

# Effective Field Theory (8.851) Spring 2013

## Homework 4

NOTE: Do either Problem 1 OR Problem 2. Then do both Problems 3 and 4.

### Problem 1) One-loop Exact $\beta$ -function in $NN$ EFT

In lecture we computed the one-loop diagram with two  $C_0(\mu)$  insertions in the offshell momentum subtraction scheme (OS) and the PDS scheme and determined the one-loop counterterm  $\delta^1 C_0 \propto \mu C_0(\mu)^2$ . From this we computed the beta function by differentiating the explicit  $\mu$ , but not the implicit  $\mu$  in  $C_0$ . To prove this is correct requires computing the counterterms for the entire bubble sum chain of  $C_0$  insertions. Lets check that this statement is consistent at two loops: Compute the two-loop counterterm  $\delta^2 C_0$  in either the OS or PDS scheme and consider how it effects the  $\beta$ -function. Note that the two-loop graph is not hard, its just a product of one-loop diagrams.

### Problem 2) Counting $NN$ operators with Group Theory

The nucleon has two degrees of freedom for spin and two for isospin, so lets think of it as a fundamental in  $SU(4)$ . Our EFT might not be  $SU(4)$  symmetric, but we can still classify all operators by their  $SU(4)$  transformation properties. The nucleon is a fermion so we need antisymmetric products of fields in our operators which restricts the allowed representations. Combine the representations for two  $N$ 's and then for two  $N^\dagger$ 's and count the number of operators (with no derivatives) from counting the representations. (Hint: of these the 15 is actually ruled out by noting that it has no spin-singlet  $SU(2)$  subgroup.) Repeat for three body operators (three  $N$ 's, and three  $N^\dagger$ 's) and then for four body operators (four  $N$ 's, and four  $N^\dagger$ 's). [Hint: the last one is the easiest.]

### Problem 3) SCET Operators with Collinear Quarks

Start with the QCD Lagrangian for a massive quark and decompose  $\not{D}$  in terms of  $n$ ,  $\bar{n}$ , and  $\perp$  components. As in lecture, write  $\psi = \xi_n + \zeta_{\bar{n}}$  where  $\not{n}\xi_n = 0$  and  $\not{\bar{n}}\zeta_{\bar{n}} = 0$  and determine which products of fields are non-zero. Keeping all the non-zero terms, integrate out the field  $\zeta_{\bar{n}}$  to generate an effective action for the massive collinear quark  $\xi_n$ .

[With power counting  $m \sim p_\perp \sim Q\lambda \ll Q$  this is the starting point to derive the action for a massive collinear quark, ie. prior to decomposing the gluon field into collinear and ultra-soft pieces and prior to distinguishing between large and small momenta. The remaining steps are the same as those discussed in lecture except that you keep the mass. The mass

terms that you have derived are important for considering how a collinear Lagrangian of light quarks  $u, d, s$  explicitly breaks chiral symmetry. They are also relevant for discussing an energetic jet initiated by a massive quark, when the jet energy  $Q \gg m$ .]

#### Problem 4) SCET Collinear Wilson Line

To get more familiar with Wilson lines lets consider the current for a  $b \rightarrow u$  transition. In QCD  $J = \bar{u}\Gamma b$ . For SCET we did a matching calculation to find the leading order current

$$J^{(0)} = \bar{\xi}_n W \Gamma h_v, \quad (1)$$

where  $W$  included terms involving the order  $\lambda^0$  collinear gluon field  $\bar{n} \cdot A_n$ . In lecture we explicitly computed the term in  $W$  with one  $\bar{n} \cdot A_n$  field and wrote down the result for any number of  $\bar{n} \cdot A_n$  fields. Do the matching computation for two  $\bar{n} \cdot A_n$  fields (by expanding QCD diagrams with offshell propagators). Verify that the result for one and two  $\bar{n} \cdot A_n$  fields agree with the momentum space Feynman rules derived from the position space Wilson line

$$W(y^+) = P \exp \left( ig \int_{-\infty}^0 ds \bar{n} \cdot A_n(s\bar{n} + y^+) \right),$$

where  $P$  is path-ordering.

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