

Chapter 2: Deriving AdS/CFT

MIT OpenCourseWare Lecture Notes

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Lecture 12

Important equations for this lecture from the previous ones:

1. Closed string mass-shell condition, equation (23) and (25) in lecture 11:

$$M^2 = \frac{2}{\alpha'} \sum_{i=2}^{D-1} \sum_{n \neq 0} n(N_n^i + \tilde{N}_n^i) - \frac{D-2}{6\alpha'} . \quad (1)$$

2. Open string mass-shell condition, equation (24) and (25) in lecture 11:

$$M^2 = \frac{1}{\alpha'} \sum_{i=2}^{D-1} \sum_{n \neq 0} nN_n^i - \frac{D-2}{6\alpha'} . \quad (2)$$

2.1.2: LIGHT-CONE QUANTIZATION (cont.)

The string-spectrum can be read-off, as each state of a string corresponds to a spacetime particle state.

Let's start with the particles content of open strings:

1. The oscillation vacuum state is:

$$|0, p^\mu\rangle , \quad N_m^i = 0 \quad \forall m, i . \quad (3)$$

The spacetime transformation of this state indicates that it should be a spacetime scalar of mass:

$$M^2 = -\frac{D-2}{24\alpha'} . \quad (4)$$

This particle is a tachyon when $D > 2$ as $M^2 < 0$.

2. The oscillation 1st excited state transforms as a $SO(D-2)$ vector under spacetime rotation:

$$\alpha_{-1}^i |0, p^\mu\rangle , \quad M^2 = \frac{26-D}{24\alpha'} . \quad (5)$$

In the light-cone gauge, while only the subgroup $SO(D-2)$ of Lorentz symmetries $SO(1, D-1)$ are manifest, all string excitations (as they are viewed as spacetime particles) must fall into the representations of the full Lorentz group. Recall that Lorentz vector particles in D spacetime dimension should have $D-1$ independent components (massive) as the irreducible representation is $SO(D-1)$ and $D-2$ independent components (massless) as the irreducible representation is $SO(D-2) \supset SO(D-1)$. Since the 1st excited state is a vector with only $D-2$ independent

component, for the consistency of Lorentz symmetries or the quantization procedure, then it should be massless:

$$M^2 = 0 \Rightarrow D = 26 . \quad (6)$$

This results for spacetime dimension is known as the critical dimension of (bosonic) string theory.

The coherent state created by many of this particle in spacetime gives the field configuration of a massless vector field A_μ , later play the roles of a gauge boson (like photon and gluon).

3. Higher excitations are all massive giving by multiplets of mass-squared spacing $\frac{1}{\alpha'}$. For example, the particle description for the 2nd excited string state (massive):

$$\alpha_{-1}^i \alpha_{-1}^j |0, p^\mu\rangle , \quad \alpha_{-2}^i |0, p^\mu\rangle , \quad M^2 = \frac{1}{\alpha'} . \quad (7)$$

This fits with the description of a massive spin 2 particles in $D = 26$ Lorentz irreducible group, form a complete multiplets of $SO(25)$ as a symmetric traceless tensor. The same consistency holds for higher spin (mass) particles.

The particles content of closed strings can also be read-off similarly:

1. The oscillation vacuum state is:

$$|0, p^\mu\rangle , \quad N_m^i = \tilde{N}_m^i = 0 \quad \forall m, i \quad (8)$$

The spacetime transformation of this state indicates that it should be a spacetime scalar of mass:

$$M^2 = -\frac{D-2}{6\alpha'} . \quad (9)$$

Similar to open string, this particle is a tachyon when $D > 2$ as $M^2 < 0$.

2. The oscillation 1st excited state is (using the level matching condition):

$$\alpha_{-1}^i \tilde{\alpha}_{-1}^j |0, p^\mu\rangle , \quad M^2 = \frac{26-D}{6\alpha'} . \quad (10)$$

Again, only for $D = 26$, this state fall into the irreducible representations $SO(D-2)$ of Lorentz group consistently (as massless $M^2 = 0$ spin 2 particles) with further decomposition to scalar (trace), symmetric traceless and anti-symmetric:

$$\sum_i^{24} \alpha_{-1}^i \tilde{\alpha}_{-1}^i |0, p^\mu\rangle , \quad \alpha_{-1}^i \tilde{\alpha}_{-1}^j |0, p^\mu\rangle , \quad \alpha_{-1}^i \tilde{\alpha}_{-1}^j |0, p^\mu\rangle . \quad (11)$$

The coherent state given from many of these particles in spacetime are the field configuration of a massless scalar field Φ (dilaton), a traceless symmetric tensor field $G_{\mu\nu}$ (graviton) and a anti-symmetric tensor field $B_{\mu\nu}$ (Kalb-Ramon, or B-field), associated with gauge symmetries.

3. Higher excitations are all massive giving by multiplets of mass-squared spacing $\frac{4}{\alpha'}$. For example, the particle description for the 2nd excited string state (massive):

$$\alpha_{-1}^i \alpha_{-1}^j \tilde{\alpha}_{-1}^k \tilde{\alpha}_{-1}^l |0, p^\mu\rangle , \quad \alpha_{-2}^i \tilde{\alpha}_{-1}^k \tilde{\alpha}_{-1}^l |0, p^\mu\rangle , \quad \alpha_{-1}^i \alpha_{-1}^j \tilde{\alpha}_{-2}^k |0, p^\mu\rangle , \quad M^2 = \frac{4}{\alpha'} . \quad (12)$$

This fits with the description of a massive spin 4 particles in $D = 26$ Lorentz irreducible group, form a complete multiplets of $SO(25)$. The same consistency holds for higher spin (mass) particles.

