, you can consider-- well, curved space. But right now we are at the high school level of string theory class, and if you go to curved space, there should be college in that level of string theory. It's beyond what we are doing right now.

AUDIENCE: That's [INAUDIBLE] curve of space, we don't have Poincare. Will we have Poincare?

HONG LIU: Yeah, then you don't have Poincare symmetry. So this is specific to this action. But this is [INAUDIBLE] general. So, of course, under this coordinate transformation, this gamma ab should transform as a tensor, as in standard [? in GR. ?] This evaluated gamma prime you evaluate at a new point, should be [? related ?] as a tensor.

OK. So you can check yourself. This essentially by constructing, but you can check yourself that the action is indeed invariant under those transformations. OK?

AUDIENCE: Is there also gauge symmetry for this, [INAUDIBLE] add a [? phase ?] to X and derivative with a it cancels.

HONG LIU: Sorry?

AUDIENCE: Is there [? any gauge's ?] theory in--

HONG LIU: Gauge in what? Gauge in which one?

AUDIENCE: X, like in equation three, [INAUDIBLE] either global [? phase ?] to X and derivative with a and--

HONG LIU: No, no, no. No, X is real here. This is our full action.

AUDIENCE: I see. OK.

HONG LIU: Yeah, these are full action. X are real. So this is invariant on that. We will not show this is [? invariant. ?] This just essentially by construction. You should check it yourself, if you are not convinced. So the third symmetry is called a Weyl scaling of gamma ab. So the [INAUDIBLE] are slightly surprising.

So under this symmetry, X mu does not change. But the metric we multiply it by overall prefactor. And this omega is arbitrary for any function. OK.

So this is a very important symmetry, so let's check that. So when you multiply the gamma by our overall prefactor, when you go to gamma ab, then that's giving you the same prefactor, but with a minus sign here. And now, in the determinant for gamma, then that give you the square of these. And then take the square root, then give you a positive power. Then they cancel. Is it clear?

So the [INAUDIBLE] power here cancels with the positive power here. OK? So this thing is invariant. On [INAUDIBLE] symmetry.

So the way to see that-- another way to understand that somehow this action-- so the same thing with this action. This is not specific to we are working with Minkowski spacetime. The same feature here only depend on these two, OK? So the same thing happens here.

So you can also see a feature why-- and you can also get a feeling why this is such symmetry, just by looking at the relation between the Nambu-Goto action with this action. So in this Nambu-Goto action, there's no gamma. OK, so in some sense you can see. You can say this is automatically invariant under this kind of symmetry, because there's no gamma.

And now we see that the equivalence-- we write down equation of motion to go from here to Nambu-Goto action. The equivalence between them, they can be related by arbitrary function lambda. And really by aperture function lambda, yeah, this is another way to see this symmetry. OK? It's another way to see this symmetry.

So in some sense, you can say that this symmetry is also required for the equivalence to the Nambu-Goto action, OK? So we can actually use-- we can also turn things around.

We can also use as a symmetry principle. OK. I see major points that we can ask. Suppose we have action. Suppose we require an action to satisfy the a, b, c, to be invariant under a, b, c. What is the most general action? OK? And then, you can essentially uniquely determine the three. OK. We use actually symmetry principles to uniquely-- actually, it's almost uniquely determined.

OK, so I will explain this "almost." So now I use this clear. Is the logic clear? So we do two ways. We first start with Nambu-Goto and then we go to this action. But then we observe this action having the symmetries. And now we ask ourselves, what is the most general action besides symmetries? OK.

And that then leads us back to this term, to this action. Except there's one more term you can add. OK. There's one other term you can add. So that's why this is almost.

So I will just immediately write it down. This is called an Euler action. So this action actually have a long history. So this is called Euler action. So this can be written as following.

And this lambda is just a constant, some arbitrary constant. And R is the Ricci scalar for gamma ab. OK, gamma ab is symmetric. It's a two-dimensional metric. We will construct the corresponding Ricci scalar.

So you can clearly check this action-- so let me call this equation four. So you can check easily that a four is invariant under a plus b under a. Because this does not depend on X. OK, of course, this is invariant under a. And this is invariant under b, because this is a covariant action, so this invariant on b. To see this is invariant under c requires a little bit of effort. Okay, requires a little bit of effort.

But if I write down the formula, then it will become clear. So under c, if you make such a transformation, say under gamma prime equal to [? exponent ?] 2 omega gamma. [? 2w ?] gamma. And then you find that minus gamma prime and R prime-- R prime means the Ricci scalar for the gamma prime-- is equal to square root minus gamma, R minus 2. The 2-- yeah, this is a formula which you can check yourself. Maybe just directly write down.

And then you see you get one more term, but this term is a total derivative. OK, so this term is a total derivative. And if we impose the right boundary conditions, or if you can see the compact worldsheets, then this term will automatically vanishes, and then this is invariant. OK? So we have shown this is invariant under everything, a to c.

So essentially, that's it. So these two terms are only-- three and four are the only terms are invariant under a to c, OK? There's no other terms.

So total action. Then. And then this is our total action. So actually some of you who knows two-dimensional gravity.

AUDIENCE: Is there a minus sign?

