HONG LIU: OK let's start. So let me just mention brief-- first let me just mention briefly, this story regarding the D-brane and the Higgs mechanism, which we discussed at the end of last lecture. So there was a P-set problem which was related to this. Yes, I expected maybe the P-set problem was a little bit tough because we did not go to many details. We did not go to many details. Yeah, so in grading we will be very flexible because we-- So let me just say a few more things about that, just to make sure--

So here is the action. If you have n-- say if you supposedly have N D-branes together-- and then in such case-- so this is a low energy defractive action for the massless degree of freedom on the D-brane for which you have a gauge field, which is a matrix, and then you also have a scalar field, a number of scalar fields, which are also n-by-n matrices. And the number of scalar fields is the same as the number of transverse dimensions.

And then we also discussed that the scalar fields can be considered as describing the transverse dynamics of the brain. Say, let's consider you have-- let's consider you have a single brane and there's only one transverse direction. Say let's call it the phi direction. There's only one direction, say phi 1 direction. Then essentially the location of the brane then essentially the expectation value of phi 1 can be considered-- say suppose this is the direction of x1, and the corresponding scalar field is phi 1. And then the expectation value of phi 1 can be considered as the location of the brane and this phi is the string which ending on this brain and come back to itself.

So now let's consider if you have two branes-- now consider you have n-branes-- again, b is only one transverse direction. I now have n-branes with one transverse direction, which again is x1. And now you can just separate them. So now let's consider the process which you separate these n-branes into-- say completely separate them in different locations. So this is x1, x2, to xn.

So this situation can be considered as a situation. So in this case, the phi is the-- phi is the m-by-n matrices. And then this can be considered as phi having an expectation value which is only in the diagonal entry, and each that diagonal entry describes the location of each D-brane.

And it's natural to think that why this is the diagonal entry because the diagonal entry describes the string, which ending on the brane itself. So this is a one-one string, and this is a two-two string, and n-string. And so the diagonal string is the excitation on the brane itself. And the off-diagonal degree of freedom corresponding to, essentially corresponding to the excitation between the string between the branes. And so it's natural that the location of the branes, and then just corresponding to the diagonal entries.

And so now we can generalize to the general dimensions. Now suppose there are two transverse dimensions-- let me call them x1 and x2-- and then now you have two scalar fields. So now the story is a little bit more complicated because when you have two transverse directions, say if you have more than two transverse directions, now you have nontrivial potential term. And under the law of the configuration, say suppose it's five, have low space time-dependent on the law of configuration, say the minimum with the potential, the mass was corresponding to the configuration, which the commutative phi a and phi b to be zero. Right now, say we have two transverse directions, so we only have two, but you can generalize to any number of transverse directions.

So that means that the phi, or the phi equals 1, or the scalar field, they have to converge with each other. That means they can be simultaneously diagonalized. That means they simultaneously diagonalized. That means we can choose a basis, say if I write the phi, the collection of all the D-branes, then I can simultaneously diagonalize them, and then have the following form.

So each x1 goes one into-- so each vector goes one into-- for example here, is x1-- I should use a different notation, so let me call the spacetime y1, y2-- and say this is x2, et cetera. So this defines the configuration, which you have one D-brane here and one D-brane at this point, and at x3, and et cetera. And again, it's only the diagonal entry that describes the location of the D-branes. The diagonal entry describe the location of the D-branes.

And these, because they are all diagonal and the commutator is zero, and these are the law of the configurations. So this is a very important lesson. It tells you that even you have a potential, still the D-brane, you can put it anywhere you want. And you have the freedom to put the D-brane-- you don't break translations image. Any questions on this?

Of course when you say, when the scalar field develops expectation value and then your original un gauge symmetry's broken, and then broken into the just in the standard story to whatever [INAUDIBLE] group allowed by this-- so if all the x1-- if all of them are different, then the remaining just be 1 to the power n. Because now you just have each brane separated, then you just have 1 to the power n. Good?

So not let's move to new stuff. So now let's say I little bit about the D-branes in super string theory. So what we said so far actually applies to D-branes in bosonic string and in superstring. Now let's say a little bit more above the D-branes.

So as we discussed before, when we quantized the open string, the D-branes in bosonic string always have a tachyon-- always have open string tachyon, no matter what D-brane you have because as we discussed before, the quantization, the zero-point energy on the string is the same no matter which dimension of the brane is et cetera. And so you always have open string tachyon.

So that means, as we said before, tachyon means that you are seeking-- if you have a scalar field which have a negative mass squared, that means you're essentially sitting at the top of the potential-- in the top of some potential. And so that's why you get the negative mass squared. And so that means you have-- this is an unstable configuration. Because if you, say, give the tachyon a little bit back, then the tachyon want to roll down, and then you move away from this configuration. You move away from this configuration.

So when you go to superstring-- so we also discussed before, for the bosonic string, the close string factor also has a tachyon, and then you can get rid of those tachyons by going to the superstring. So similarly, going into the superstring, the D-branes of certain dimensions-- not for all dimensions, but only for some dimensions-- do not have tachyons. So they are stable. So their lowest modes are just massless modes. They don't have negative mass square modes.

