

Effective Field Theory (8.851) Spring 2013

Homework 1

Problem 1) Matching with Massive Electrons

Consider QED with electrons and photons. For photon momenta q^μ much less than m_e we can integrate out the electrons.

a) Calculate the QED one-loop photon vacuum polarization diagram with dimensional regularization and $\overline{\text{MS}}$, and expand $\Pi(q^2)$ in q^2/m_e^2 .

[Note: The first term in the expansion motivates matching onto a theory without electrons at a scale $\mu \sim m_e$ rather than $\mu \sim 1 \text{ TeV}$ so that a large logarithm does not influence the perturbative expansion in α .]

b) Write down a Lagrangian with a gauge invariant photon operator that reproduces the second term in the expansion. Use your calculation from part a) to determine the Wilson coefficient of the operator at this order in α .

c) What QED symmetry(s) forbid dimension-6 operators with three field strengths from ever appearing?

d) At dimension-8, operators are generated which give light-by-light scattering. Determine the number of α 's in their coefficients. Then use dimensional analysis in the low energy effective theory to numerically estimate the size of the $\gamma\gamma \rightarrow \gamma\gamma$ cross section for 5 keV photons.

Problem 2) Right Handed Neutrinos and Proton Decay

a) Consider adding a right-handed singlet neutrino N_R to the standard model. A Majorana mass M is allowed, and the most general set of gauge invariant operators with dimension ≤ 4 are

$$\mathcal{L}_N = \bar{N}_R i \not{\partial} N_R - \frac{M}{2} \bar{N}_R^c N_R - \frac{M}{2} \bar{N}_R N_R^c + (g_\nu \bar{N}_R H_i^T L_{Lj} \epsilon^{ij} + h.c.), \quad (1)$$

where $N_R^c = C(\bar{N}_R)^T$ is the charge conjugate field, $C = i\gamma_2\gamma_0$ (in the Dirac representation), H is the higgs doublet, L_L is the left-handed lepton doublet with ν_L and e_L , and $h.c.$ denotes the hermitian conjugate. Take the mass M to be large compared to the electroweak scale and integrate out the right handed neutrinos at tree level. Show that the leading term reduces to the form of the dimension-5 standard model operator we discussed in class.

b) Construct a dimension-6 operator built out of standard model fields that is Lorentz and gauge invariant, and allows the proton to decay because it violates baryon number.

Problem 3) Electric Dipole Moments

Electric dipole moments violate CP symmetry. Lets explore where this could come from.

- a) Write down an operator that couples two up-quarks and a photon which can contribute to the neutron electric dipole moment, d_n . Check that your operator violates CP and is gauge invariant under the U(1) for electric charge.
- b) With the full standard model gauge group it is not possible to write down a dimension five operator analogous to the one you have in a) without breaking the gauge symmetry. Explain why. There is a dimension-6 operator that contributes to the neutron electric dipole moment. What is it? (Feel free to cite literature to help answer this question.)

Problem 4) Field Redefinitions

Consider the following Lagrangian for a scalar field

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4 + \frac{\eta g_1}{6!}\phi^6 + \frac{\eta g_2}{3!}\phi^3\partial^2\phi \quad (2)$$

where $\eta \ll 1$. In lecture we considered the same Lagrangian, but with different numerical prefactors. We then made a field redefinition $\phi \rightarrow \phi'$ which eliminated the last term, giving a new Lagrangian \mathcal{L}' .

- a) Consider this field redefinition suitably modified for the numerical factors in Eq. (2). Show at $\mathcal{O}(\eta)$ that the field redefinition gives the same form for \mathcal{L}' , as applying the $\eta = 0$ equations of motion to the last term. Determine the new λ' and g_1' parameters in \mathcal{L}' in terms of the parameters in the original Lagrangian \mathcal{L} .
- b) Demonstrate the equivalence of the 4-point and 6-point functions to $\mathcal{O}(\eta)$ by explicit computation of the tree level Feynman graphs with \mathcal{L} and \mathcal{L}' . Use your result from a).
- c) Draw the diagrams you would need to compute to demonstrate the equivalence of the 4-point functions at one-loop and $\mathcal{O}(g_2\eta)$. Sketch how the equivalence would work by taking zero 3-momenta on the external lines and working out results for the graphs, up to the overall symmetry factors. That is, you are free to determine how the correspondence works out by simplifying loop integrals, and can introduce symmetry prefactors “ S_i ” for the various diagrams that you do not have to work out explicitly.
- d) With the original field redefinition determine the terms induced at $\mathcal{O}(\eta^2)$ in \mathcal{L}' , and then show that a second field redefinition can be used to again eliminate any terms proportional to the equations of motion.

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