HONG LIU: Yeah. Maybe let me put the minus sign here. Good. So some of you who know the two-dimensional gravity will immediately point out, you actually don't need to add this term. Because this is actually a total derivative. So this is a property of-- I only make a claim here. So this thing is a total derivative.

So you can check yourself. Go back, go to your favorite GR book, go to your favorite GR book, and we'll call it the scalar, Ricci scalar in 2D. And then you find actually the square root of gamma times i is actually a total derivative.

So it will not affect the equation motion, et cetera. But it is important. Otherwise I would not have talked about it. Because when you go to the Euclidean signature, that's the standard way we evaluate the path integral.

So when you go to Euclidean signature, this is only-- saying that this is a total derivative is only a local description. It's actually a local description. In fact, this is actually a topological term. So locally, you can write it as a total derivative. But if you put on the topological [? nontrivial ?] manifold, then you cannot do it globally.

And so, in fact, in the Euclidean signature, it's a-- actually I don't know whether this is due to Euler. This Euler action is actually equal to lambda times chi. And the chi is equal to [? pi-- ?] yeah, which is defined to be-- so now it's R. It's actually 2 minus 2h, OK? And the h is the genus of the surface.

AUDIENCE: [INAUDIBLE].

HONG LIU: No, this is only in 2D.

AUDIENCE: This is only true in 2D.

HONG LIU: Yeah, in 2D. So in 2D, this-- even though this is locally a total derivative, but it's not the global total-- But you cannot write a total derivative globally. And so we integrated over a nontrivial manifold. But manifold with a nontrivial topology-- actually, this is [INAUDIBLE]. It actually gives you this Euler number.

So example, if you integrated on a sphere, then you find that this number is 2, because [? when ?] h equal to 0. And if you evaluate it on the torus, then you find that this is 0, et cetera. OK. So one second. So in the Euclidean path integral, so Euclidean path integral, this S minus-- so this exponential S Euler term-- just give you exponential [? s ?] lambda chi, OK?

So now you may ring a bell, because this is precisely the [INAUDIBLE] term I added before when I say, when we sum over different genus. You have the freedom to add [? mechanical ?] potential, lambda times the Euler number. And actually this is dictated by the symmetry. And if you include the most general terms are consistent with those symmetries, then you are required to include this Euler term. And then you'll have automatically weight, that you would automatically have a weight for different genus, OK?

But for this theory, the lambda is actually constant. So the choice of lambda is still arbitrary, but this does give you a nontrivial weight. So this is weight-- different genus. Yeah. Any questions about this, yes?

AUDIENCE: So if you have [? a nontrivial ?] topology, wouldn't the boundary term from the symmetry also be nontrivial? So wouldn't the symmetry break?

HONG LIU: Sorry?

AUDIENCE: If you have a nontrivial topology, what is the total derivative boundary term, total derivative of the symmetry transformation also?

HONG LIU: No. No, they are different kind of-- no, there's no global issue here. This is just a ordinary function. You have to write down this explicit to know it. Yes.

AUDIENCE: So the claim was that [? efficient ?] work, no matter what value [INAUDIBLE] input. So this is why you were able to put in that factor at the sum. We could make lambda equal 0, in which case, we don't have it.

HONG LIU: Yeah. Lambda is not dictated by symmetry, so lambda is just a free parameter here. But in principle, if I determine it exists, yeah. Just lambda equal to 0 is a special case. Yes.

AUDIENCE: [INAUDIBLE].

HONG LIU: Yeah, in the Minkowski case it's a little bit tricky to talk about topology, [? et cetera. ?] So that's why we always, when we do the path integral, we always go to Euclidean signature. And for example, later when we quantized the action, which we work with at Lorentzian signature. And we only care, say when we solve the equation of motion, but we only care about the local structure. Then we can just ignore this term because this does not contribute [? to ?] local structure.

Good. So to summarize, from the two-dimensional-- so again, we can signal this as a two-dimensional field theory coupled to in the curved spacetime. From that two-dimensional perspective, this is a global symmetry. OK, so a is a global symmetry. Because the symmetry does not depend on the sigma and the gamma, and the sigma and tau.

So this should leave to conserve charge, conserve current, conserve charge and conserve current. OK? So we're going to see later this indeed plays a very important role in physics. And the b and the c are what we call the gauge symmetries, or local symmetries, because they depend on sigma and tau. They have [? arbitrary ?] dependence on sigma and tau. There and here can also depend on sigma and tau. So this is what we call the gauge symmetries.

So for those of you who have started Quantum Field Theory II or even Quantum Field Theory I, for those of you who have quantized [? QED, ?] you know that the gauge symmetries are not [? general ?] symmetries. They just tells you they are redundant degrees of freedom. OK? And they just redundantly [? with the ?] freedom.

And then when you quantize the theory, then you need to rid of those redundant degrees of freedom first. Because otherwise you will have a consistency problem. And the process of getting rid of redundancy problems-- those redundancies are called gauge fixing. OK, so that's what we'll get to a little bit later.

So in principle, you can just proceed to do the path integral quantization as we did before. And the path integral quantization is conceptually simple and geometrically elegant. But it does not give you, say, string theory spectrum. And because it's a path integral, it's [INAUDIBLE]. If you really want to find what are the spectrums of the string theory, you actually have to do the standard canonical quantization. So that's what we are going to do next, OK?