In particular-- so this I will-- this is a long story. This is a somewhat long story which I will not explain here, but I will just make the statement. In particular you can show that for those stable D-branes in superstring, they always carry a conserved charge. So I will explain where does this conserved charge come from, and they always carry some conserved charged. And which is also a reason why they are stable.

And the second feature is that the worldvolume theory is supersymmetric. So in addition to this phi-- so in addition to this say A alpha phi we are looking at here, A alpha and phi a, which we already see in the bosonic string, they are also massless fermions. So the open string excitations also include massless fermions. They also include massless fermions. And in particular, the low energy theories, all of them together, is the supersymmetric version of this. It's a super-Yang-Mills theory, supersymmetric version of this. And it's some kind of super-Yang-Mills theory.

So now let me elaborate a little bit on the first point. They carry a conserved charged. Let me elaborate a little bit on the first point. So maybe go to the one, elaborate more on one. So let me just remind you again, the bosonic part of the of massless closed superstring spectrum, which we briefly mentioned before-- so this is so-called type II string, so-called type II string. When I say the superstring I always mean type II string.

So if you quantize the superstring, then you find that they are again, you find the metric, you find the graviton, then you find that there are this metric tensor, and you find that there are scalar fields just as what we did in the bosonic string. But they have some additional massless modes. Depending on your quantization or how you treat the fermions in the superstring, then there can be, say IIA or IIB, there are two types of superstring. And for IIA you have one additional 1-form field and additional 3-form field. And for IIB, you have additional scalar fields, and additional 2-form fields, and additional 4-form fields. In particular, this is so-called self-dual. I will explain that in a little bit. In the IIB, contain a self-dual 4-form.

And this additional field are called the Ramond-Ramond field, typically. Just a name, it's called the Ramond-Ramond fields. And so this called the Ramond-Ramond fields.

AUDIENCE: C1 and C3?

HONG LIU: C1 and C3, that's right. They're all called Ramond-Ramond fields. Yeah.

So the story is essentially the same, very similar as to what we did for the bosonic string. For the bosonic string you only have x on one sheet, when you quantize x then you find the graviton, you find those B, you find the phi, and in the superstring you pull some additional fermions on the world-sheets, and those fermions give you additional modes, and then you will have those additional-- they will also give rise to additional massless modes in the spacetime, and those are this Ramond-Ramond fields. They are also massless fermions, which I will not write here. Yes?

AUDIENCE: So why is it that type IIA and type IIB string theory, they appear to be very, very different, which seems bad. It just seems like there's this arbitrary choice about how you go about doing the mathematics and you get this spectrum of particles or this spectrum. So who do you get around this problem?

HONG LIU: This is not the problem. This is a fact of life. This is just what you find. This is just what you find.

AUDIENCE: So which one is right?

HONG LIU: They both can be right, and they both can be wrong. And our goal is just to find all possible theories are there. And this just allow the quantizations. A lot consists in the quantizations. And both of them give you consistent quantum gravity.

And if I have time, if this is a string theory class, if this were a string theory class, then I would explain that actually secretly they are equivalent, secretly they are equivalent. So in some sense, they are not that different. Anyway, for your purpose, at the perturbative level, when we can see that the string coupling to be small, then they appear to be different, and yeah, we have two types of strings here.

And so let me mention a little bit about those forms. All of these are anti-symmetric potentials, so these anti-symmetric potentials are generalizations of the Maxwell fields, of the Maxwell field A mu.

So mathematically, you can write-- say for example, mathematically you can write the gauge field A mu as the so-called 1-form, so-called 1-form, and then A mu, it just coefficients of this 1-form. And then the field strings is just so the derivative, it's just exterior derivative of this 1-form. And then the Lagrangian will be just minus one quarter F mu mu, F mu mu construct from this procedure.

So you can just general-- mathematically straightforward to generalize this you have dimension of forms. So for example, we can consider n-form, which is the n-component tensor, but all indices are fully anti-symmetric with each other. So all the indices are fully anti-symmetric with each other. So these are fully anti-symmetric. If they depend on the conventions, sometimes we also put the one over n-factorial here.

So you can define such objects, such as the generalization of A, and the corresponding field strings for this C, we also called F. So the field strings for C is just a n plus 1-form, which is exterior derivative of dC. And then this is n plus 1-form. Again, this is a fully anti-symmetric tensor now with n plus 1 indices.

And then you can write down your Lagrangian similarly by generalizing these, you can write s1 over-- let me write it here-- you can write the Lagrangian s minus 1 over 1/2 times n factorial-- 2 times n factorial, not 2n factorial-- 2 times n factorial F mu1, mu n plus 1, F mu1 mu n plus 1. So F is n plus 1-form, so it's a tensor of n plus 1 indices, fully anti-symmetric fully anti-symmetric. So those fields essentially have this kind of structure, essentially have this kind of structure. And say the 1-form is very similar to our Maxwell. So this is a 3-form and this is a scalar, then just a massless scalar, then this is a 2-form whose field string will be a 3-form, and then this is a 4-form factor.

