8.821/8.871 Holographic duality

MIT OpenCourseWare Lecture Notes

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Lecture 9

Reminder from last lecture

Recall that each Feynman diagram can be considered as a partition of a genus-h surface. The scattering amplitude of n particles on genus-h surface can be written as

$$f_n^{(h)} = \sum_{\text{all Feynman diagrams of genus h}} G = \sum_{\text{all possible triangulations of genus h surface}} G$$

Here G represents the expression for each diagram. Similarly in string theory, we have n-string scattering process

$$F_n^{(h)} = \int_{\text{genus h surfaces with n boundaries}} DXe^{-S_{string}} = \sum_{\text{all possible triangulations of genus-h surfaces with n boundaries}} e^{-S_{string}}$$

If we could identify G with some $e^{-S_{string}}$, we will then have:

$$\begin{aligned}
\text{a large N gauge theory} &= \text{a string theory} \\
\frac{1}{N}\text{ expansion} &= \text{perturbative expansion in } g_s \\
\text{large N limit (classical theory of glue balls)} &= \text{classical string theory} \\
\text{single-trace operators (glue balls)} &= \text{string states}
\end{aligned}$$

In fact this identification is difficult

1. G is expressed as products of field theory propagators integrated over spacetime, there is no obvious connection to $e^{-S_{string}}$. Note that the action S_{string} gives a map from the world sheet Σ to the target space $\mathcal{M}$ (spacetime manifold)

   $$(\sigma, \tau) \to X^\mu(\sigma, \tau)$$

   In such a map, we can make choices of spacetime manifold $\mathcal{M}$, the specific forms of the action S_{string}, we can also have "internal" d. o. f. living on the world sheet with no immediate spacetime. For example, it can be superstrings, including fermions on the worldsheet.

2. String theory is formulated in the continuum, while the Feynman diagrams at best has a discrete version (triangulation of the manifold). Even if there exists a connection, we expect that the geometric picture of G to emerge only in the strong coupling limit, *i.e.* when the Feynman diagrams with many (infinite) vertices dominate.

3. For simple theories, like matrix integrals or matrix quantum mechanics, one could go pretty far in relating them to some low-dimensional string theory, see *e.g.* Sec. II in Ref. [1]. But in general it is not possible for higher dimensions.

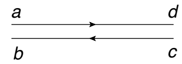
Generalizations:

1. We have so far been restricted to matrix-valued fields, *i.e.* fields in the adjoint representation of $U(N)$ gauge group. One could also include fields in the fundamental representation (quarks)

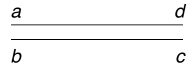
   $$q = \begin{pmatrix} q_1 \\ \vdots \\ q_n \end{pmatrix}$$

   e.g. , vacuum diagrams now include loops of quarks, which can be classified topologically by 2d surfaces with boundaries, then it corresponds to a string theory with both open and closed strings.

2. So far we considered $U(N)$ gauge group,

$$\langle \Phi^a{}_b(x) \Phi^c{}_d(y) \rangle =$$ 

   If instead, we consider $SO(N)$ or $SP(N)$, then there is no difference between the two indices of the fields

$$\langle \Phi_{ab} \Phi_{cd} \rangle =$$ 

   The corresponding Feynman diagrams will live on non-orientable string theories, and it corresponds to non-orientable string theories.

Now take *e.g.* large N generalization of QCD in (3+1)d Minkowski spacetime. Suppose $\frac{1}{N}$ expansion can be described by a string theory, what can we say about it?

The simplest guess would be a string theory in (3+1)d Minkowski space

$$ds^2 = -dt^2 + d\vec{x}^2 = \eta_{\mu\nu} dX^\mu dX^\nu$$

We can consider Nambu-Goto action

$$S_{NG} = \frac{1}{2\pi\alpha'} \int_\Sigma dA$$

or the Polyakov action which is equivalent to S_{NG} classically. But this does not work:

1. Such a string theory is inconsistent for $D \neq 10, 26$, where D is the spacetime dimension.
2. Take a string theory in 10d with $\mathcal{M}_4 \times \mathcal{N}$, where $\mathcal{N}$ is some compact manifold. Such a theory contains a massless spin-2 particle (graviton) in $\mathcal{M}_4$, which is not present in Yang-Mills theory.

To solve the problem, we can either think about more exotic string actions or consider other target space. Actually there are hints for considering a 5d string theory:

1. Holographic principle
   String theory necessarily contains gravity, to be consistent with holographic principle, such a gravity theory should be in 5d.
2. The consistency of string theory itself
   It needs to include a Liouville mode which behaves as on extra dimensions.
3. Geometrization of renormalization group flows (pure hindsight).

Now consider a string Y in 5d spacetime. It should at least have all the symmetries of 4d YM theories, *e.g.* translations, Lorentz symmetries etc. *i.e.* consider

$$ds^2 = a^2(z) \left[dz^2 + \eta_{\mu\nu} dX^\mu dX^\nu \right] \tag{1}$$

which is the most general metric consistent with 4d Poincare symmetries. For a general gauge theory, not more can be said. But if a theory is conformal, or simply scale invariant, Eq. 1 should be the AdS metric. This is simple to see. If Eq. 1 is invariant under scaling transformation

$$X^\mu \to \lambda X^\mu$$

Then we must have $z \to \lambda z$ and $a(\lambda z) = \frac{1}{\lambda} a(z)$, which means $a(z) = \frac{R}{z}$ with R constant.

At last, to close this chapter, we make a list of the history, of the discovery of the holographic duality.

1974 (continued)	lattice QCD (Wilson), confining strings
1993-1994	holographic principle (t' Hooft, Susskind)
1995	D-branes (Polchinski)
1997 June	need 5d string theory to describe QCD (Polyakov)
1997 Nov	AdS/CFT(Maldacena)
1998 Feb	connection between holographic principle and large N gauge theory/string theory duality (Witten)

References

[1] Igor R. Klebanov, arXiv:hep-th/9108019