Now we are going to quantize the string in the canonical way. There are many ways of doing it. I will use the simplest way, so [INAUDIBLE] the simplest way, also conceptually simplest way. It's called the light-cone quantization. And the meaning of this word will be clear.

So the basic picture will emerge from this quantization is that each oscillation mode-- so if you have a string, then you can oscillate, right? Because that's what the string does. It can oscillate. So each oscillation mode, so the bottom line is that we will show that each oscillation mode of the string gives rise to a spacetime particle.

OK. From the string point of view, it looks like some kind of oscillation of string. But from the spacetime point of view, it's like a spacetime particle. In particular, if you have a closed string, then that will give rise, say, graviton. Say one particular mode will give you graviton and then many other particles. Actually, we have an infinite number of particles from string because there's infinite number of vibration modes with a string.

And if you have an open string, say the string which does not close, and then actually then you find the gauge field-- a rise in one of the vibration modes. Say this can either be photon or gluon, et cetera. OK.

Yeah, so this is the thing we like to show. Yes.

AUDIENCE: [INAUDIBLE].

HONG LIU: Oh, there are an infinite number of particles. So because the string can oscillate, just any string oscillate in principle in an infinite number of ways. And if each oscillation mode [? cause ?] one into a particle, then you can have an infinite number of particles.

AUDIENCE: [INAUDIBLE].

HONG LIU: Oh yeah, oh yeah. They're completely different from gravitons and protons. They're just completely different particles.

AUDIENCE: So [INAUDIBLE] particles [INAUDIBLE]. What other particles in the same class with gravitons?

HONG LIU: Depend on what you mean by class.

AUDIENCE: Well, can you give a second [INAUDIBLE]? In the same line as graviton, [? what else can you do? ?]

HONG LIU: Oh, for example, mass will spin two particles. Mass will spin three particles.

AUDIENCE: Everything will spin more than one?

HONG LIU: It can also have a scalar particle, [? massive ?] scalar particles, but they're all massive.

AUDIENCE: That'd be boson?

HONG LIU: They can be fermions. In this theory they can only be bosons. But we can play tricks to get fermions.

AUDIENCE: So does this predict that-- so if you were to have a string theory model which would reproduce the standard model, this basically says that at higher and higher energies that we expect to always see new types of particles or something?

HONG LIU: Yeah, that's right.

AUDIENCE: Interesting.

HONG LIU: But we will see. You need to go to a specific time-- we already go to a specific energy scale to see.

AUDIENCE: OK. We can see how they're spaced.

HONG LIU: Yeah, yeah. That's right. Yeah, but that space is determined by some energy scale, and that scale can be very high. OK, so now we will quantize this theory using canonical quantization, OK? So by canonical quantization-- so let me remind you, this is the procedure. So this is a procedure we typically explain, say, in the second day of free quantum field theory. And so we are doing the same thing here.

So the quantization procedure is the following. The first step is you write down equation of motion. OK. And in the case which you have gauge symmetries, then we fix the gauge symmetry.

And the third is find the complete set of classical solutions. OK. And the four is you promote the classical field on the worldsheet, worldsheet field, say the X or gamma to quantum operators. OK. So the quantization procedure is that you promote them to quantum operators, which satisfy the so-called canonical [? computation ?] relation. And the previous equation of motion then becomes operator equations for those operators. OK?

In particular, then the classical solution you find in three becomes the solutions to operator equations. And in particular, the parameters we use to parameterize your classical solutions-- so those become, say, the creation and annihilation operators. Just as what we do in the free field theory quantization.

And then the last step, once you have the creation and annihilation operators, then you can just find the spectrum. By acting the creation and annihilation operators on your vacuum, then you can find the spectrum. So find the spectrum.

So this is the standard procedure. Yeah, so this is almost exactly the same as how we quantize our free scalar field theory. In Quantum Field Theory I, the only tricky thing is now, this is a system with gauge symmetries. That's the only tricky thing here. Any questions here?

So now let me just again say some boundary conditions we are going to use.

AUDIENCE: [INAUDIBLE].

HONG LIU: Sorry?

AUDIENCE: What are parameters in classical solutions permutations. [INAUDIBLE].

HONG LIU: What other--

AUDIENCE: Parameters in classical solutions in case of [INAUDIBLE].

HONG LIU: It's different modes for your classical waves. The amplitude for your different classical waves.

AUDIENCE: [INAUDIBLE]?

HONG LIU: Yeah, it is like that, yeah. For scalar field theory, yeah, just for harmonic oscillator, you can write a cosine. a explains your i omega X plus a star, [INAUDIBLE] omega X. So a is the parameter, and a and a star become creation and annihilation operator. It's the same thing here.

So we will consider both closed and open strings. So suppose closed and open strings. So if you have a closed string, then, just by convention we always take sigma to go from 0 to 2 pi. So the closed string means it's closed. Then we take this from to 0 to 2 pi. That means whatever function should be a periodic function of 2 pi.

OK, should be a periodic function of 2 pi, because it's closed, OK? So the same thing with gamma. Same thing with gamma, OK?