And then just by definition, just like here, there's a gauge symmetry because F equal to dA, here, because this is F equal to dC. So there's a gauge symmetry because I can add C n, the total derivative, of any n plus 1, n minus 1-form. So I can make a gauge symmetry, so lambda n minus 1 can be any n minus 1-form. And because of this square equal to zero, now F is invariant. So F n plus 1 is invariant. So if you put this into here, and because this is already a total derivative, and so F is invariant.

So there's a great symmetry, then there will be a conserved charge associated with those n-forms. You can define conserved charge associated with those n-forms. So so far any questions? Yes.

AUDIENCE: And maybe, so there's a C field that they denoted fermions of bosons.

HONG LIU: Bosons, it's all bosons.

AUDIENCE: But it looks-- so this anti-symmetric has nothing to do with fermions?

HONG LIU: No, this is just the indices. This just indices which are anti-symmetric. This is not the location, spacetime location. This is just indices.

It's the same thing, I think, F mu mu. F mu mu, the indices are anti-symmetric. It's the same thing here. Are the people comfortable with this differential form notation? Yes.

AUDIENCE: [INAUDIBLE]?

HONG LIU: Bosons, they're all bosons.

AUDIENCE: And where are the partners of those?

HONG LIU: Sorry?

AUDIENCE: Hb and phi, where are their partners?

HONG LIU: So far I did not write them down. So they are also fermionics. They are also massless fermions. We will not need to worry about them. So they are massless fermions in this.

AUDIENCE: [INAUDIBLE]?

HONG LIU: Right.

AUDIENCE: With superstrings.

HONG LIU: Yeah those just come from, because when you go to the superstring, you have additional fermions on the world-sheet. And then they can give you additional massless modes essentially come from them. The story, the real story is a little bit more complicated because actually in the superstring, even those come from the fermions, world-sheet fermions. Yeah but anyway vehicle you have at the formula for the world-sheet, then you have more possibilities then that give rise to those modes. That's the long story.

AUDIENCE: Those modes are also for bosons, not for fermions?

HONG LIU: No, these are all of the spacetime bosonic fields, which are low energy excitations of the strings. And how they arise on the world-sheet, whether they come from fermions or from bosons on the world-sheet, it's a separate issue. And, in fact, all of them actually arise from world-sheet fermions, but they are spacetime bosons. Yes.

AUDIENCE: [INAUDIBLE] of these Ramond-Ramond fields?

HONG LIU: Yeah, you just workout their repetitions under their say, Lorentz symmetry. They're all integer spin-- they're all integer spin-- they're all integer spin repetitions of the Lorentz group. And not like fermions, then will be half integer. They are generating repetitions of the rotational group.

AUDIENCE: So they are like spin 2, spin--

HONG LIU: No, normally we don't call them spin 2 or spin three. When we call spin 2, we mean a fully symmetric indices, symmetric and traceless.

AUDIENCE: But anti-symmetric, we would never call them--

HONG LIU: Yeah, we don't call them spin. Just the repetitions of the Lorentz group, it's-- it's just some repetitions of it's integer. Good?

And in the case of the Maxwell, we know that for the 1-form, for the case of the Maxwell, its source, where for A, its source is a point particle. So we can couple the-- so the point particle interact say with this extender vector potential in the following way. So suppose a particle following some trajectory described by x mu tau, and then this particle, we are intact with this vector potential in the following way. This is familiar from E and M. So you just integrate essentially with A along the trajectory of the particle.

And you can write this in the more compact mathematical form. You just say you integrate along the trajectory and A is a 1-form, and you just integrate this 1-form along the trajectory. And so this is so-called pull back of this 1-form A to C. So this is the mathematical language.

Similarly, we can generalize to these higher dimensional forms. So 1-form, naturally coupled to a point particle-- which can be considered as a zero-dimensional object. So a p plus 1-form, then naturally-- I should say is fermion. Then a p-dimensional object that naturally couples-- just as a generalization of this-- to a p plus 1-form.

As follows, you can just-- the worldvolume of a p-dimensional object is a surface of p plus 1 dimension. So the worldvolume of p-dimensional objects-- p plus 1 dimensions because there's also time. You also move in the-- it's a p-dimensional object moving the time, then in the p plus one dimension. So let me call that worldvolume sigma. And then the naturalization of this will be this, the coupling between a p-dimensional object to a p plus 1-form will be like this. Just you integrate this p plus 1-form on the worldvolume of this p-dimensional object.

And again, this is the pull back of C to sigma, C plus 1 to sigma. And let me just write it in a more explicit form. So here I use tau to parametrize the trajectory along the [INAUDIBLE]. So here, let me use psi to parametrize the worldvolume coordinate of this brane. Then this pull back means mu 1, mu p plus 1, partial x mu 1, partial psi 1, partial x mu p plus 1, partial psi p plus 1. And x mu psi one, psi zero to psi p describe the embedding of sigma in spacetime. Any questions on this?