At low energy $E \ll \frac{1}{\alpha'}$, the dynamics of A_μ and $G_{\mu\nu}$ should be governed by Maxwell and Einstein theory. This is confirmed by explicit calculations of scattering amplitudes of these particles in string theory ($g_s \ll 1$) in a nontrivial spacetime (A_μ and $G_{\mu\nu}$ background only). Also the interactions is dominated by massless fields exchange, as the massive modes are decoupled from the low energy perspective. For closed strings, the effective field theory after integrating out the massive degrees of freedom (and also excluding tachyon) and matching the interactions (for tree level only), at leading the order:

$$S_{eff} = -\frac{1}{2\kappa_0^2} \int d^{26}x \sqrt{-G} e^{-2\Phi} \left(R - 4G_{\mu\nu} \partial^\mu \Phi \partial^\nu \Phi + \frac{1}{12} H_{\mu\nu\lambda} H^{\mu\nu\lambda} \right) , \quad H_{\mu\nu\lambda} = \partial_\mu B_{\nu\lambda} + \partial_\nu B_{\lambda\mu} + \partial_\lambda B_{\mu\nu} . \quad (13)$$

This action can also be found starting with the Polyakov stringy action with nontrivial background (of massless fields, the graviton, dilaton and Kalb-Ramon) and calculating correlation on a spherical worldsheet with insertions (the QFT interpretation of this stringy topology is the sum of all tree-level Feynman diagrams contribution) and let the running (β -function vanish) which is expected as the worldsheet theory of string should be conformal. One then arrives at a set of equation of motion, which can be derived from the above form of S_{eff} .

For different purposes, sometime the Einstein metric is used instead of string metric get the Einstein-Hilbert form:

$$\tilde{G}_{\mu\nu} = e^{\frac{\Phi}{6}} G_{\mu\nu} \rightarrow S_{eff} \supset -\frac{1}{2\kappa_0^2} \int d^{26}x \sqrt{-\tilde{G}} \tilde{R} . \quad (14)$$

The average value of the dilaton field is related to the gravitational constant:

$$\langle \Phi \rangle = \Phi_0 , \quad 2\pi\alpha' e^{\Phi_0} = \kappa_0 , \quad g_s = e^{\Phi_0} . \quad (15)$$

This also indicates that, although string theory has no dimensionless coupling in the theory, but different vacua can have different effective string couplings g_s .

The QFT treatment for quantum general relativity at loop-level is badly divergent (nonrenormalizable), hence does not describe a consistent UV physics. String loop diagram, however, is expected to be UV finite (from the topology nature of the perturbative diagrams), and therefore one can hope that at nonperturbative string theory is a consistent theory of quantum gravity.

Let's take a quick look at perturbative superstring theory, to be precise, type II superstring theories. After fixing the worldsheet metric $\gamma_{ab} = \eta_{ab}$, the superstring action becomes (Ψ is the worldsheet fermion and γ^a is the γ -matrices):

$$S = -\frac{1}{4\pi\alpha'} \int d^2\sigma \left(\partial_a X^\mu \partial^a X_\mu + i\bar{\Psi} \gamma^a \partial_a \Psi \right) . \quad (16)$$

There are different quantizations exist (with no tachyon), and IIA and IIB are the name of perturbative superstring theories of future interests. Both of them require the critical dimension to be $D = 10$.

The massless closed superstring (bosonic) fields in spacetime are:

1. For IIA superstring:

$$\Phi , \quad G_{\mu\nu} , \quad B_{\mu\nu} , \quad A_\mu , \quad C_{\mu\nu\lambda}^{(3)} . \quad (17)$$

A_μ and $C_{\mu\nu\lambda}^{(3)}$ are the RR fields.

2. For IIB superstring:

$$\Phi , \quad G_{\mu\nu} , \quad B_{\mu\nu} , \quad \chi , \quad C_{\mu\nu}^{(2)} , \quad C_{\mu\nu\lambda\sigma}^{(4)} . \quad (18)$$

χ , $C_{\mu\nu}^{(2)}$ and $C_{\mu\nu\lambda\sigma}^{(4)}$ are the RR fields.

The low energy effective field theories of these superstring theories are IIA and IIB supergravity.

By studying the consistencies of stringy interaction, 5 different superstring theories were found in 10 dimensions (the critical dimension, as all the fields' configuration the theories perturb around is trivial; this number of dimensionality can be changed if the fields' configuration is nontrivial and one gets noncritical string theory with less than 10 dimensions). The low energy effective description of these superstring theories perfectly match with the only 5 known anomaly-free SUGRA theories up to date, which originates from the only 11D SUGRA. Along with the discoveries of nonperturbative structures in superstring theories, all these theories can be related by a network of dualities and compactifications, and the relation underlying them all is M-theory. Up to date, M-theory is the only candidate for the theory of everything. There are attempts to formulate M-theory in a certain background configurations, for example, M(atric) theory.

Although perturbative stringy is not the focus of this course, it should be mentioned as we move on as one reason why string theory is so desirable as a similar perturbative framework with QFT (integration of all configuration) that consistently includes gravity (contains a massless spin-2 particles in the theory, which always result in general relativity as a low energy effective description): finiteness at all perturbative orders, as the result of CFT on worldsheet. It should be noted that the summation of all perturbation orders doesn't need to be convergent, and the nonperturbative results (up to some ambiguities) can be guessed from the analytical continuation trick. This divergence is actually seen as a feature of perturbative QFT, since all calculations in QFT give asymptotic series of orders in perturbations (loop-orders). This can be seen schematically as the number of Feynman diagrams is increasing factorially as the number of loops going up, so, after the loop-order $\sim 1/g$ (g is the QFT's coupling) the contribution at higher loop orders start to diverge.

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