But if you have open string, by convention we take sigma goes to 0 to pi. So one end of the string is at sigma equal to 0. The other end is at sigma equal to pi. So for open strings, so here is sigma equal to 0. Here is sigma equal to pi. So the sigma is directly on the string. So we need to-- for open strings, then, because you have boundaries, then we need to supply boundary conditions at the end points.

Good. So now with this set-up, we can now just follow that five steps, OK? We can just follow that five steps and quantize the string. So do you want to have a break?

OK, let us start. So we just follow these steps. First let's write down equation of motion. OK, first we just do a variation of gamma ab. So gamma is the worldsheet metric from the point of view of two-dimensional field theory. The gamma is the worldsheet metric. When you do evaluation over gamma, essentially what you get is the stress tensor for this worldsheet theory, OK?

So when you do the [? evaluation over ?] gamma, essentially, it's just the statement that the stress tensor had to be 0. OK. And then you can work this out explicitly. And the stress tensor, which I call Tab.

OK. So I just [? did the evaluation ?] of that action with respect to gamma ab. This is what you get You find this thing has to be equal to 0. And I just call this thing Tab. And this Tab is essentially the stress tensor of this. If you [INAUDIBLE] the scalar field theory, so this Tab is essentially the stress tensor of this scalar field theory. OK? Yeah.

AUDIENCE: If we add the topological term and said, there's a Ricci scalar there, there must be Einstein's tensor equal to Tab.

HONG LIU: No, no, no. Because there's a total derivative term in two dimensions-- total derivative term never contribute to equation of motion. Yeah.

AUDIENCE: So but if we calculated the Einstein tensor for that R [INAUDIBLE] get 0.

HONG LIU: You will get 0, yeah.

AUDIENCE: Sorry, so you just said that from the equation of motion, that gamma ab can get gamma ab equals 1/2 lambda hab. Is that it?

HONG LIU: Yeah. It's the same thing. It's the same thing. But this is a very good question. Let me do that here. Say if I write gamma ab equal to 1/2 lambda hab, I can write this lambda explicitly. Then I can modify this by hab on both sides and this means hab gamma ab equal to lambda.

OK, that means lambda is equal to gamma ab times hab. And so this equation, I can also write it as gamma ab is equal to 1/2 hab gamma ab hab. Oh, gamma cd.

Yeah, which you can show is equivalent to that equation. OK. Or maybe I should do gamma ab. No, if you do that one, it doesn't matter. I can do-- to be exactly the same as that equation, let me just do both sides. I can track with gamma ab. Let me see, maybe I can [? track ?] both sides. Let me think. So that is [? HCD ?] so that is h gamma ab, times that thing, is equal to--

Yeah, you can-- just take this equation. You get this equation. Then you can show that equation is equivalent to this equation, OK? [INAUDIBLE] right now, they're not reading the identical way, but they can show the equivalence. So yeah, actually that's your homework.

So now if you look at equation of motion for X mu, look at the equation of motion for X mu, then this is just [INAUDIBLE] your standard scalar field. So this just give you-- OK, and we call this equation six.

So these are all of your equation of motion. If you have open string, then when you do the evaluation of X, you have to do integration by parts. Then in the standard [? story, ?] you have to have a boundary term. OK? But you actually have to be careful about that boundary term. For the open string, you will get a boundary term, which is given by delta mu gamma sigma b partial b X mu, evaluated at sigma equal to 0 and pi, should be equal to 0. OK?

So this just come from where you get this second-order equation, you always have to integration by parts when you do the evaluation. That just comes from the boundary term of that [? variation, ?] evaluated at the end point of the string you have in the sigma direction. OK. Is this clear? Good.

Again, I will not go through all the algebraic details. You should check those details yourself. You should check them, and if you find mistakes, it's greatly encouraged. Yeah, I will add to your points. Yeah, if you find mistakes, then I will add your points. If you have found enough mistakes, then you may not need to do the [? PSAT ?] anymore.

So this tells us we can actually impose two kinds of boundary conditions. Say delta X mu have to be 0. So evaluated at sigma equal to 0, pi, equal to 0. Sigma 0 or pi-- it does not have to be the same.

Or this thing is 0, gamma sigma b. The reason the sigma here, because we are interested only in the boundary term in the sigma direction. The boundary term in the time direction we [INAUDIBLE] care about. Because the boundary term in the time direction, we always assume in the far past and the far future, nothing happens.

We always assume that. But you cannot assume a boundary condition at the spatial condition. You have-- yeah, in the spatial boundary, you have to be careful, because we have a finite-- [? a special ?] extension. OK, so that's why this sigma.

So a lot of the possible boundary conditions, gamma sigma b, times partial b X mu, so evaluated at sigma is equal to 0 or pi, equal to 0. So this is normally called the Dirichlet boundary condition. This is normally call the Neumann boundary condition. The reason this is called Dirichlet boundary condition because if that X is fixed at the end point, it means that the value of X are not allowed to vary. So this is normally called the Dirichlet, OK?

And for the Neumann boundary condition, you only constrain the derivative. And the value of X can be anything. And you only constrain the derivative. So this is called the Neumann boundary condition.

So for simplicity, later we will treat both of them. But for now, let me just look at this one, because this one is slightly simpler. So for the moment, let me just consider the Neumann boundary condition, OK.