So it tells you that-- so this just tells you that the

AUDIENCE: There is [INAUDIBLE] p plus 1 back there could say zero because of p because of notation.

HONG LIU: Let me just do zero.

AUDIENCE: Yes, so here the p-dimensional object in the simple case is just like the point particle is a zero-dimensional object. So that's what the object is.

HONG LIU: That's right. The zero-dimensional object naturally couple to 1-form in this way. And then a p-dimensional object naturally couple to a p plus 1 dimensional this way.

So let me just make, in the Maxwell case, the gate symmetry here implies an electric charge is conserved, OK, the electrical charge is conserved. So for a p-dimensional object coupled to a p plus 1 dimensional form, and because of this gate symmetry and then this charge is also conserved. In particular, the object with minimal charge must be a stable object because there's nothing to decay to.

So again, after generalization of this case, so in the standard story, in the standard electromagnetism, we can have an electrically-charged particle, we can also have a magnetically-charged particle. We can have a magnetically-charged particle. In particular, you can talk about the magnetic monopole in quantum mechanics and et cetera, even though we have not observed them. So similarly one can generalize that concept to higher dimensions.

So let me first just remind you of how we define the magmatic-- the mathematical way of how we define the magnetically-charged object EM. So EM-- so because F is a 2-form, we can dualize F-- so this is Hodge dual. We can define another form, which I call F tilde, which is related to original F by Hodge dual. So this is a 2-form, the Hodge dual of a 2-form in the four dimension is another 2-form. And then I can write it as the total derivative with another 1-form potential.

And then the object-- so you know that the under this dual, so F 0,1 is mapped to F tilde 2, 3. So the electric field is mapped to the magnetic field, and similarly, the magnetic field here is mapped to the electric field here, so under this Hodge dual, under this Hodge dual. I should not just write this. So after some minus sign we'll not keep track of, and under this Hodge dual, you essentially say the electric field now I call magnetic field.

So now object coupled to a tilde, then it's magnetically charged. Because in terms of A tilde, such objects they generate electric fields, and from our original F point there is a magnetic fields. So there will be a magnetically-charged object. So essentially that's how we think about the magnetic moment pole, EM.

So this can also be generalized to this n-form case. So one can also dualize an n-form. Is that we introduce use a lot of C tilde, which is the dual of dC. So dC is the field strength for C. So this is n-form. And the dC is the field strength of C, and then I do a Hodge dual, and then I define to be dC tilde. I define to be dC tilde.

And the dC tilde-- so this is n-form. When you do a field strength become n plus 1-form. And when you do the Hodge dual, when you take the Hodg-- dual, the Hodge dual of n-form is D minus n, OK? So this is D min n minus 2.

So this should be a D minus n minus 2-form because when you take the D, then that makes into D minus n minus 1-form, and this Hodge dual take this n plus 1-form into D minus n minus 1 form. So you map your n-form to a D minus n minus 2-form. So essentially, it's the integration of the electric-magnetic dual of E and M.

So now you can couple a D minus n minus 3-dimensional object to C tilde. And in terms of C, in terms of regional C-- so this is a magnetic object, OK? Just the same as we define magnetic object or magnetic monopole for E and M. So is the clear? Yes.

AUDIENCE: In order to write F tilde and A tilde, do we have to assume that there are no electric charges?

HONG LIU: That's a good point. No, you don't have to-- so you do this, you have to assume that the equation of motion is satisfied. You have to assume the equation of motion is satisfied, then you can write this procedure.

AUDIENCE: So if dF is equal to J, then that is non-zero. So if there are--

HONG LIU: So you have the exact the same situation with E and M. It's just identical situation with E and M. It's just whatever you do in E and M you do it here.

AUDIENCE: So I thought that we could only write F equals to dA because there were no magnetic monopole?

HONG LIU: No, so you can do that, now you have to introduce some similarity, so that's why the magnetic monopole have a Dirac string. That's why the magnetic monopole have a Dirac string, then you have this Dirac quantization condition, et cetera. So as you would then, say either in E and M, or in the quantum mechanics, say in the graduate quantum mechanics, or E and M.

So are people familiar with the concept of magnetic monopole and the Dirac-- so-called Dirac string and Dirac quantization condition? Yeah, I saw some nodding heads. So you're not familiar with it, then look at, for example, in one chapter of Jackson have a very detailed discussion of magnetic monopoles.

But the best, actually, was Dirac's original paper. It was really beautiful, very, very beautiful. But Jackson has a rather pedagogical discussion. Jackson's in actual dynamics, that's a rather--

AUDIENCE: Does he discuss the differential forms, though, or no?

HONG LIU: I don't remember. Oh, you mean Jackson?

AUDIENCE: Yeah.

HONG LIU: I don't remember. Yeah, it's the fact that you can allow the magnetic monopole then if you allow the certain singularities in the vector potential. And that's the so-called Dirac string.

So you don't have a thing to do. So the idea is that you-- what you were saying is right, the fact that you could write F as dA means there's no magmatic source, but you can introduce a magnet source if you allow certain singularities, and that's what Dirac realized. Good.

So now, this other gauge fields, in type IIA and type IIB string, and they can give rise to extended objects which couple to them. And it turns out the extended object cover to those Ramond-Ramond fields, are precisely D-branes. And they are precisely D-branes, and because they couple to those gauge fields, and they are stable objects. At least the minimally-charged ones are stable objects. So this is related to the statement that actually in superstring you actually find some stable branes.

And the other dimensions-- so let me just list them just to-- So with this preparation, then in IIA, so we can have the following electric and fully magnetic object. See in the IIA we have a C mu1. We have a 1-form. So that means there's a natural object coupled to it-- electric object coupled to it. Turns out this is a D0-brane. A zero-dimension object coupled to a 1-form, this is D0-brane.

And as a 1-form in 10 dimension-- so in superstring we have 10 dimension-- is dual to-- so D is 10 minus 1 is 9, minus 3 is 6, and a then the magnetic object will be a D6-brane. So similarly we have a 3-form, and then this gave an electric D2-brane, and a magmatic D4-brane. So if you just follow this rule.

So for the type IIB, so let me now first start with this C2. So this gives you a D-string one-dimensional object coupled to it for the D-string. And then the mathematical object will be a five-dimensional object. It's called a D5-brane. So this string is a D1-brane.

So now, let's look at this 4-form. So the so-called self-dual-- so sometimes we put a plus here to indicate this is a self-dual. So let me now explain what this self-dual means.

So self-dual means that from C4, as we construct an F5-- and construct an F5 from this C4-- to be a self-dual form means that this forms satisfies the condition that F5 equal to F5 dual. So this is a self-dual. Just means this is a constraint that this C has to satisfy. So this is called a self-dual 4-form. If you look at repetitions of the Lorentz symmetry, you'll find that this is actually a repetition. And so for this C, you have to satisfy this self-duality condition.

Then now, for object which coupled to the C, would be a naturally D3-brane because the three-dimensional object coupled to a 4-form, so it would be a D3-brane. And this D3-brane, because of this self-duality condition, we have to be both electrically and magnetically-charged because this is a self-dual form. So this must source both electric flux and the magnetic flux to be consistent with self-duality.

And then finally, you can also have a scalar field in the type IIB. And the scalar field, now eventually you will couple to so-called the D minus 1 dimensional brane. Because scalar field, by itself, is zero dimension. It's already zero dimensional form. So following this convention we will couple to a D minus 1 dimensional brane. And that obviously not make much sense, but actually can make sense when you go to Euclidean signature, and then it turns out this scalar fields couple to something called the instanton, which I will not go into there. Something called D-instanton.

So the bottom line is that the object, they are charged under [INAUDIBLE] fields, they are all D-branes. They are all D-branes, and they come into those dimensions. And any D-brane-- of course you can define D-branes with other dimensions.

So you can consider in IIA a D3-brane. But in IIA, a D3-brane will not be a stable object because there's not conserved charge for you to couple to. So even though in IIA you can consider a D3-brane, but it's not a stable object, and actually, when you quantized the spectrum on the D3-brane in IIA, then you find there's a tachyon. Again there's a tachyon in the worldvolume that indicates that's not a stable object. But those branes don't have a tachyon.

And in particular, on those branes, these are all supersymmetric field series. For example, the most important one is this D3-brane because then you have four dimensions of worldvolume, then you actually have four-dimension theory on this D3-brane, and that's what gives you so-called super-Yang-Mills theory. So on these branes, on these stable branes, all of the theories-- super-Yang-Mills theory, in particular for D3, it's given by a so-called n equal to four super-Yang-Mills theory in four dimensions.

AUDIENCE: So what you're saying is for any sign table, does it mean dissolving--

HONG LIU: Yeah, it can decay. It can decay.

AUDIENCE: But you lower dimension of brane?

HONG LIU: It's possible for it to decay into lower dimensional brane, and it can also for it just to decay into closed string modes, radiated closed string modes.

AUDIENCE: But, I mean the dimensions of these two objects are different. It sounds like if branes decay to a string, it will become infinite string.

HONG LIU: Yeah, that's right, ud type density of strings. It's actually a beautiful subject to discuss those branes, how they decay. Yeah, it's a nice subject, but it's way out of our discussion here. Any questions? Yes.

AUDIENCE: As to four, if we consider stacks of such branes, do we get higher-ranked age groups?

HONG LIU: Yeah, that's right. Exactly. It always give you n. Good, so that concludes our discussion of the D-branes from the point of view as the rich boundary conditions.

So now, I want to take a slightly different perspective on the D-branes. From this perspective, from the perspective the object, which are charged under those generalized gauge fields. And now I want to view those D-branes. I have some objects, some solitons, if you want some solitons, which are charged under those generalized gauge fields. I want elaborate from this perspective. I talk about the different perspective.