So now the second step is fixing the gauge. Second step is fixing the gauge. So we have two gauge freedom. One is to change the coordinates. And the one is to do a scaling. OK. So before we fix the gauge, let's do a little bit of counting. So here I can change coordinates by arbitrary two functions, OK? So here I essentially have two arbitrary functions with freedom of the gauge freedom.

And here, I have one arbitrary functions for both gauge freedom, because I can choose omega [INAUDIBLE] arbitrarily. And if you look at gamma ab, the worldsheet metric-- so ab is from 0 to 1. So this also has three freedoms, three degrees of freedom.

So that means actually, in [INAUDIBLE] theory, at least you can expect by taking account those degrees of freedom, taking account of these three gauge degrees freedom, you can set this gamma ab to some fixed metric. Because there's only three freedom here, and we have three gauge freedom. And you can completely fix them.

So [? not ?] let me erase it now. Now let me do the second step. Gauge fixing using this different morphism of b, so these have two parameters. You can show that you can just make a coordinate transformation. Take any gamma ab, you can show that by using a coordinate transformation, you can transform the gamma ab to the following form.

OK. So I will not prove it here, but I'll let you go back to try to convince yourself of this fact, OK? Just by using the coordinate transformation starting with generic gamma ab, you can make a coordinate transformation so that in the new coordinates, the gamma ab is given by a prefactor times the Minkowski metric. And now we can use the c, that guy, because we have the freedom to change the gamma ab by arbitrary function. And then we can just choose that function to be [INAUDIBLE] to this guy. Then we can set gamma ab just equal to eta ab. OK. So we can use these two freedom to set the gamma ab with eta ab. Yes?

AUDIENCE: So you're saying that we can [INAUDIBLE] gauge [? toward ?] locally on the worldsheet. [INAUDIBLE] gauge transform the metric [? globally ?] [INAUDIBLE].

HONG LIU: That's right. Yeah, we can transform it locally into [? flat ?] metric. And then, indeed, there is a global issue of which I will not worry about now. Yeah, because the question which I'm interested in, say, the spectrum only [? consists ?] of the local questions. For example, if you do the path integral, then you don't have to worry about the global issues.

AUDIENCE: Why? Why is it that when you [INAUDIBLE]?

HONG LIU: Yeah, because you need to integrate over all possible configurations. And you don't want to do the overcounting, or count mass. Yeah.

AUDIENCE: So we don't have to work this out. So why does that mean we don't have to?

HONG LIU: Yeah. So let me give you an example. For example, if you do the genus g surfaces, and then say let's do Euclidean again. It's easier. Do Euclidean.

Indeed, you can locally transform to eta ab the actual set of discrete parameters. And then you have to be careful about those discrete parameters, et cetera. Yeah, just those discrete parameters have to do with global issues, et cetera.

AUDIENCE: I see what you're saying.

HONG LIU: Yes?

AUDIENCE: [INAUDIBLE].

HONG LIU: Yeah, just imagine you have some two-dimensional surface. Right now let's not consider about the topology. Right now just consider. So in this case, for the closed string, the simplest thing-- you can see this [? sitting ?] there. So this is sigma direction. This is the tau direction. Yeah. Yeah, for the closed string it's [INAUDIBLE]. But the open string, roughly just like this.

And right now we don't consider complicated topology. We just consider local question. Then we can consider the simplest topology.

AUDIENCE: Yeah, sorry, I didn't quite get the path integral being--

HONG LIU: Yeah, don't worry about it.

AUDIENCE: Don't you have to integrate around both sides of every [? pole that ?] you have in the--

HONG LIU: Yeah, you have to integrate over everything. You have to be careful about everything.

AUDIENCE: So global [? effect is a factor. ?]

HONG LIU: Yeah.

AUDIENCE: It does matter. OK.

HONG LIU: Of course.

AUDIENCE: I thought you were saying the other.

HONG LIU: No, no, no. I'm saying the global effects are very important when you do the path integral, and if you want to do the path integral correctly. But here we are interested in the spectrum. And this is a local question.

OK, so now with this gauge fixing, we can see how the equation of motion simplifies. First we have fixed all gauge freedom, OK? So let's look at the six first. So six just become a free scalar equation. Because when gamma ab become a Minkowski metric, this is just like a scalar field, OK? This is like a scalar field.

So this we can write explicitly. So this is a free scalar field. And let me call this somehow-- oh, yeah, let me call this seven. This one seven, this one eight, just to be consistent with my notes. Then I call this nine.

But if we still have to worry about the five, OK, so the five can now also be written explicitly. So first thing you can check is that T0,0 is equal to T1,1, so the diagonal elements are the same. It's given by 1/2 partial tau X mu partial tau X mu, plus partial sigma X mu partial sigma X mu equal to 0.

And T0,1 component will give you partial tau X mu [? times ?] partial sigma X mu equal to 0. So I'm just writing everything explicitly. So these are the three components of that equation, when I set gamma ab equal to eta ab.

You can easily check yourself. Just plug that in. OK? So the reason that T0,0 is equal to T1,1 is because we actually have a scaling symmetry. So in the case when this action-- yeah, this is a free massless scalar field. It's a scaling symmetry. And you know that when you have a scaling symmetry under the stress tensor, the [? trace ?] of the stress tensor always vanishes, so that's why T0,0 and T1,1.