So D-brane has math, has tension. Also D-brane carry-- so now we only consider those stable D-branes. So they have a mass, they also carry those charges, conserved charges. So that means there will be flux coming out-- electrical or magnetic flux coming out of those branes.

So that will deform the spacetime because the gravity will deform spacetime. And so now, we'd like to find out what are the spacetime around those D-branes. How do they deform spacetime?

So for example, say it consider charge as a total example of this. Say, let's consider just again in the Maxwell theory, in Einstein-Maxwell theory, consider, for example, a charged particle sitting at the origin of a four-dimensional spacetime, Minkowski spacetime, for example. So if you include both GR and E and M, then the dynamics of this series controlled by so-called Einstein-Maxwell-- so you will have a Einstein theory and the Maxwell. So the dynamics of such particles should be captured by them. And so this captures the gravity due to the particle. This captures E and M due to the particle.

So from here, you can work out how does this particle deform spacetime from the Einstein equation? So the equation motion of this system would be, you have a standard Maxwell-- so I will be careful about the minus sign-- and also the Einstein equation, the stress tensor. So j is just the-- if I have a charged particle sitting there, there's no current, there's only a charged density. And the stress tensor contains two parts. It's the stress tensor due to the particle and also the stress due to the electromagnetic fields. Because this excited the electrogmagnetic fields.

And the particle part only have a zero component, so you only have a mass. There's no momentum. So this is just a-- maybe I should write r. I think I'm using the notation of r.

So the particle part of the stress tensor would be just the data function. The only non-zero component are zero-zero component. You only have energy density and just given by the data function of the mass. So from here, by solving those equations we see those forces then you can work out how a charged particle deforms your spacetime.

So let me just say I'm a bit further, for example you can easily solve the Einstein equation, we all know the solutions, so you just have a coulomb potential, and in other words you have electric flux surrounding the s2 around the point particle, you go to q. Again, this is the dual of F. You have electric field, when you dualize that hyperspatial component, then that gives you the-- so this a Gauss node, OK? This is a Gauss node.

And you can also work out the metric surrounding this charged particles. So because of the spacetime symmetric, spherical symmetric, then you can actually write down the spherical symmetric ansatz for the metric around the particle. So from the Einstein equation, you can find out what's-- you can work out what is f(r) and h(r). Some of you may have already done this exercise before.

So if your q is zero, what answer would you get? That's right. So if the q is zero, then you just get the Schwarzschild metric. And it's the exactly the metric say produced by a song, et cetera, far away. So here we have a charge, then what you get is something called the Reissner-Nordstrom metric. And when you have a charge having electromagnetic fields. So you will get so-called Reissner-Nordstrom metric.

AUDIENCE: So that's if q equals 0? If q equals 0?

HONG LIU: No, q larger equal to zero is the Reissner-Nordstrom. And when q equal to zero, then you just get the Schwarzschild That's how you calculate the precession. That's what you use to calculate the precession of, say, of the mercury.

Similarly, you can also do this for magnetically-charged particle. So the only difference for magnetically-charged particle, the difference is that this equation now becomes just directly magnetic flux equal to G. Say for example, if the magnetic charge is G, and then you will get this. Then instead you have this, and then you have yeah-- We have a magnetic flux. And again, you could plug into any equation you solve, you get the same Reissner-Nordstrom metric. It's also a Dirac quantization telling you that the qg should be quantized in integers of 2 pi n-- so n equal one, two, et cetera.

So this is the familiar story in the GR, so if you want to look at how the gravity and the E and M surrounding the charged objects, and so you can generalize this story immediately to those branes, which are charged-- which have their own mass and charged under some generalized gauge fields. So you can generalize that immediately. So you can generalize procedure immediately to higher-dimension objects.

So you can just, as a single exercise in GR, you can just carry out this procedure, just replace this to some higher-dimensional form, under the source is some higher-dimensional object, then you can work out the solutions. And that was a simple exercise which actually Horowitz and Strominger-Dietz, in the early '90s, in 1991, it's a trivial generalization you can do, why not do it? But turns out that's a very, very important step to do for the reason which I will explain.

So for simplicity, let me just talk a little bit about doing this for this D3-brane, also for definite entities, you could do it for any of this. And let me just say it for D3-brane. So now consider the D3-brane-- so D3-brane is part of the type IIB theory, so of course, we don't know to really do this in the full type IIB string, so we will do the low energy limit of the type IIB sting. So we will do this for the D3-brane. In the low energy limit of type IIB stream, which is so-called type IIB supergravity.

So it doesn't matter that you don't learn anything about type IIB. Supergravity, and essentially, that's just some generalization of this equation. Type IIB supergravity is some generalization of this equation.

But nevertheless let me just introduce some notations just to be-- So type IIB supergravity-- we will do this in the law of limit of the IIB string. So this is the law energy effective theory for massless modes of IIB string. So it has the form as what we had before. You have 1 over 6 pi, T pi GN. But not this is a 10-dimensional Newton constant, so this is a 10-dimensional theory equal to 10. And you have the Einstein term, and then you have forms associated with all of this flux, generalized gauge fields, et cetera, fermions, et cetera.

And as we said before, the relation with string theory, the Newton constant should be proportional to gs squared. So you can actually work out the prefactor precisely. So let me just write down the prefactor. So the relation with the string is-- just on the dimensional ground, so this is the Newton constant in 10 dimensions, so this would have dimension eight in terms of the lens have dimension eight. So the right-hand side I must be upper prime to the power 4 because this is only lens scale in string theory.

And then the prefactor, it turned out to be-- so you can work out the prefactor, so turns out 16 pi GN is equal to 2 pi to the power 7. Say, when you work out the law energy effective theory, you can work out this precise factor.

And the regime of the validity of this type IIB supergravity is that the gs is much, much smaller than 1. The string [INAUDIBLE] should be much, much smaller than 1. So that just tells-- this is a statement the string loop correction should be small because the when the string become big, the loop correction become more and more important. The loop correction would be more and more important from the point of view of gravity this quantum gravitational corrections. And so when G string become big, the quantum gravitational corrections-- so spacetime will fluctuate more. And then of course you cannot trust the classical gravity in that region.

So if you want to trust the classical gravity, you want to have small coupling, or translate into this language, the Newton constant has to be small. OK? The Newton constant has to be small. That means the gravity's weak. But in real life, gravity's weak, so it's OK.

And so this means the quantum correction is small, so you can trust the classical gravity. So you can trust classical gravity. And then we also want the energy-- no energy, so we want energy squared to be smaller then one over alpha prime. Because when energy squared becomes more than one over alpha prime, then the massless string modes become important because all of the massless string modes have mass square of one over alpha prime. And if we only want to concentrate on the massless modes, and then you don't want to excite them. And also you want the curvature of the spacetime-- here we're considering curve spacetime-- so you want the curvature to be much smaller than one over alpha prime.

So this gravity, essentially, when you consider the mass of this particle, you consider the particle limit of the spacetime. So this can be considered-- as you can see, the spacetime curvature-- the spacetime curvature radius is much much-- so alpha prime is essentially it gives us the rough lens scale of a string. And then when the curvature is much, much smaller than one over alpha prime, that means that the sight of the string is tiny compared to the curvature radius, the typical size of your system.

And that you can roughly treat them as point particles, and that's what the gravity theory does. In the supergravity theory it's all point particles. And so this means we decouple massless modes. So you can count on massless modes.

So whenever you take a limit, you always have to take the dimensionless number small. So the dimensionless number is alpha prime times curvature, of alpha times energy squared. But in reality, we often-- when we work with a theory, we often just keep our curvature, or our energy scale as what we want. And then in talking about this limit, we just say, we take alpha prime goes to zero limit. We say we take alpha prime goes to zero limit. So that means a law of energy limit. Even though, legitimately, this is not the right thing to say because this is the dimensional prime integral.

And so the regime where supergravity works, can be formulated, say, as alpha prime goes to zero and the g string goes to 0. So in this regime, now you can work with supergravity, and now you can now generalize this story to the branes. Now generalize this story for branes.

So now let me say a few things about the D-branes-- D3-branes. So they D-brane, D3-brane's charged under C 4 plus-- process and we said this C 4 plus is required to be self-dual. That means any source of C 4 plus must carry both electric and magnetic charge. So that means that for the D3-brane-- say if we define an electric charge to be-- So now, so the D3-brane is a three-dimensional object and the transverse direction is a six-dimensional because this is 3 plus 1 including time, so the transverse direction is six-dimensional.

So we can similarly just think about the D3-brane. So if we ignore the spatial direction along the brane, then the D3-brane, the transverse direction then can be just considered a point in R6, OK? So a way to think about the D3-brane, the surrounding of the D3-brane, it's an analog of a point in R6, in the six-dimensional space. Is this clear? This is going to be very important.

And then the surrounding this point, the sphere will S5. Just like in the electric case, just like the E and M case, this is the S2 surround the point particle, and this S5 is the sphere, which is surrounding the whole D3-brane. And then this D3 can have electric and magnetic flux through this S5.

And so electric charge would be just the 5-form running through it. And the magnetic charge would be just-- right. Minus the dual of 5-form is 1-form-- just make the relation between these two. And now because of self-duality condition, then they must be the same. So that means for the D3-brane, the electric and magnetic charge must be the same.

And also for higher-dimensional object--

AUDIENCE: Excuse me, why should they be the same? Why not--

HONG LIU: It's because of F5, you get F5 star. It's because of that. Because of that self-dual condition.

So electric and magnetic charge of the D3-brane must be the same. It must carry both electric and magnetic charge. And now, this Dirac quantization condition also works for higher-dimensional objects.

And again, it's a very simple exercise to do it-- very simple exercise to generalize Dirac's argument to higher dimension. Again, it's just related to how you treat those higher-dimensional forms, et cetera. Again, it's a very simple step, but it's a very important step.