So for the open string, we also have to worry about the boundary conditions. And so this just become the Minkowski metric. And the sigma sigma, only the sigma sigma is non-zero. So you just get the partial sigma X mu equal to 0. OK, so this should be evaluated at sigma equal to 0 and pi.

So right now, we only consider Neumann boundary condition. And let me call it 12. So this is for Neumann. Good? Any questions on this? Yeah.

AUDIENCE: For this Dirichlet boundary condition, it's a [? valuation. ?] So you already may say the [? valuation ?] is arbitrary by [INAUDIBLE] can be arbitrary. So it seems like only Neumann boundary condition is valid.

HONG LIU: Yeah, this is your choice. This is your choice. So normally in the field theory, what we do-- indeed, normally in the field theory, yeah, this is your choice. In the normal field theory, where you consider, say, an infinite space, we just assume everything goes to 0 at infinity. And we don't even need to worry about the boundary condition. Then the boundary term automatically vanishes. And so that's why, normally, in the field theory in the infinite space, we never worry about the boundary conditions because we made that assumption.

So here, because the string has a finite [? lens, ?] it's a finite boundary. And then you actually have a freedom to introduce both. Yeah. You do have the freedom to introduce both.

Good? So let me emphasize again. So now in this gauge, we just have a free scalar field. So this is a standard wave equation in one plus one dimension. But we are not quantizing a free scalar field theory, because we still have to solve those things. We have to solve these two equations. And these are constraints. Because in this equation, in principle, you completely solve the equation. We know that this is a wave equation. You already solved X completely.

But these two equation, coming from the gamma equation of motion, they impose a [? longitudinal ?] constraint we have to satisfy. And those constraints actually last [INAUDIBLE] because they are non-linear. This is a linear equation, but these are non-linear constraints. And this is the 10, 11-- I'll normally call them Virasoro constraints. Virasoro was a person. So this is the Virasoro constraints. OK? And so this allows me to constrain the equation [INAUDIBLE].

So now let's first solve the easy equation. So let's go to the third step, trying to solve the equation of motion. And now we have fixed the gauge. And now we try to solve the equation of motion, OK? So for people at your level, I can write down a solution of line immediately.

So I can have a constant, of course, satisfy this equation. I can have a linear term in tau, which of course will satisfy this equation. For closed string, in principle, I can also have a linear term in sigma, but that won't satisfy the periodic boundary condition. So that's not allowed. And then I can add the traveling wave solutions. So this is a left-moving wave, a right-moving wave, and this is a left-moving wave.

OK? So this is a full set of solutions to-- so x mu are arbitrary constants. x mu and v mu are arbitrary constants. And XL and XR are arbitrary for closed strings. So XL and XR just are independent, should be independent periodic functions of period 2 pi. OK?

Because they have to be. So XL and XR are function of single variable. And they have to be period in sigma means they have to be periodic. So these are the periodic function. OK, is this clear to you? Do I need to explain this equation? Good. Yes?

AUDIENCE: Why a function of tau minus the sigma?

HONG LIU: It's because you have a traveling wave. Because first we can immediately see this satisfies this equation. And then you need to convince yourself these are the only solutions, in the sense that-- yeah, because these are the two arbitrary functions. There's no other choices anymore. Any more questions on this?

AUDIENCE: [INAUDIBLE].

HONG LIU: Also not, for other reasons. Not for the Neumann boundary condition. Not for this boundary connection. And now we are come to open string.

So for open string, you can then be-- again, we have these. We can check that if you have a linear sigma term [? it will be ?] incompatible with this boundary condition, so we don't worry about that. So we don't worry about that. And then, from this equation, then plug that into this equation, we conclude that the XL prime tau should be equal to XR prime tau at sigma equal to 0.

And XL prime, tau minus pi, should be equal to XR prime, tau plus pi, at sigma equal to pi. OK, so you just plug this into here. So the first two terms does not matter, and then you just have the last two terms. So the last two terms, then if you evaluate it at sigma equal to 0, then you just get this equal to that. And if you evaluate it at sigma equal to pi, then find this, OK? Yeah.

AUDIENCE: Why is it that you don't package those original two terms into the right and left functions? So why is that you write x mu plus v mu tau?

HONG LIU: Because those equation don't have this form. They're not in the form of tau plus sigma or tau minus sigma. No, this is just tau. This is not tau plus sigma. Now, I can [? pack it ?] in that form. It doesn't matter. I'm just saying here I'm not doing this. Yeah. What I'm saying-- these are trivially periodic functions. And yeah, I [? just separate ?] that. OK?

So now, from here, you can find that-- so this equation tells you that XL is essentially equal to XR, OK? So up to a constant we can absorb into X mu, for example. Then these two functions could be the same. And if we take these two function to be the same, then this function tells us this is a function-- the second line tells us this is periodic in 2 pi. OK?

Yeah, I actually think I missed a minus sign here. So if I take the sigma derivative, yeah, I can put a minus sign here. But these two conclusions still are right.

So now for both open and closed string, you have solved nine completely. Solved nine completely. Any questions about this?