So that tells mu that's for D3-brane, which is self-dual, which carry both electric and magnetic charge, then you have to satisfy this quantization condition. Then that means the minimal single D3-brane, the minimally-charged object, the charge must be square root of 2 pi. And for n of them, it must D3 equal to q3 equal to 2 pi n.

So now we have to specify the analog for E and M, those conditions. Now we also have to specify the mass of the D3-brane. So as we discussed before, tension over the D3-brane, tension over D-brane should be proportional to one over gs. That you also did in your P-sets.

And again, by doing a precise calculation, you can work out the precise prefactors. It turns out actually to be the following form. It's the q3 divided by 16 pi GN. And the GN is just gs squared, so if you translate into-- so for n D3-brane, that would be n divided by 2 pi cubed gs alpha prime squared.

So this gs and alpha prime squared we know on general grounds. So gs we'd-- one over gs we know on general ground, and alpha prime squared just from dimensional analysis. Because this is a three-dimensional object, the tension we'll have mass dimension 4, so that's why you have one over alpha prime squared here. And then the other 2 pi cubed from precise calculations. And you can also write it in this form in terms of Newton constants.

And the reason I write this form because this is a very special case. And you see that the tension is actually precisely related to the charge and divided by the square root of Newton's constant. And it turns out that only special objects have this kind of property. And this is so-called BPS objects. I will not go to there. It's related to those branes that are supersymmetric.

Anyway, so these are the mass over the-- essentially, this is the mass of the brane, and these are the charge of the brane. When you specify the charge, than those fields, then those flux that you uniquely determined, and then the C4 potential uniquely determined, up to gauge symmetry are determined. And then now you can plug into the generalization of the Einstein equation to work out the geometry.

So again, the symmetry here-- I won't have time for the bottom line, so that's OK. So the symmetry preserved by the D-brane, by D3-brane as we said before, similar to all D-branes is you have a Poincare symmetry along the brane direction, and then you have rotational symmetry in the transverse direction along the brane. So the transverse we have six dimensions. So there's SO(6) directions you can rotate in the-- and then you have Poincare symmetry in the (1,3) direction.

So based on these symmetries you can write down what the metric looks like. You can write down answers for the metric. Just like right here, you can write down the answers for the metric of a point particle based on symmetry. So here, if you can write down the answers for the metric based on symmetry.

So first, you should have a part which is preserved Poincare symmetry. Means that this part can only be this form. Say along the brane direction, can only be this form times some other factor which depends on r, so r is the transverse radius from the brane.

And then, the other dimension must be spherical symmetric, so you can have a lot of factors. And then the r squared d omega 5 squared. So this is s5 surrounding the brane, and then this is the radial direction. So this is the only form of the metric.

And then essentially, again, as in the charged particle case, you need to determine these two functions. Which you can plug in the Einstein equation to determine these two functions. So let me just write down the form of those functions, then we discuss what they mean next time. Some how, this year I'm consistently much, much slower than what I did before. Maybe I'm explaining physics better.

Anyway, so now it's just a mechanical exercise. You plug in these answers into the Einstein equation, then you can find the two equations for f and h, then you can solve it. In the old days, when Horowitz and Strominger did it-- maybe they did not have Mathematica yet, but nowadays you can do it. Using Mathematica, if you have the right program, you can do it in five minutes. But in the old days-- they were friends with Wolfram, maybe they already have the program at that time.

Anyway, so it was a [INAUDIBLE] exercise to do at the time. So you find actually the answer is very simple. Turns out, both f and h, they just-- can be written in the following form. H is just a harmonic function. And R to the power 4 is given by some constant GN, Newton's constant times the D3-brane tension times n. And it can also be written as 4 pi gs N alpha prime squared.

So turns out that those functions are very simple. They just square root of a harmonic function. So this is a harmonic function for the transverse R6. So if you solve a coulomb problem, and that will be the solution. And this is the harmonic-- so this is the generalization of 1 over R potential. So this is just generalization of 1 over R potential. And then f and g is very simply related to this by square root.

And this constant R to the power 4, so this is not curvature, this is just some constant. I'm using the standard notation. And then this can be-- this is just related to the mass of the brane. Mass of the brane, of course, is proportional to n. Yeah.

AUDIENCE: In this case, is the mass, the charge is it massless?

HONG LIU: Sorry?

AUDIENCE: The charge is massless?

HONG LIU: Sorry?

AUDIENCE: Is the charge considered on the D3-brane brane massless or massive?

HONG LIU: No. Charge are just charge. What do you mean by charge is massless? This is just the charge carried by this D3-brane.

Anyway, so this R4 is just like GN times the tension, N have the tension. We have n objects. So this is what you're familiar with the Schwarzschild black hole. Schwarzschild metric just using constant times mass, so this is just generalization of that. The Newton constant times the tension, and you have n-branes, we just multiply n. And you write it-- if you write everything from gs, this is a proponent to gs squared, this is a proponent to one over gs, so and that, the thing's proportionate to gs. OK. OK we'll stop here.