OK, but we still have to solve 10 and 11. So what 10 and 11 does is to impose some nontrivial constraints between XL and XR and x mu and v, et cetera. I'd just say plug in this most general solution into 10 and 11. And we'll impose some constraints on those functions. So I urge you to try it a little bit yourself. And those equations are not nice equations. There are lots of nice people. Yeah, there are lots of nice equations.

And so at this stage, we face a decision. So when you see this 10 and 11 are hard to solve, then you have a decision to make. Yeah, whenever you have something that you don't know how to do immediately, you have a decision to make. We have some decision to make. Or you just give up.

And first-- so one option is that we could just quantize this theory. We know how to quantize this free scalar field theory. We just quantized this free scalar field theory first. And then we can construct the Hilbert space, et cetera. And then we can impose those constraints at the quantum level. We can impose those constraints at the quantum level. Yeah, so this is one option.

The second option is that you find some ingenious way to solve those equations so that you can actually find independent variables. OK, because those are constraint equations, means some degrees of freedom allow the independence from others. So the alternative is, I guess, you just solve it. And finally, independently, with freedom, and then quantize only [? those ?] independently with freedom.

OK. So the first option, this stage is easy, because you can quantize the first [? gauge ?] field theory. But the process will impose the constraint at a quantum level. It's actually rather tricky. So I will not go there. So I will do the second method, [? rather ?] we try to solve this constraint, find some trick to solve this constraint, and then quantize independent degrees freedom. Is it clear?

So I will take the second approach. So this is the idea of the light-cone quantization, OK? So this is the idea of going to the light-cone gauge.

So this is an ingenious way to solve those constraints. OK? And it was first developed by four people, including our colleague here, Goldstone, Jeffrey Goldstone, who was among the first people to quantize this theory, using the light-cone gauge. Actually they quantized the Nambu-Goto action. They did not quantize this theory. They quantized the Nambu-Goto action using the light-cone gauge.

So the key idea is the following. He said after fixing this gauge, after fixing gamma ab, you go to eta ab. There are actually still some residue gauge freedom. We actually have not fixed everything.

OK. By residue gauge degrees freedom-- so if you fix the gauge completely, then that means that any operation which previously-- what we did b and c-- when you act on gamma, we will take this away from this configuration, OK? So by residue degrees freedom, it means I can still find the combination of coordinate transformation and the Weyl scaling so that after those operations, I'm still going back to this metric. OK, so is it clear what we mean by the residue gauge freedom? Good.

So to see this I have to introduce so-called the light-cone coordinate on the worldsheet, introduce sigma plus/minus equal to square root 2 tau plus/minus sigma. And then the worldsheet metric can be written as minus d tau square plus d sigma square-- because we take the Minkowski metric. So this can be written as minus 2 d sigma plus, d sigma minus.

So now here is the key observation is that this metric is preserved by the following coordinate transformation. I take sigma plus goes to sigma tilde plus, which is only a function of sigma plus. And sigma minus goes to some function sigma tilde minus, some new coordinate sigma tilde minus, which is only a function of sigma minus.

So if you've plugged this in here, then you'll find under this coordinate transformation, we get this ds tilde square. If you just plug this in here, this is, say, 2d sigma tilde plus, 2 sigma tilde minus. You go to new coordinates. And [? these ?] new coordinates expressed in terms of original coordinates, you get 2f prime sigma plus, g prime sigma minus, and d sigma d minus.

So now the key is that under this coordinate transformation, you only transform your metric by an overall prefactor. And then you can get rid [? of it ?] by Weyl scaling. It is preserved by this, followed by Weyl scaling. Because this coordinate transformation only changed your metric by an overall prefactor. And any overall prefactor you can get rid of by Weyl scaling.

OK, is this clear? So we actually still have some residue of gauge freedom left. OK? So now we can use this freedom to do something good for us. So these new coordinates, say, in this new tilde coordinate, tau tilde, then have the form of square root sigma plus tilde plus sigma tilde minus. OK.

So this have the form 1 over 2 f sigma plus tau, and g sigma minus tau, say tau minus sigma. So I have this form. So in this new coordinate, yes, so you can change the coordinates.

So you're allowed to make this kind of coordinate change. So you are allowed to make this kind of coordinate change. Yes, so the residue degrees freedom means we are allowed to make this kind of coordinate change, OK? And that means-- and you see, this is precisely the combination of this form, a left-moving wave plus a right-moving wave.

So that means we can use this freedom. We can choose. So this means we can choose f and g so that tau is given by any combinations of this X-- the solutions of X, OK? Because solutions of X precisely have this form. So what we will do is we will-- maybe should I erase this? Yeah, let me erase this because I think you all know this.

So I did a level of the equation of motion. You have the freedom. [? You're going to ?] choose tau tilde to be any combinations of those X, OK? Because those X precisely have that freedom. And the smart choice is what we call the light-cone gauge-- we can choose so that tau tilde is proportional to so-called X plus. And X plus is defined by square root of 2, say, the 0-th component of X plus, say, one of the spatial directions. OK, say, take one.

So to fix this gauge, to fix to fix this residue gauge freedom, I have the freedom to choose tau tilde to be some combination of X. And this is the combination that we will choose. So this is what we call the light-cone gauge. So I will suppress this tilde. So in the light-cone gauge means we go to a coordinate so that tau become equal to X plus, and up to some constant which I called V plus. OK. So V plus is some constant. Is this clear to you?

And this is so-called light-cone coordinate in the target space. So this [INAUDIBLE] identify the worldsheet's time with the light-cone time in the target space. Or in other words, X plus equal to V plus, tau. This is what we call light-cone gauge.

So why do we want to do that? And the ingenious thing is the following. It's that if you write X mu, so we can write X mu as X plus, X minus, and Xi. And the i come from 2 to D minus 1. So X plus and X minus is X0 and X1. So that's right in this form. OK, then you can easily check yourself, or you maybe already know it, is that this particular contraction is given by 2X plus, X minus, plus Xi squared.

In particular [INAUDIBLE], just as here, which you have this off-diagonal structure. This kind of contraction has this off-diagonal structure in X plus and X minus. And this turns out to be key. Yes.

AUDIENCE: Do you mean for that to be 0 and 1, and then that to be plus 1?

HONG LIU: Sorry?

AUDIENCE: Do you mean for that to be 0 and 1?

HONG LIU: So these two are equivalent to X0 and X1. Let me include the rest, starting from i equal to 2 to D minus 1. So X mu have X0, X1, then what I would call Xi, which i start from 2 to D minus 1. I just renamed these two to be X plus and X minus. Just change this two to X plus and X minus.

AUDIENCE: Oh, I see. OK.

HONG LIU: Yes? Yes.

AUDIENCE: So when we do another one, [? I thought ?] the Weyl scaling's already fixed when you're trying to fix [INAUDIBLE].

HONG LIU: No, no. I'm just saying I do these two operation, but in sequence. So I do that operation, and then I choose the Weyl scaling so that precisely cancels this. Of course, if you just do a single Weyl scaling, this will be violated. But if I do that two operation together, then this will be preserved. Yeah, I have to do that two together. Yeah, if you do a single one, this will be invalidated.

OK, so this is the key point. So we will see that this structure will play a key role. So this is the smart thing which our friend Jeffrey did. This is in fact the ingenious thing of our friend Jeffrey and his friends [? did. ?]

So now let's look at these two equations, OK? Now let's look at these two equations. So the first thing-- so now let's look at 10 plus 11. So first let's note, because of this, partial tau X plus is equal to V plus, and partial sigma X plus is equal to 0. OK?

Because there's no sigma dependence there, only tau dependence, which isn't in here. And then let me just plug it in. So let's look at here. Let's forget about the 1/2. You can also think about the 1/2, doesn't matter.

So look at plus and minus structure here. Use this here. Then you get minus 2 V plus, partial tau X minus, then plus partial tau Xi squared. This is for the first term.

In the second term, the X plus and the X minus part is 0 because of these. Because of this and because of the off-diagonal structure here, OK? Do you see it? So the first term here would be partial tau minus 2, partial sigma X plus, partial sigma X minus plus partial sigma Xi squared. So this would be 0. But our friend here is 0, because of that is 0. So now we can erase it.

So now let's change this to the other side. OK. Let's change this to the other side, and then change this sign. So the equation I get is the following. OK, so this is equation 10. So now let's look at equation 11.

In 11, so we have partial tau X plus, then partial sigma X minus. Then you go to partial tau Xi and partial sigma Xi And then this one just give us V plus.

So that two equation become these two equation. And let me call this 14. This is 15. So is it clear to you, these equations? So if it's not immediately clear to you, just check it yourself. OK. Urge you to do the check, because then you will see the magic. Then you will see the magic and the beauty of this trick.

AUDIENCE: [INAUDIBLE].

HONG LIU: Sorry?

AUDIENCE: Can you tell X minus [INAUDIBLE] sigma X plus?

HONG LIU: Yeah, that's 0 because partial 0 X plus is 0. So that's why there's only one here, not two. Because the other term is 0. OK, so now the beauty of these two equation is that they express the X minus solely in terms of Xi. So 14 and 15 tells you, is that X minus can be fully solved in terms of Xi. Because this equation give you the tau derivative of X minus, this equation give you the sigma derivative of X minus, and all expressed only in terms of Xi.

So the X minus is completely solved, completely expressed, in terms of Xi. And then we have solved those constraints [? similarly ?] [INAUDIBLE] constraint explicitly, OK? So right now we are doing it a little bit fast. I urge you to go through the algebra yourself to appreciate the beauty of this step.

So we conclude that independent degrees freedoms are only Xi. Because X plus we already know by fixing the gauge. And X minus now is completely solving in terms of Xi, so the independent freedom are only Xi's. And this will make our life very easy. Because Xi are just free scalar fields. Xi-- just free scalar field with the right kinetic term.

You have also get rid of X0, because we said that [? before ?] X0, we have a [? long-sign ?] kinetic term. But now we are only independent-- [INAUDIBLE] only Xi's, and they have the right number of kinetic terms. They have the right sign for the kinetic term. And they are just free scalar fields. So we can quantize them as what you did in your first day of Quantum Field Theory class. And then you have quantized the string theory. And then you have quantized string theory.

OK, yeah. I think I'm going very slow here. Maybe I'm a little bit too emotional. Anyway, so maybe let's stop here. The next time, then we will just quantize those things. And those equations still will give us some very important information, and then we will quantize those things. And then we will be able to see that the string theory contains gravity. It contains gravitons, OK?